Psychoanalysis of Centrally Symmetric Gravitational Field Using a Centrally Symmetric Metric

Muhammad Sajjad Hossain, M. Sahabuddin and M. A. Alim

***Abstract*—The current study is conducted to solve some quantities, λ, μ and ν which are functions of the radial coordinate, R and time coordinate τ, employing a centrally symmetric metric. In this study, a number of new cosmological solutions have also been calculated. The physical significance of the analysis is disscused in detailed. The results of this investigation illustrate that the centrally symmetric gravitational field with constant density automatically takes the shape of the steady state like universe.**

***Index Terms*—Ricci tensor, general relativity, Einstein field equations, cosmology.**

I. Introduction

A centrally symmetric gravitational field can be produced by any centrally symmetric distribution of matter[1]. The centrally symmetry of the field means the space time metric, that is, the expression for the interval ds , must be the same for all points located at the same distance from the center. In Euclidean space this distance is equal to the radius vector. But in a non- Euclidean space, in the presence of a gravitational field, there is no quantity which has all the properties of the Euclidean radius vector. Therefore the choice of a 'radius vector' is now arbitrary. Fiedler and Schimming [2] performed the singularity-free static centrally symmetric solutions of some fourth order gravitational field equations. The fourth order field equations proposed by Treder with a linear combination of Bach's tensor and Einstein's tensor on the left-hand side admit static centrally symmetric solutions which are analytical and non-flat in some neighborhood of the centre of symmetry. Vyblyi [3] considered the general centrally symmetric exterior solutions of the field equations of the relativistic theory of gravitation for the static Schwarzschild and Reissner–Nordstrom static fields. Particular interior solutions are found and they are fitted to the exterior solutions. Genk [4] investigated the strong centrally symmetric vacuum field in the relativistic theory of gravitation. In this investigation, an exact static solution of the equations of the relativistic theory of gravitation in vacuum for the case of spherical symmetry that generalizes Fock's harmonic interval and also radial motion of test particles in the generalized harmonic metric are considered. The condition that the field be physical is used to find the region of applicability of that solution. The authors presented that the picture of the motion of the particles for a distant observer differs fundamentally from the one usually adopted (in Fock's metric) and does not correspond to the picture of asymptotic slowing down of the collapsing body. The energy density of the gravitational field in this case was calculated. Very recently, Samsonov and Petrov [5] made an investigation on the physical interpretation of the central symmetric gravitational-field singularities. The authors showed that the existence of singularity in the centrally symmetric gravitational field, which is interpreted as a surface with unusual physical properties, follows from equations for the action and the energy of a test particle not using Einstein equations and their solutions. In addition, a black hole is treated as a physical model of the singularity in question.

To the best knowledge of the authors, no attention has been paid to solve the quantities and using a centrally symmetric metric. The centrally symmetric gravitational field with constant density automatically takes the shape of the steady state like Universe. This centrally symmetric gravitational field also satisfies the rigorous theorem known as the Birkhoff's theorem [6], which state that "any spherically symmetric vacuum solution of Einstein equation is necessarily the Schwarzschild solution that is, static". This theorem implies that if a spherically symmetric source like a star undergoes pulsations or changes its shape, while maintaining the spherically symmetry, it cannot radiate any disturbances in the exterior, namely, Schwarzschild exterior [7] solution can be used to describe the outside metric for several situations as spherically symmetric star is either static or it undergoes radial spherically symmetric gravitational collapse [8].

II. Mathematical analysis

The 'spherical' space coordinates $r,\ \theta,\ \varphi$ are considered for this present investigation. Most of the general centrally symmetric expression for ds^2 [9] is as follows:

$$\begin{aligned} ds^2 = h(r,t)\,dr^2 + k(r,t) \\ \left(\sin^2\theta d\varphi^2 + d\theta^2\right) \\ + l(r,t)\,dt^2 + a(r,t)\,drdt \end{aligned} \qquad (1)$$

where $a,\ h,\ k,\ l$ are certain functions of the radius vector 'r' and the time 't'. But because of the arbitrariness in the choice of a reference system in the general theory of relativity, we can still change the coordinates to any transformation which does not destroy the central symmetry of ds^2,this implies, we can transform the coordinates r and t according to the formula $r = f_1(r', t')$ and $t = f_2(r', t')$, where f_1, f_2 are functions of the

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new coordinates r' and t'. We make use of the two possible transformation of the coordinates *r, t* in the element of interval. Firstly, make the coefficient *a(r,t)* of *drdt* vanish, and secondly make the radial velocity of the matter vanish at each point (because of the central symmetry the other components are not present). After that, *r* and *t* can still be subjected to an arbitrary transformation of the form $r = r(r')$ and $t = (t')$.

We now assume the radial coordinate and time coordinate by R and τ and the coefficients *h, k, l* by $-e^{-\lambda}$, $-e^{\mu}$, e^{ν}, respectively [where λ, μ, and ν are functions of R and τ]. Then the equation-(1) gives

$$ds^2 = c^2 e^{\nu} d\tau^2 - e^{\lambda} dR^2 - e^{\mu}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) \quad (2)$$

The energy momentum tensor for perfect fluid is as follows:

$$T_{ij} = (p+e)u_i u_j - p g_{ij}$$

where p is the pressure and e is the energy density.
In the commoving reference system the components of the energy- momentum tensor are:

$$T_0^0 = \varepsilon, \quad T_1^1 = T_2^2 = T_3^3 = -p$$

The non-zero metric tensor can be found from equation (2) as follows

$$\left.\begin{array}{ll} g_{00} = c^2 e^{\nu} & g^{00} = c^{-2} e^{-\nu} \\ g_{11} = -e^{\lambda} & g^{11} = -e^{-\lambda} \\ g_{22} = -e^{\mu} & g^{22} = -e^{-\mu} \\ g_{33} = -e^{\mu}\sin^2\theta & g^{33} = -e^{-\mu}\sin^{-2}\theta \end{array}\right\} \quad (3)$$

Now we are interested to find out the non-zero values of 'Γ'as follows,

$$\Gamma^i_{jk} = \frac{1}{2} g^{il}\left(g_{il,k} + g_{kl,j} - g_{jk,l}\right) \quad (4)$$

Substituting $i = j = k = 0$ in equation (4), one may get

$$\Gamma^0_{00} = \frac{\dot{\nu} c}{2} \qquad \text{[Using equation (3)]}$$

Using equations (3) - (4), the following results can easily be found

$$\left.\begin{array}{l} \Gamma^0_{10} = \Gamma^0_{01} = \frac{\nu'}{2}, \qquad \Gamma^1_{01} = \Gamma^1_{10} = \frac{\dot{\lambda} c}{2} \\ \Gamma^0_{11} = \frac{c^{-1} e^{\lambda-\nu}\dot{\lambda}}{2}, \; \Gamma^0_{22} = \frac{c^{-1} e^{\mu-\nu}\dot{\mu}}{2}, \\ \Gamma^0_{33} = \frac{c^{-1} e^{\mu-\nu}\dot{\mu}}{2}\sin^2\theta \end{array}\right. \quad (5)$$

$$\Gamma^1_{11} = \frac{\lambda'}{2}, \; \Gamma^1_{22} = -\frac{e^{\mu-\lambda}\mu'}{2}, \; \Gamma^1_{33} = -\frac{e^{\mu-\lambda}\mu'}{2}$$

$$\Gamma^2_{02} = \Gamma^2_{20} = \frac{\dot{\mu} c}{2}, \; \Gamma^2_{12} = \Gamma^2_{21} = \frac{\mu'}{2}$$

$$\Gamma^3_{03} = \Gamma^3_{30} = \frac{\dot{\mu} c}{2}, \; \Gamma^3_{31} = \Gamma^3_{13} = \frac{\mu'}{2} \quad (6)$$

$$\Gamma^3_{23} = \Gamma^3_{32} = \cot\theta, \; \Gamma^2_{33} = -\sin\theta\cos\theta, \; \Gamma^1_{00} = \frac{c^2 e^{\nu-\lambda}\nu'}{2}$$

The Ricci Tensor is defined as

$$R_{jk} = -\Gamma^i_{jk,i} + \Gamma^i_{jk,i} - \Gamma^m_{jk}\Gamma^i_{mi} + \Gamma^m_{ji}\Gamma^i_{mk} \quad (7)$$

With the aid of equations (5), (6) and (7), the Ricci Tensor takes the form as below:

$$\begin{aligned} R_{00} = {} & \frac{c^2\dot{\nu}^2}{4} + \frac{c^2\dot{\lambda}^2}{4} + \frac{c^2\dot{\mu}^2}{2} - \frac{c^2 e^{\nu-\lambda}\nu'\mu'}{2} \\ & + \frac{c^2 e^{\nu-\lambda}\nu'\lambda'}{4} - \frac{c^2 e^{\nu-\lambda}\nu'^2}{4} \\ & - \frac{c^2 e^{\nu-\lambda}\nu''}{2} + \frac{c^2\ddot{\lambda}}{2} - \frac{c^2\dot{\nu}^2}{4} \\ & - \frac{c^2\dot{\nu}\dot{\lambda}}{4} - \frac{c^2\dot{\nu}\dot{\mu}}{2} + c^2\ddot{\mu} \end{aligned} \quad (8)$$

$$\begin{aligned} R_{11} = {} & -\frac{e^{\lambda-\nu}\ddot{\lambda}}{2} - \frac{e^{\lambda-\nu}\dot{\lambda}^2}{4} - \frac{e^{\lambda-\nu}\dot{\lambda}\dot{\mu}}{2} \\ & + \frac{e^{\lambda-\nu}\dot{\lambda}\dot{\nu}}{4} - \frac{\lambda'\nu'}{4} + \frac{\nu'^2}{4} \\ & - \frac{\lambda'\mu'}{2} + \frac{\mu'^2}{2} + \frac{\nu''}{2} + \mu'' \end{aligned} \quad (9)$$

$$\begin{aligned} R_{22} = {} & -\frac{e^{\mu-\nu}\ddot{\mu}}{2} - \frac{e^{\mu-\nu}\dot{\mu}^2}{2} + \frac{e^{\mu-\lambda}\mu''}{2} - \frac{e^{\mu-\lambda}\mu'^2}{2} \\ & + \frac{e^{\mu-\nu}\dot{\mu}\dot{\nu}}{4} - \frac{e^{\mu-\nu}\dot{\mu}\dot{\lambda}}{4} + \frac{e^{\mu-\lambda}\mu'\lambda'}{4} \\ & - \frac{e^{\mu-\lambda}\mu'\nu'}{4} - 1 \end{aligned} \quad (10)$$

$$\begin{aligned} R_{33} = {} & -\frac{e^{\mu-\nu}\ddot{\mu}\sin^2\theta}{2} + \frac{e^{\mu-\lambda}\mu''\sin^2\theta}{2} \\ & - \frac{e^{\mu-\nu}\dot{\mu}\dot{\nu}\sin^2\theta}{4} - \frac{e^{\mu-\nu}\dot{\mu}\dot{\lambda}\sin^2\theta}{4} \\ & - \frac{e^{\mu-\nu}\dot{\mu}^2\sin^2\theta}{2} + \frac{e^{\mu-\lambda}\mu'\nu'\sin^2\theta}{4} \\ & - \frac{e^{\mu-\lambda}\mu'\lambda'\sin^2\theta}{4} + \frac{e^{\mu-\lambda}\mu'^2\sin^2\theta}{2} \\ & - \sin^2\theta \end{aligned} \quad (11)$$

$$R_{10}=\mu^{/}-\frac{c\dot{\mu}\nu^{/}}{2}-\frac{c\dot{\lambda}\mu^{/}}{2}+\frac{c\dot{\mu}\mu^{/}}{2} \qquad (12)$$

On the other hand the Ricci scalar is calculated as follows:

$$R=-e^{-\lambda}\nu^{//}+e^{-\nu}\ddot{\lambda}+2e^{-\nu}\ddot{\mu}+\frac{e^{-\lambda}\lambda^{/}\nu^{/}}{2}-\frac{e^{-\nu}\dot{\lambda}\dot{\nu}}{2}$$
$$-\frac{e^{-\lambda}\nu^{/2}}{2}+\frac{e^{-\nu}\dot{\lambda}^{2}}{2}+\frac{3e^{-\nu}\dot{\mu}^{2}}{2}-\frac{3e^{-\lambda}\mu^{/2}}{2}$$
$$-e^{-\nu}\dot{\mu}\dot{\nu}-2e^{-\lambda}\mu^{//}+e^{-\nu}\dot{\mu}\dot{\lambda}+e^{-\lambda}\mu^{/}\lambda^{/}$$
$$-e^{-\lambda}\mu^{/}\nu^{/}+2e^{-\mu}$$

The Einstein field equation is defined as

$$R_{ij}-\frac{1}{2}g_{ij}=-\frac{8\pi k}{c^{4}}T_{ij} \qquad (13)$$

The following results can be found taking suitable values of i and j in equation (13) as:

$$\left.\begin{aligned}R_{00}-\frac{1}{2}g_{00}&=-\frac{8\pi k}{c^{4}}T_{00}\\R_{11}-\frac{1}{2}g_{11}&=-\frac{8\pi k}{c^{4}}T_{11}\\R_{22}-\frac{1}{2}g_{22}&=-\frac{8\pi k}{c^{4}}T_{22}\\R_{33}-\frac{1}{2}g_{33}&=-\frac{8\pi k}{c^{4}}T_{33}\\R_{10}-\frac{1}{2}g_{10}&=-\frac{8\pi k}{c^{4}}T_{10}\end{aligned}\right\} \qquad (14)$$

Therefore the field equations are obtained from equations (8)-(12) and (14) as follows:

$$e^{-\nu}\left[\frac{\dot{\mu}^{2}}{4}+\frac{1}{2}\dot{\lambda}\dot{\mu}+e^{-\mu}\right]-e^{-\lambda}\left[\mu^{//}+\frac{3\mu^{/2}}{4}-\frac{1}{2}\lambda^{/}\mu^{/}\right]=\frac{8\pi k\varepsilon}{c^{4}} \qquad (15)$$

$$e^{-\lambda}\left[\frac{\mu^{//}}{2}+\frac{\nu^{//}}{2}+\frac{\mu^{/2}}{4}+\frac{\nu^{/2}}{4}-\frac{\lambda^{/}\mu^{/}}{4}+\frac{\mu^{/}\nu^{/}}{4}-\frac{\lambda^{/}\nu^{/}}{4}\right]$$
$$+e^{-\nu}\left[-\frac{\ddot{\mu}}{2}-\frac{\ddot{\lambda}}{2}-\frac{\dot{\mu}^{2}}{4}-\frac{\dot{\lambda}^{2}}{4}-\frac{\dot{\mu}\dot{\lambda}}{4}+\frac{\dot{\mu}\dot{\nu}}{4}+\frac{\dot{\lambda}\dot{\nu}}{4}\right] \qquad (16)$$
$$=\frac{8\pi k}{c^{4}}p$$

$$e^{-\nu}\left[\frac{\ddot{\mu}}{2}+\frac{\dot{\mu}^{2}}{2}-\frac{\dot{\mu}\dot{\nu}}{4}+\frac{\dot{\mu}\dot{\lambda}}{4}\right]$$
$$-e^{-\lambda}\left[\frac{\mu^{//}}{2}-\frac{\mu^{/2}}{2}+\frac{\mu^{/}\lambda^{/}}{4}-\frac{\mu^{/}\nu^{/}}{4}\right] \qquad (17)$$
$$+e^{-\mu}-\frac{1}{2}=\frac{8\pi k}{c^{4}}p$$

$$e^{-\lambda}\left[\frac{\mu^{/2}}{4}+\frac{1}{2}\mu^{/}\nu^{/}\right]$$
$$-e^{-\nu}\left[\ddot{\mu}+\frac{3\dot{\mu}^{2}}{4}-\frac{1}{2}\dot{\mu}\dot{\nu}\right] \qquad (18)$$
$$-e^{-\mu}=-\frac{8\pi k}{c^{4}}p$$

$$2\dot{\mu}-\dot{\mu}\nu'-\dot{\lambda}\mu'+\dot{\mu}\mu'=0 \quad \text{as } e^{-\lambda}\neq 0 \qquad (19)$$

The symbol prime denotes differentiation with respect to R while dot represents differentiation with respect to t.

The Einstein's field equation for the metric g_{ij} in the presence of matter can be defined as folows:

$$R_{ij}-\frac{1}{2}g_{ij}R=-\frac{8\pi k}{c^{4}}T_{ij} \qquad (20)$$

Equation (20) gives,

$$G_{j}^{i}=-\frac{8\pi k}{c^{4}}T_{j}^{i} \qquad (21)$$

Differentiate equation (21) with respect to i, we get [5]

$$0=-\frac{8\pi k}{c^{4}}T_{j,i}^{i}$$

For conservation the relation is as follows:

$$T_{j,i}^{i}=0$$

i.e. $T_{i,k}^{k}=0$

$$\text{Gives,}\quad \frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^{k}}\left(\sqrt{-g}T_{i}^{k}\right)-\frac{1}{2}\frac{\partial g_{lk}}{\partial x^{i}}T^{kl}=0 \qquad (22)$$

Therefore for $k=0$ the equation (22) gives the following result:

$$\frac{c}{2}\left(\dot{\lambda}+2\dot{\mu}+\dot{\nu}\right)\varepsilon+2\dot{\varepsilon}-\frac{c}{2}\left(\dot{\nu}\varepsilon-\dot{\lambda}p-2\dot{\mu}p\right)=0$$

$$\text{i.e.}\quad \dot{\lambda}+2\dot{\mu}=-\frac{2\dot{\varepsilon}}{p+\varepsilon} \qquad (23)$$

If we consider the density is constant everywhere, i.e. e = constant so that $\dot{e}=0$, the equation (23) becomes,

$$\dot{\xi}+2\dot{\mu}=0$$

$$\text{i.e.}\quad \xi+2\mu=f\left(t\right) \qquad (24)$$

But in the beginning of the universe i.e. at time $t = 0$, the equation (24) becomes,

$$\xi+2\mu=0 \qquad (25)$$

We need one more equation which is provided by the equation of state $P = P(e)$, in which the pressure is given as a function of the mass-energy density. This equation of state is as follows:

$$P=(\xi-1)e, \quad \text{where } 1 \le \xi \le 2 \tag{26}$$

Now for $k = 1$ the first part of equation (23) is given by,

$$-\frac{1}{2}\left[\left(\lambda^{/}+2\mu^{/}+\nu^{/}\right)p+2p^{/}\right]$$

And the 2nd part of the equation (23) can be expressed as,

$$-\frac{1}{2}\left[\nu^{/}\varepsilon-\lambda^{/}p-2\mu^{/}p\right].$$

Hence for $k = 1$ the result of equation (22) can be written as:

$$-\frac{1}{2}\left[\left(\lambda^{/}+2\mu^{/}+\nu^{/}\right)p+2p^{/}\right]$$
$$-\frac{1}{2}\left[\nu^{/}\varepsilon-\lambda^{/}p-2\mu^{/}p\right]=0$$

So, we get,

$$\nu^{/}=-\frac{2p^{/}}{p+\varepsilon} \tag{27}$$

If p is known as a function of e then equation (23) can be integrated in the form

$$\lambda+2\mu=-2\int\frac{d\varepsilon}{p+\varepsilon}+f_1(R) \tag{28}$$

where the functions $f_1(R)$ can be chosen arbitrary in view of the possibility mentioned above of making arbitrary transformations of the form $R = R(R^{/})$. Now, at the initial moment $t = 0$ the equation (28) gives $f_1(R)=0$

III. Findings

Case-I:

When $\xi = 1$, $p = 0$ i.e. for dust like sphere or the pressure less matter, the equation (28) becomes,

$$\lambda+2\mu=-2\int\frac{de}{e}$$

i.e. $e=e^{-\sqrt{\xi+2\mu}}$, when $\xi+2\mu=0$.

Therefore, e = constant (29)

Case-II:

When $\xi = \frac{4}{3}$, $P=\frac{1}{3}\varepsilon$ i.e. for radiation dominant universe, the equation (28) becomes,

$$\lambda+2\mu=-3\int\frac{de}{2e}$$

i.e. $e=e^{-\sqrt[3]{\xi+2\mu}}$, when $\xi+2\mu=0$.

Therefore, e = constant (30)

Case-III:

When $\xi = 2$, $P=e$ i.e. for stiff fluid, the equation (28) becomes,

$$\lambda+2\mu=-2\int\frac{de}{2e}$$

i.e. $e=e^{-(\xi+2\mu)}$, when $\xi+2\mu=0$.

Therefore, e = constant (31)

Again, for the equation (27), we get the following cases:

Case-I:

When $\xi = 1$, p= 0 i.e.for dust like sphere or the pressure less matter, the equation (27) becomes,

ξ = Constant (32)

Case-II:

When $\xi = \frac{4}{3}$, $P=\frac{1}{3}\varepsilon$ i.e. for radiation dominant universe, the equation (27) becomes,

$$e\,\alpha\frac{1}{e^{4\xi}}.$$

This implies,

e = constant (33)

Case-III:

When $\xi = 2$, $P=e$ i.e. for stiff fluid, the equation (27) becomes,

$$e\,\alpha\frac{1}{e^{2\xi}}$$

This implies,

e = constant (34)

IV. Conclusion

We choose the interval ds^2 in the form of equation (2), there still remained the possibility of an arbitrary transformation of the time of the form $t = f(t)$ like (24). Such a transformation is equivalent to adding to ν an arbitrary function of the time, and with its aid we can always make $f(t)$ in equation (24) and vanish if we consider $t = 0$. And so without any loss in generality, we can get $\xi+2\mu=0$. Note that the centrally symmetric gravitational field with constant density in each solution from equations (29) to (34) automatically takes the shape of the steady state Universe. Since the Universe is expanding, the principle demands that new matter must be created to maintain a constant density of the Universe [10]. The most remarkable feature of the theory is that the new matter (believed to be hydrogen atom) is supposed to be created out of nothing in a creation field called the C field, that is, Matter requires to be continuously created in the Universe according to this theory [11].

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