Study on Reliability Model of Multistate Phased Mission System Based on BDD

Tao Hu and Chun Hui Yang

***Abstract*—In engineering application, there are many systems whose mission often involves several different tasks or phases that must be accomplished in sequence. At the same time, some systems are composed of multistate components, which have different performance levels where one cannot formulate an "all or nothing" type of failure criterion. At first, the Multistate Fault Tree (MFT) is introduced to describe the multistate systems, then the method is put forward which can transform the MFT into Binary Decision Diagram (BDD). Secondly, a new algorithm is presented which combines the characters of multistate and multiphase and can obtain a single model of all phases. Finally, the case study shows that the method is available and can reduce the complexity of the PMS model.**

***Index Terms*—Phased mission system, binary decision diagram, multistate fault tree.**

I. Introduction

The traditional reliability theory depends on binary logic to describe whether a product can perform the specified functions, so the impacts of partial failure of component or system on system performance are overlooked. Therefore, the reliability analysis model built on this basis is often much different from actual condition, and unable to satisfy the analysis of increasingly complex system [1]. In multiple engineering fields, there are multiple states (different failure levels) between ideal work state and complete failure state of system and its components. Thus, the division of system and its components into binary states, that is, success and failure, is too simple, and may even lead to fatal error.

Fault tree is constructed based on the binary states of event, that is, success and failure. Hence, fault tree method faces a lot of difficulties in studying and evaluating multistate systems.

Phased mission system (PMS) consists of multiple different phases which are discontinued and not overlapped in terms of time [2]. Due to the limitations in modeling complexity, description capability and processing method, etc., all the PMS methods based on mathematic model simplify a problem under certain assumptions, so as to simplify a model and realize the deduction and computation of the model [3]-[5]. Among them, each unit in the system has two states. Thus, most of methods must satisfy the assumption of two states, that is, success and failure.

In this paper, multistate fault tree is employed to describe multiple states of a system, and convert multistate fault tree into the corresponding BDD, so as to obtain the minimum cut-set and compute reliability and security. On the basis of phased generation, a new algorithm for one-time generation of phased task system BDD is put forward, so as to effectively reduce the analytic complexity of phased mission system.

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II. Multistate Fault Tree and Its BDD Algorithm

Similar to fault tree, multistate fault tree [6] (MFT) stands for all the combinations of states for each component. All these states lead to a specific state of system in the tree structure. The top event S_i in the tree means that the system is in the state of s_i. Top event S_i can generate a set of events caused through logic gates. Except top event S_i, all other events can be defined as follows:

1) Bottom event: It means that a component is in a specific state.
2) Intermediate event: It can connect the combinations of other intermediate event and initial event through logic gates.

Each event can be represented by the logic value 1 at the node. Otherwise, the logic value at the node is 0.

Each gate has several inputs and one output. The input of a gate may be initial event, or the output of another gate. If and only if all the inputs of a "AND" gate are the logic value 1, the output is logic value 1. If and only if one or several inputs of an "OR" gate is/are the logic value 1, the output is the logic value 1. If and only if the *k* or several inputs in a gate of "*k* out of *n*" is/are the logic value 1, the output is the logic value 1.

As a component may be only in the same state at any time, the state variables of the same component are not independent from each other. Apart from the relation of Boolean algebra among these variables, there are also three other relations to deal with the dependency among variables.

1) The component CA cannot be in two states at the same time:

$$A_i \cap A_j = 0 \quad i \neq j \tag{1}$$

2) The component CA must be in any of its states:

$$\bigcup_{i=1}^{n} A_i = 1 \tag{2}$$

3) A supplemental rule is deduced from the above two rules:

$$\overline{A_i} = \bigcup_{k=1, k \neq i}^{n} A_k \tag{3}$$

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It must be noticed that these constraints are only suitable to represent Boolean variables of the same component in different states.

Like general fault tree, multistate fault tree is converted into BDD in the similar way. When recursive method is employed in conversion, the rules on converting general fault tree into BDD is improved based on the constraints of different states on the same component. Thus, the rules on converting multistate fault tree of system into BDD are as follows:

$E_1=ite(X, (E_1)_{x=1}, (E_1)_{x=0})= ite(X, G_1, G_2)$

$E_2=ite(Y, (E_2)_{y=1}, (E_2)_{y=0})= ite(Y, H_1, H_2)$

◇ stands for logic operation "AND" or "OR". Let $I_2=(H_2)_{X=1}$, there is:

$$ite(X,G_1,G_2)\Diamond ite(Y,H_1,H_2)=$$

$$\begin{cases} ite(X,G_1\Diamond H_1,G_2\Diamond H_2) & index(x)=index(y) \\ ite(X,G_1\Diamond I_2,G_2\Diamond E_2) & \\ \quad X,Y \text{ stand for different states of the same component} & \\ ite(X,G_1\Diamond E_2,G_2\Diamond E_2) & \text{others} \end{cases} \quad (4)$$

As Boolean variables representing different states of the same component are not independent from each other anymore, the common BDD algorithm cannot be used again to solve the BDD with dependence. Hence, a new algorithm should be developed based on such dependences.

By observing the BDD generated according to the above rules, we may discover that the 1-line of variable *X* connects variable *Y*, while variable *Y* belongs to a component different from variable *X*. In addition, the BDD with *Y* as the root contains no other variable that belongs to the same component as variable *X*. Nevertheless, the solution must be conducted by taking different ways for two states of 0-line, which are the 0-line connecting the variables of different components and the 0-line connecting the variables of the same component.

Assuming that BDD G is $G=ite(X, G_1, G_2)=X\,G_1+\overline{X}\,G_2$, in which $G_2=ite(Y, H_1, H_2)=Y\,H_1+\overline{Y}\,H_2$,

Then,

$$\Pr\{G=1\}=$$

$$\begin{cases} \Pr\{G_2=1\}+\Pr\{X=1\}\left(\Pr\{G_1=1\}-\Pr\{I_2=1\}\right) & \\ X,Y \text{ stand for different states of the same component} & \\ \Pr\{G_2=1\}+\Pr\{X=1\}\left(\Pr\{G_1=1\}-\Pr\{G_2=1\}\right) & \text{others} \end{cases} \quad (5)$$

In which, $I_2=(H_2)_{X=1}$. We may take the same way to compute $Pr\{G_1=1\}$, $Pr\{G_2=1\}$ and $Pr\{I_2=1\}$ till the convergent node, such as, $G_i=1$ or $G_i=0$, is obtained.

1) $G_i=1$ implies that the system or subsystem represented by G_i is always in the current state.
2) $G_i=0$ implies that the system or subsystem represented by G_i is never in the current state.

The construction and computation cost of BDD is not much different for MFT and general fault tree. There is only a difference that several 0-lines are involved in the computation of I_2 in Equation (5), but only a 0-line is used in the general fault tree. However, the number of 0-lines involved is not larger than the number of component states.

III. BDD of Multistate Phased Mission System

In a phased mission system, the sub-mission in each phase can be corresponding to a single fault tree and its BDD. However, computation will be more complex if the BDD of each subsystem is analyzed separately and the subsystems are then combined based on their algebraic relation to obtain the BDD of system. Therefore, this paper proposes a new algorithm to generate the BDD of the whole system at one time. The main procedure of this algorithm is as follows:

Step 1. Establish the relation between the missions in each phase and the functional unit.

By analyzing the relations between the missions in each phase and the functional unit, the fault tree for missions in each phase is constructed. After that, the fault trees are converted into failure trees according to the rule $\overline{A\cap B}=\overline{A}\cup\overline{B}$. The failure of functional unit is taken as a mode of fault.

Step 2. Generate the BDD of phased mission system.

A. Sequence the Indexes of Bottom Event

The sequence of variables is very important to the generation of BDD. The size of BDD (e.g. number of nodes) depends much on its sequence.

In a phased mission system, the fault tree of the whole system consists of several separate fault trees according to certain mathematic relations, so the bottom event sequencing method for separate fault tree is not suitable anymore. Hence, two principles are put forward in converting failure trees of phased mission system into bottom events of BDD as follows:

1) Bottom events are firstly sequenced according to the level of *mission* phase. The functional unit for higher level of mission phase is placed before the functional unit for lower level of mission phase, which is corresponding with reverse PDO (phased dependent operation).
2) The missions in the same phase should be sequenced by placing more *frequent* bottom events first.

B. Generate the BDD of Phased Mission System

1) Based on the information and failure trees provided in the computation, write the structure expression for the whole mission system.
2) Convert the failure trees obtained in the previous step into ite structure by employing the recursive method

For convenience of conversion, a method is put forth to reduce the number of BDD variables generated prior to conversion. When a functional part is repeatedly applied in multiple sub-mission systems in multiple phases, it has the following characteristics:

If $\overline{Y}$ appears in n phases, it is marked as $\overline{Y_1},\overline{Y_2},\cdots,\overline{Y_n}$ respectively. According to the previous sequence of indexes for bottom event, the sequence indexes for such n bottom events shall be $\overline{Y_n},\cdots,\overline{Y_2},\overline{Y_1}$. To a non-repairable part, if $\overline{Y_n}=0$, that is, the functional part *Y* operates normally in the n^{th} phase, *Y* operates normally in the previous (*n*-1) phases, that is, $\overline{Y_1}=0$, in which, *i*=1,2,…*n*. Thus,

$$\overline{Y}_n + \overline{Y}_{n-1} + \cdots + \overline{Y}_2 + \overline{Y}_1 = ite(\overline{Y}_n, 1, 0) \quad (6)$$

If simplification is not employed, the BDD can be still obtained for this system. However, it is much complex to obtain the BDD without simplification. Thus, Equation (6) can simplify the reliability model of phased non-repairable mission system to effectively reduce the complexity of computation.

Step 3. Compute the reliability of phased mission system.

The paths with leaf node of "1" are traced back on the generated BDD to obtain the non-intersect expression of this mission system. After the fault rate of each component is substituted into the expression, the reliability of the multistate phased mission system can be computed.

IV. Case Analysis

When a weapon system executes a mission, its command control system [7] can mainly perform it in three phases: the first phase is information integration. In this phase, the system mainly completes the search and positioning of enemy target and passes the information of position to next phase. The second phase is command control. In this phase, the main mission is to complete the analysis and processing of position data transmitted from the previous phase, and set the parameters of the weapon based on analysis results, so as to ensure that the weapon can find and hit the target accurately after launching. The third phase is attack, mainly including weapon launching and guidance. The command control system has 3 states, that is, success, failure and degradation.

This weapon system is mainly composed of five functional subsystems { H , X_1 , X_2 , Y , Z }. Among them, H , X_1 and X_2 are used in the first phase, X_1 and X_2 are used in the first and second phases, Y is used in the second and third phases and Y and Z are used in the third phase. In the first phase, the subsystem operates normally on the premise that at least X_1 or X_2 is functional and H operates normally. In the second phase, it is required that at least X_1 or X_2 is functional and Y operates normally. In the third phase, it is required that Y and Z are functional at the same time. It must be pointed out that functional part Z has three states { Z_0, Z_1, Z_2 }, and contains two components A and B : A has three states { A_0, A_1, A_2 } and B has three states as well { B_0, B_1, B_2 }. When A and B are both in the state of "0", Z is in the state of Z_0 ; when at least A or B is in the state "1" and both are not in the state of "2", Z is in the state of Z_1 ; when at least A or B is in the state of "2", Z is in the state of Z_2 . When Z is in the state of Z_0 , system fails; when Z is in the state of Z_1 , system is degraded; when Z is in the state of Z_2 , system operates normally. The fault rate of each functional part in each phase is presented in the following table.

The occurrence probability of each status is:

$$P(A_0) = P(B_0) = 0.05$$
$$P(A_1) = P(B_1) = 0.1$$

Fault trees are drawn and converted into failure trees as shown in Fig. 1, in which i={0,1,2}.

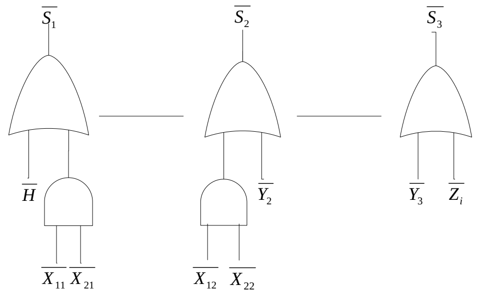

Fig. 1. Failure tree of command control system.

TABLE I: Fault Rate of Functional Unit λ(10^{-6}/ h)

Unit	Mission Phase		
	1	2	3
H	500	-	-
X_1	1000	500	-
X_2	1000	500	-
Y	-	200	100
Mission Time	4	1	2

In this case, this command control system divides mission into three phases, so it is a phased mission system. Moreover, some components have multiple states. Thus, it is also a multistate system. When analyzing and solving this issue, the component Z is analyzed firstly. The expressions of Boolean function in different states of Z are as follows:

$$Z_0 = A_0 B_0$$
$$Z_1 = A_1 B_0 + A_1 B_1 + A_0 B_1$$
$$Z_2 = A_2 B_0 + A_2 B_1 + A_2 B_2 + B_2 A_0 + B_2 A_1$$

When the sequence of indexes is index (A_0)< index (A_1) < index (A_2)< index (B_0)< index (B_1)< index (B_2), the above structure function in Equation 3.5 is converted into the corresponding ite structure as follows:

$$Z_0 = ite(A_0, B_0, 0)$$
$$Z_1 = ite\left(A_0, B_1, ite\left(A_1, ite\left(B_0, 1, B_1\right), 0\right)\right)$$
$$Z_1 = ite\left(A_0, B_2, ite\left(A_1, B_2, A_2\right)\right)$$

Based on the ite structure expression in different states, the BDD can be drawn for each state as presented in Fig. 2.

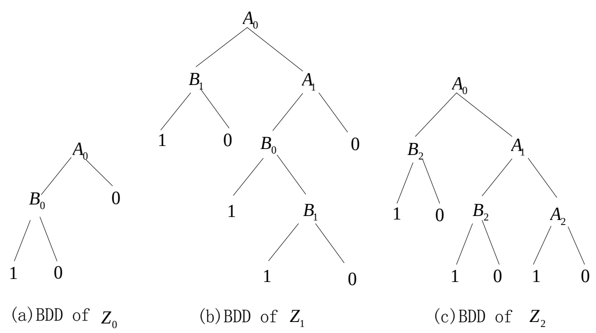

Fig. 2. BDD model of Z in each state.

By tracing back all the paths with the leaf node of "1", the non-intersect expression of minimum cutest in each state may be as follows:

$$Z_0 = A_0 B_0$$

$$Z_1 = A_0 B_1 + B_1 \overline{B_0} A_1 \overline{A_0} + B_0 A_1 \overline{A_0} = A_0 B_1 + A_1 B_1 + A_1 B_0$$

$$Z_2 = A_0 B_2 + A_1 B_2 + A_2$$

Therefore, $P(Z_0)$=$P(A_0)P(B_0)$=0.0025

$P(Z_1)$=0.02

$P(Z_2)$=0.9775

By then, the analysis on function parts with multiple states is completed. Now, the construction of BDD model for phased mission process is presented hereinafter.

Firstly, the optimum sequence of indexes for bottom events in this process is determined as follows according to the rules for determining the sequence of indexes for bottom events in fault tree of phased mission process: index($\overline{Y_3}$)<index($\overline{Z_1}$)<index($\overline{Y_2}$)<index($\overline{X_{12}}$)<index($\overline{X_{22}}$) <index($\overline{X_{11}}$) <index($\overline{X_{21}}$)<index(H)

When this command control system operates normally, Z is in the state of Z_2. At this time, the structure function of this system is as follows:

$$\overline{S_2} = \overline{H} + \overline{X_{11}}\,\overline{X_{21}} + \overline{X_{12}}\,\overline{X_{22}} + \overline{Y_2} + \overline{Y_3} + \overline{Z_2}$$

Considering the non-repairable nature of phased mission system, the above structure function can be converted into ite structure as follows:

$$\overline{S_2} = ite\left(\overline{Y_3}, 1, ite\left(\overline{Z_2}, 1, ite\left(\overline{H}, 1, ite\left(\overline{X_{12}}, \overline{X_{22}}, 0\right)\right)\right)\right)$$

The BDD in normal state is presented in Fig. 3:

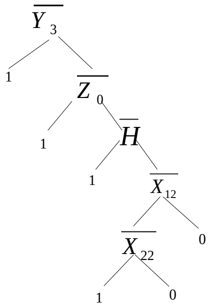

Fig. 3. BDD of command control system.

Based on Fig. 3, the non-intersect expression for this state is written as follows:

$$\overline{S_2} = \overline{Y_3} + Y_3 \overline{Z_2} + Y_3 Z_2 \overline{H} + Y_3 Z_2 H \overline{X_{12} X_{22}}$$

By substituting fault rate into the equation, it can obtain:

$$\Pr\left(\overline{X_{12}} = 1\right) = \Pr\left(\overline{X_{22}} = 1\right) = 4.496 \times 10^{-3}$$

$$\Pr\left(\overline{Y_3} = 1\right) = 3.960 \times 10^{-4}$$

$$\Pr\left(\overline{Z_2} = 1\right) = 0.0225$$

$$\Pr\left(\overline{H}\right) = 2 \times 10^{-3}$$

Thus, $P(\overline{S_2})$=0.02486. Hence, the mission reliability of system is $P(S_2)$=1-$P(\overline{S_2})$=0.97514. Similarly, the probability of system degradation is $P(S_1)$=0.01996. Thus, the probability of system failure is $P(S_0)$=0.00490. As revealed in the analysis, the reliability of this system is 0.975.

V. Conclusion and Prospects

This paper employs BDD method to study and analyze the phased mission system with multiple modes of failure, and constructs a single BDD model to describe a phased problem, so as to simplify the modeling process. Through the case study on the application of this model in a command control system, its accuracy is proved.

In the engineering applications, system operation is featured by dynamic structural change. Along with the change of structure, the corresponding BDD model should also be reconstructed. The future research will focus on the application of BDD in constructing the reliability model of dynamic system for analysis.

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