Effectiveness of Mutual Information to Select Appropriate Indices for Selection of Design Ground Motions

Tauqir Ahmed and Honda Riki

***Abstract*—Selection of design ground motion for nonlinear dynamic analysis is a difficult task because of its unpredictability and sensitivity to various uncertain parameters. Particularly, indices based design ground motion selection procedures may lag in this aspect because complexity and unpredictability of nonlinear response cannot be represented by a unique index. To improve the reliability of index based design ground motions selection procedures, it is proposed to use multiple indices, because it is helpful to consider a variety of aspects of ground motions, which may not be consider if we use a unique index. Selection of appropriate index/indices for selection/synthesis of design ground motions from a list of available indices is a critical issue and discussed in this paper. It is proposed to use the mutual information for the evaluation of indices, because it evaluates the information shared by two variables. The effectiveness of mutual information to select the appropriate indices for selection of design ground motions is verified through a numerical simulation. In numerical simulation, two dimensional three bays five floor moment resisting concrete frame is used as a target structure. A set of 450 ground motions records from past seismic activities are used to show the effectiveness of the approach in context of real ground motions. Advantage of using mutual information over conventional approach is discussed. The results from the study indicate that mutual information can serve as a useful tool for selection of appropriate indices for selection of design ground motions.**

***Index Terms*—Mutual information, dynamic nonlinear response, feature indices, coefficient of covariance.**

I. Introduction

Selection of design ground motions is an important and influential aspect of seismic design. The advancement in ground motion simulation approaches and development in network of seismographs in the last few decades result with a large number of ground motions data bank. Consideration of such ground motions for the design of structure will increase the reliability of seismic performance of structure. Meanwhile, practically, it is required to select a limited number of ground motions for the design of structures.

Index based design ground motions selection procedures are getting acceptance due to simplicity in application. For example, authors have proposed a scheme for synthesis of design ground motion, considering the uncertainty of structural and seismic uncertainties [1], [2]. In the proposed approach, feature indices are used to represent the set of possible ground motions. Different index consider different aspect of the seismic performance. Appropriate index can lead to selection of good design ground motions.

In order to select appropriate indices, we need to quantitatively evaluate the efficiency of an index or combination of indices to represent the severity of structural damage due to a ground motion. In general, this will be helpful to improve the performance of index based design ground motion selection/synthesis procedure, especially the procedure proposed by the authors [1], [2].

II. Objectives

Knowing the importance of having the appropriate indices for the performance-enhancement of index based design ground motion selection producer, we set the objective of this study to propose a mean for the selection of index or a combination of indices, which is relatively efficient to represent the damaging capabilities of ground motions, considering stochastic nature of structural characteristics and complexity of nonlinear response. To meet the required objective, we propose to use mutual information (MI) for the evaluation of effectiveness of indices. The advantage of using MI over conventional covariance is elaborated.

III. Evaluation of Performance of Indices for Selection of Ground Motions

For the evaluation of performance of index, we consider the relationship between the probability of structural damage and the index values. In conventional approach, coefficient of covariance is utilized to evaluate the relationship quantitatively.

A. *Coefficient of Covariance*

Coefficient of covariance, which is given for two variables X and Y as

$$C^R(X,Y) = \frac{C(X,Y)}{\sqrt{Var(X)Var(Y)}} \tag{1}$$

where $C(X,Y)$ denotes the covariance. Conventionally, a higher value of coefficient of covariance between the index value and the probability of structural damage validate the superiority of index or combination of indices. This is helpful when a linear relationship exist between the index value and the probability of structural damage, but due to complexity of nonlinear response and variety of combinations of indices, the linear relationship is not the most likely option. We need an alternative approach that can consider influence of nonlinearity.

Manuscript received May 1, 2012; revised August 30, 2012.
Tauqir Ahmed is with the Department of Civil Engineering, The Univesity of Tokyo, Japan, on leave from University of Engineering and Technology Lahore, Pakistan (e-mail: tauqir@ip.civil.t.u-tokyo.ac.jp).
Riki Honda is with the Department of Civil Engineering, The Univesity of Tokyo, Japan (e-mail: rhonda@civil.t.u-tokyo.ac.jp).

DOI: 10.7763/IJET.2012.V4.463

B. Selection of Indices Based on Mutual Information

MI is a measure of information shared between two independent variables. For two discrete random variables X and Y, MI can be defined as

$$MI(X,Y) = \sum_{y\in Y}\sum_{x\in X} p(x,y)\log\left(\frac{p(x,y)}{p(x)p(y)}\right) \quad (2)$$

where $p(x)$ and $p(y)$ are probability distribution function of X and Y respectively, and $p(x,y)$ is joint probability distribution function of X and Y. Let I denotes the MI value between an index k and probability of structural damage P, then I quantifies the amount of information that k have about the P. It is expected that indices with higher MI value will be more appropriate to evaluate the probability of structural damage. In combination of indices the amount of information is enhanced due to inclusion of different aspects of structural damaging capabilities of ground motion, so combination of indices yield superior results as compared to a single index. In that context, we propose to use the MI to quantitatively evaluate the effectiveness of indices.

Different from covariance, MI value will be appropriate regardless of type of correlation between k and P. The advantage of using MI value over conventional approach is elaborated by a numerical simulation

IV. Numerical Simulations

We discuss the quantification of effectiveness of indices. Indices are used to select the design ground motions out of a given set of possible ground motions. Effectiveness of indices to evaluate the damaging capabilities of ground motions, which is a prerequisite to select the design ground motion, is evaluated by MI and conventional approaches.

A. Possible Ground Motions

To formulate the set of possible ground motions, 450 ground motions records from past earthquake events are obtained from K-NET [3]. It would be possible to generate such ground motions using numerical techniques. We use actual ground motion records, in order to discuss the applicability of the presented scheme to real ground motions. In order to verify the applicability of the proposed scheme under the wide range of variation, ground motions are selected without considering the ground conditions. The ground motion records are factored so that their peak ground acceleration values are ranging between 600cm/sec2 to 800cm/sec2.

B. Structural Model and Uncertainty of Material Properties

Design ground motions are selected for a two dimensional, three bay, five-story moment resisting concrete frame, elevation of the frame is shown in Fig. 1, sectional details are shown in Table I, here after referred as target structure. The dead load for the nonlinear analysis is contributed by the weight of members; beam, columns, concrete slab and weight of floor finishes. Nonlinear dynamic analysis is completed by using OpenSees [4].

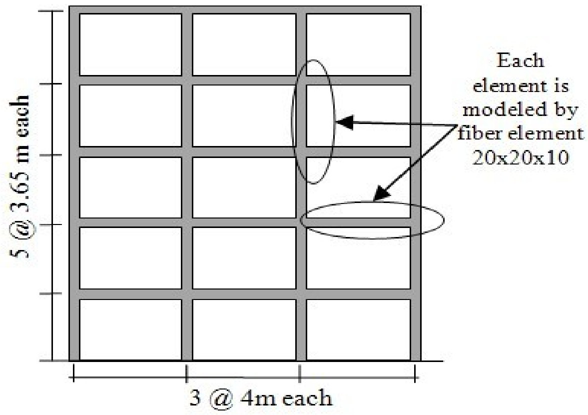

Fig. 1. Elevation of concrete frame.

TABLE I: DETAIL OF BEAM AND COLUMN SECTIONS.

Section	Width [cm]	Depth [cm]	Reinforcement
Column	38	38	19mm dia. 22 bars uniformly distributed on all faces
Beam	30	38	Top. 19 mm dia. 7 bars Bot. 19 mm dia. 7 bars

C. Structural Model and Uncertainty of Material Properties

Design ground motions are selected for a two dimensional, three bay, five-story moment resisting concrete frame, elevation of the frame is shown in Fig. 1, sectional details are shown in Table I, here after referred as target structure. The dead load for the nonlinear analysis is contributed by the weight of members; beam, columns, concrete slab and weight of floor finishes. Nonlinear dynamic analysis is completed by using Open Sees [4].

Beams and columns of frame are modeled by using unidirectional steel and concrete fibers models, which are characterized by stress strain relationships. We used models available as recipes in Open Sees, to characterize stress strain curve for the fibers of concrete and steel. Steel02 [4], [5] is used to model the stress strain behavior of steel fibers. The stress strain curve for steel is shown in Fig. 2. In this model we can control the transition from linear to nonlinear stage. Parameters to model stress strain curve for steel are summarized in Table II.

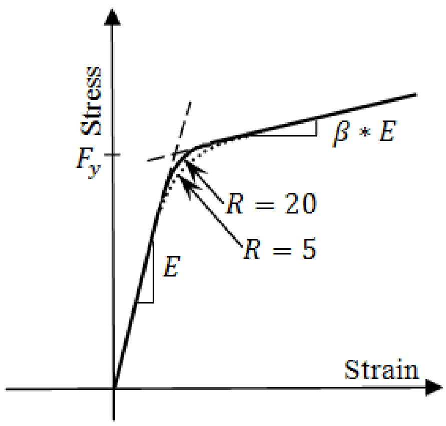

Fig. 2. Stress strain model for steel02 [4], [5].

TABLE II: PARAMETERS OF STEEL02 MODEL USED IN SIMULATION.

Parameter	Value
F_y	250 Mpa (subjected to uncertainty)
E	200,000 Mpa (subjected to uncertainty)
β	0.18
R	18

Similarly, material model Concrete02 [4] of OpenSees is used to characterize the stress strain behavior of confined and unconfined concrete fibers. In this model we can also consider the tensile strength of concrete in modeling.

Analysis shows that natural period of the concrete moment resisting frame under consideration is 0.65sec. It shows good agreement with the value 0.64sec, which is estimated by using empirical relationship given in UBC-97 in equation 30-8 [6].

In order to consider the fluctuation of material properties, we assume material properties of elements are independent stochastic variables. Yield strength of steel, modulus of elasticity of steel and compressive strength of concrete are considered as stochastic variables. OpenSees calculates the strain of each fiber against the deformation of member. Such strain of columns is used to quantify the effect of ground motion on structure.

D. Quantification of Damage of Structure

Let us assume that a ground motion, say ground motion1, is more damaging than another ground motion, or ground motion2, if the number of damage components due to ground motion1 is more than that by ground motion2. By means of that definition of structural damage, percentiles of ground motions based on damage of target structure is calculated.

E. Indices and Combination of Indices

Eight indices and their twenty eight combinations are used to evaluate percentile of ground motions. Four of them are response values of the bilinear single degree of freedom (SDOF) systems whose natural period corresponds to the first mode of the target structure, such as displacement response (D1), velocity response (V1), acceleration response (A1) and dissipated energy (E1). Remaining four indices are those of the SDOF system corresponding to the second mode of target structure. They are displacement response (D2), velocity response (V2), acceleration response (A2) and dissipated energy (E2) . Based on similar nature of indices, indices and their combinations are categorized into five groups (Table III). Group of indices are sorted in ascending order of expected performance of indices, e.g. indices related to response of bilinear SDOF corresponding to first mode (group-3) are expected to superior then second mode indices (group-1) and combination of them (group-2).

Percentiles of ground motions are also calculated by using index values, while joint probability is used to calculate the percentile for the case of multiple indices. Percentile of a ground motion indicates the rank of ground motion in the set of ground motions. Distribution of aforementioned percentile for the case when displacement response of SDOF corresponding to first mode is used as an index, are plotted in Fig. 3. From this distribution, MI value is calculated to show the goodness of an index by using Eq. 2. For comparison, coefficient of covariance is also calculated by using Eq.1. This process is repeated for each combination of aforementioned groups and results are presented in Fig. 4.

TABLE III: INDICES BASED ON SDOF CORRESPONDING TO FIRST AND SECOND MODE OF TARGET STRUCTURE

Group-1	Disp. Vel. Acc. Dissipated Energy of SDOF second mode.
Group-2	Combinations of Disp. Vel. Acc. Dissipated Energy of SDOF second mode.
Group-3	Disp. Vel. Acc. Dissipated Energy of SDOF first mode.
Group-4	Combinations of Disp. Vel. Acc. Dissipated Energy of SDOF first mode.
Group-5	Combinations of Disp. Vel. Acc. Dissipated Energy of SDOF first and second mode.

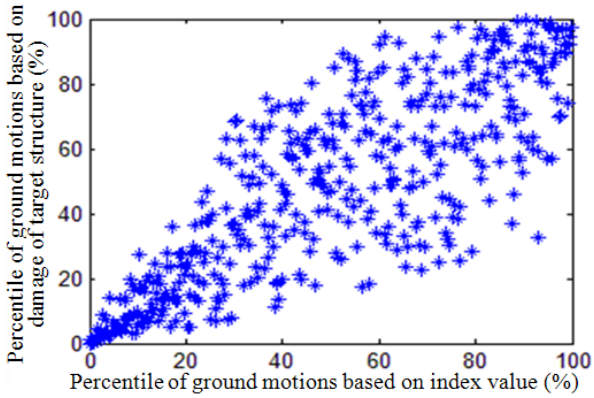

Fig. 3. Distribution of percentile of ground motion based on damage of target structure and based on index value.

V. SIMULATION RESULTS

The MI value between the percentile of ground motions based on damage of target structure and percentile of ground motions accessed by the indices are plotted in Fig. 4 (a). Fig. 4 (b) shows the corresponding coefficient of covariance values. The values of MI is shown by (*), while average value of MI for each group is shown by (o) and connected by dash line in Fig. 4(a). Similarly, values of coefficient of covariance are shown in Fig. 4(b). The comparison shows the followings.

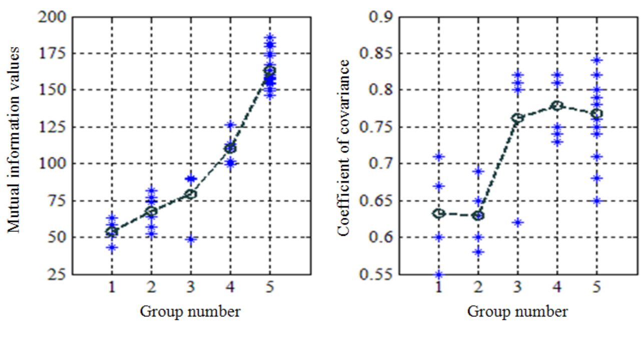

Fig. 4. (a) Mutual information and (b) Coefficient of covariance between the percentile of ground motion based on probability of damage and percentile of ground motion from indices.

1) The groups of indices are arranged in ascending order of expected performance. Increasing trend of MI in Fig. 4 (a) is harmonic with the expected performance of indices. For example, group-5 is expected to be the best among the five groups because information is accumulated due to combination of indices from both first and second

mode responses. For that group-5, MI values attain the maximum value, while coefficient of covariance is insensitive to this aspect. The values of covariance coefficient are not consistent with expected performances of indices.

2) Group-4 of indices is combination of the indices used in group-3, and accordingly the MI for Group-4 is higher than the group-3. While, more information is accumulated in group-5, because it is combination of indices of group-3 and group-1. Fig. 4(a) shows that MI for group-5 is higher than the MI for group-4. This shows that MI is efficient to select the informative indices.
3) For each group of indices, as compared to values of covariance coefficient, the MI values are less scattered, indicating that MI is more reliable than coefficient of covariance in this case.

This verifies that MI is efficient to evaluate the effectiveness of indices, higher the value of MI batter the index or combination of indices will be.

VI. Conclusions

Quantification of effectiveness of indices to depict the damage capabilities of ground motion is a crucial issue of earthquake engineering and it requires a detailed investigation. Authors proposed to use MI for the quantification of effectiveness of indices, as MI is a measure of amount of information which is shared by two independent quantities. With the help of numerical simulations, it is shown that MI is efficient to quantify the effectiveness of indices for selection of design ground motion.

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