# Time Dependent Ratchetting of Thin Cylindrical Shell Due to Axial Temperature Variation Using Visco-Plastic Model

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Abstract—Purpose of the present study is to predict thermal ratchetting deformation of thin cylindrical shell, in the framework of unified Chaboche visco-plastic constitutive model. Strain controlled cyclic characteristics and uniaxial monotonic loading at different strain rate is analyzed for SS316 L. Thermal ratchet load of 550 °C is applied cyclically along the axis of the cylinder to predict the deformation pattern accounting strain rate dependence of the material operating at high temperature condition as in Prototype Fast Breeder Reactor (PFBR). The effect of axial temperature variation is simulated by cyclically varying the heat transfer properties at the interacting surfaces of cylindrical shell. To achieve comparable accuracy and better convergence, semi-implicit plasticity integration approach is implemented in UMAT code. Cyclic hardening and strain rate dependence of material is compared by the experimental results.

*Index Terms*—Progressive deformation, thermal ratchetting, visco-plastic, chaboche model.

## I. INTRODUCTION

Ratchetting deformation due to sodium free level variation is one of the critical phenomena in reactor assembly of the Prototype Fast Breeder Reactor (PFBR). Axial temperature variation due to Sodium free level, during normal operations and other operating conditions depend upon the temperatures of hot and cold pools. During normal operation, the upper cylindrical portion of the inner vessel in the vicinity of hot pool sodium free level is highly affected by level variations.

Cyclic loading, induced by level variations in PFBR main vessel shown in Fig.1, result into complex material behavior leading to progressive deformation. It was reported that speed of level variation affects the deformation pattern [1]. Unified visco-plastic constitutive theory for time dependent formulation is considered to investigate deformation behavior of smooth cylindrical portion of main vessel of PFBR. Cylinder of dimension 500 mm outside diameter (OD) and thickness of 1 mm is considered for the analysis. Since progressive deformation in austenitic stainless steel structure includes plastic straining with cyclic hardening property, it is essential to involve sophisticated constitutive model to simulate inelastic behavior. Classical ratchetting model by J. Bree [2], discussed the evolution of ratchetting phenomenon due to primary and secondary load combination however thermal ratchetting in reactor components can occur by secondary stresses alone. There are various literatures

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available on thermal ratcheting for high temperature reactor components [3]. Visco-plastic constitutive model is implemented in user defined subroutine with semi-implicit integration technique for plasticity calculation. Transient thermal loading of 550 % is achieved by FILM subroutine.



Fig. 1. Typical sodium free level variations in PFBR main vessel.

#### II. CONSTITUTIVE EQUATIONS

In this section, a constitutive model is shown describing temperature dependent ratchetting of material. The main equations of unified visco-plastic theory considered for the present analysis are reproduced here as below:

$$\varepsilon_{ij} = \varepsilon_{ij}^{In} + \varepsilon_{ij}^{e} + \varepsilon_{ij}^{T} \tag{1}$$

$$\varepsilon_{ij}^e = D_{ijkl}^{-1} \sigma_{kl} \tag{2}$$

$$\dot{\varepsilon}_{ij}^{In} = \sqrt{\frac{3}{2}} \left\langle \frac{F_y}{K} \right\rangle^n \frac{S_{ij} - \alpha_{ij}}{\left\| S_{ij} - \alpha_{ij} \right\|}$$
(3)

$$\dot{\varepsilon}_{ij}^{T} = C_{ij} \dot{T} \delta_{ij} \tag{4}$$

$$F_{y} = \sqrt{1.5(S_{ij} - \alpha_{ij})(S_{ij} - \alpha_{ij})} - R$$
(5)

where  $\mathcal{E}_{ij}$ ,  $\mathcal{E}_{ij}^{In}$ ,  $\mathcal{E}_{ij}^{e}$ ,  $\mathcal{E}_{ij}^{T}$  and  $\dot{\mathcal{E}}_{ij}^{In}$  are total strain inelastic strain elastic strain, thermal strain and rate of inelastic strain respectively;  $\dot{\mathcal{E}}_{ij}^{T}$  is the thermal strain rate and  $D_{ijkl}$  is the matrix of elasticity; *K* and *n* are material parameters representing the viscous characteristics.  $C_{ij}$  is the coefficient

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of thermal expansion.  $S_{ij}$ ,  $\alpha_{ij}$  and R represents deviatoric stress, back stress and isotropic deformation resistance, respectively.  $\langle \bullet \rangle$  is Macaulay's bracket and means that: as

$$\langle x \rangle = 0$$
 for  $x \le 0$  and  $\langle x \rangle = x$  for  $x > 0$ .

The rate of change of back stress describing kinematic hardening discussed by Chaboche and Rousselier [4] is considered and shown below:

$$\dot{X} = \frac{2}{3}Cd\varepsilon^{P} - \gamma X\dot{P}$$
(6)

where  $C, \gamma$ : material parameters defining kinematic hardening,  $\dot{P}$ : accumulated plastic strain rate. The evolution of isotropic hardening variable is given by:

$$\dot{R} = b(Q - R)\dot{P} \tag{7}$$

where, Q is asymptotic value which corresponds to a regime of stabilized cycles and b indicates the speed of stabilization. The analysis is performed by implementing the concept of radial return technique with Implicit Scheme to ensure the stability during plastic deformation. Newton's method of integration for effective plastic strain increment is used while all other quantities are integrated explicitly with self developed FORTRAN coding. In implicit scheme, momentum balance or equilibrium equations require the determination of Jacobian that comprises the tangent stiffness matrix and load stiffness matrix. Since the tangent stiffness matrix is dependent on material behavior hence constitutive model chosen for material behavior should be realistic so as to give accurate results for strain calculation.

#### **III.** ANALYSIS AND SIMULATION

There are several hardening rules available for ratchetting estimation. Each hardening rule has different advantages and disadvantages. Work done by Igari and co-authors [5] for evaluation of thermal ratchetting due to axial temperature variation, implementing different hardening rules were reasonably conservative and established a fact that the prediction of the thermal ratchetting is strongly dependent on the constitutive model selected and completely accurate prediction of ratchetting strain is not always guaranteed by any of the models. At high temperature, such as 550 °C, viscosity of the material and its effect on time dependent ratchetting should be addressed in detail. Further, the implementation of hardening rules with visco-plastic formulation in FEM package is another difficulty in using Implicit/Explicit technique for plasticity integration. It is quite challenging for complex plasticity hardening to implement fully implicit technique in the program. A simpler approach is to implement a semi implicit scheme [6] of plasticity integration where Newton's method is used to evaluate effective plastic strain increment implicitly however all other quantities (isotropic, kinematic hardening variable, stress etc.) are updated explicitly. Progressive inelastic deformation produced due to temperature variation along the axis of the thin cylindrical shell can cause contraction or expansion depending upon loading conditions and geometry of the specimen [1]. Further, the loading also affects the elastic-plastic behavior of the structure such that the structure achieve the steady state of saturating cyclic plastic straining called as shakedown and fails due to low cycle fatigue. Alternatively, if the structure undergoes progressive deformation such that there is increase in plastic strain cycle by cycle, the structure gets collapsed after gross plastic deformation and is known as Ratchetting.

#### A. Uniaxial Monotonic and Cyclic Loading Analysis

In this study, the unified visco-plastic constitutive model is first used to simulate the uniaxial time-dependent behavior under tensile loading of SS-316L stainless steel at room temperature by introducing a combined nonlinear kinematic hardening model with a static recovery term. Then, the capability of the model, to simulate the uniaxial time dependent ratchetting is discussed by comparing with the corresponding experiments of the material at room temperature with different strain rate. Further, the proposed model is implemented into finite element code ABAQUS to compare cyclic hardening behavior of the material at high temperature.

In order to analyze time dependent thermal ratchetting behavior, due to cyclic thermal loading caused by free level variations in sodium pool, let us consider the work done by Kang and co-author [7] to evaluate viscous parameters. Uniaxial tensile test data at different strain rate is referred for validation of the developed UMAT code. Monotonic loading at different strain rate of 2.0 E-4/s and 2.0 E-3/s is applied at room temperature. The strain rate dependence of SS316L at room temperature is shown in Fig. 2. It is found to be in reasonable agreement with the experimental value.

Due to cyclic variation in temperature along the axis of cylinder, secondary stresses are developed which when exceed the state of elastic stress, material hardening occur in case of SS316 L. Formulation of combined hardening [8] in the present model is done in UMAT code to take care of material hardening property.



Fig. 2. Comparison of tensile stress strain curve at room temperature at different strain rates of 2.0 E-4/s and 2.0 E-3/s.

Strain controlled uniaxial cyclic loading test at 6.67E-4/s strain rate [1] is compared with the present analysis in Fig. 3, to validate the results. Slight overestimation of stress is

observed but the range of stress is matching and is in reasonable agreement with the experimental analysis. Stabilized loop is shown, so as to identify the maximum range of stress developed after hardening due to visco-plasticity combined with nonlinear kinematic and isotropic hardening in the material at high temperature.



Fig. 3. Comparison of cyclic stress-strain behavior of SS 316 L at strain rate of 6.67E-4/s at 550 °C.

# B. Multi-Axial Thermal Ratchetting Analysis

Identified material constants for visco-plastic constitutive equations are shown in Table I.

TABLE I: IDENTIFIED MATERIAL CONSTANTS		
Material constants	At room temperature	At 550 °C
С	131200 (MPa)	98400 (MPa)
γ	860	880
b	38	8.7
Q	123 (MPa)	119 (MPa)
k	138 (MPa)	64 (MPa)
Κ	42	38
n	5.6	2.2

## 1) Thermal loading:

A smooth thin hollow cylinder of 500 mm (OD), height 600 mm and 1 mm thickness is analyzed in ABAQUS by imposing fifteen cycles of thermal loading.

The test cylinder was heated when moving up and down into the pool. FILM subroutine is used to take care of time dependent temperature and heat transfer properties and material subroutine UMAT is formulated such that it will take care of visco-plastic, isotropic and kinematic hardening behavior of material. The temperature profile at different instances of 40 seconds is shown in Fig. 4.



## Fig. 4. Temperature profile at the instances of 40 seconds.

#### 2) Residual radial deformation

The progressive inelastic deformation at the end of each cycle is measured for fifteen loading cycles and it is recorded for 50 mm of level variations in 160 seconds. The deformations after fifteen cycles, is estimated to be 0.56 mm (outward radial deformation). Fig. 5 shows residual radial displacement of cylindrical shell at various instances of time (at each cycle).



Fig. 5. Profile of residual displacements at different instances of time (for 15 cycles).

#### IV. RESULTS

The strain controlled cyclic characteristics and uniaxial monotonic loading at different strain rate is analyzed for SS316 L and material constants are identified. Based on the identified material constants, thermal ratchetting analysis is done. As observed in the present work, the following results can be inferred from Fig. 2 and Fig. 3: (a) At room temperature, the material exhibits remarkable rate dependence in monotonic tension, and the responded stress at smaller strain rate is much lower than that at higher one. (b) Slight over stress is noted compared to the experimental result for stabilized stress-strain loop at high temperature of  $550 \,$ °C. Uniaxial cyclic stress- strain behavior of SS 316 L to recognize hardening is shown in Fig. 6.



Fig. 6. Stress-strain hysteresis loop at strain rate of 6.67E-4/s.

Multi-axial ratchetting of thin hollow cylindrical shell is predicted in terms of radial deformation. The total residual deformation accumulated at the end of fifteen cycles due to axial temperature variation using visco-plastic constitutive model as in Fig. 5 is 0.56 mm radially outward. Progressive ratchetting with no sign of saturation in strain accumulation up to fifteen cycles is observed.

# V. CONCLUSION

The study shows the analysis of inelastic cyclic behavior of austenitic steels under axial temperature variation, which is essential for life prediction of high temperature structural component for PFBRs. The accuracy of life prediction is highly dependent on constitutive models selected for material behavior and the realistic material properties/constants required for analysis. Thermal ratcheting studies of thin hollow cylindrical shell have been carried out using self developed UMAT subroutine in ABAQUS with time dependent visco-plastic formulation. The analysis indicated that SS 316 L material exhibits combined isotropic and kinematic hardening with considerable strain rate dependence even at room temperature. Progressive deformation prediction capability can be improved by considering realistic thermal loading, visco-plasticity, age-hardening, time-recovery and creep in the constitutive model.

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