Estimation of Dynamic Properties of Honeycomb Paperboard and Parameters Identification

Zhu Dapeng, Zhou Shisheng and He Ruichun

***Abstract*—A system identification technique is presented for estimation of parameters associated with the dynamic model of the mass loaded honeycomb paperboard system. The honeycomb paperboard is modeled as a linear material with viscoelastic property, whose constitutive law is expressed by an exponential hereditary relaxation kernel. The free response of the mass loaded honeycomb paperboard system is expressed as the sum of complex exponentials, the system poles and residues are identified by the modified Prony method, the parameters of the dynamic model are identified by a substitution strategy. An experiment system is fabricated to record the free response of the mass-material system. Finally, the parameters identification technique is applied to the experimental data to obtain the relevant stiffness, viscous and viscoelastic parameters associated with the system. Variations in values of these parameters as function of static load level is also investigated and presented. Honeycomb paperboard dynamic properties model and the parameters presented in this paper can provide theoretical and design basis for the proper use of honeycomb paperboard in packaging.**

***Index Terms*—Honeycomb paperboard, Modified Prony method, Parameter identification, Viscoelastic property.**

I. INTRODUCTION

Honeycomb paperboard is a kind of sandwich panel, it is made up of three parts: the upper and lower liners, between which is the honeycomb core. All parts are made of reusable paper. Because of its specific structure, it has many advantages over other materials, such as the high strength-to-weight ratio, high stiffness-to-weight ratio, light weight, the ease to be processed, and it is recyclable, reusable and biodegradable. Because of these advantages, honeycomb paperboard has been used in many fields. In resent years, because of the environment protection concerns and the command for reducing the plastic wastes, people began to use honeycomb paperboard in protective packaging as the cushion material to substitute the foam. The realization of the potential of honeycomb paperboard as an important cushion material has inspired a close scrutiny of its properties.

Guo and Zhang[1] investigated the cushion properties and vibration transmissibility properties of honeycomb paperboard with different thickness by a series of experiments, the experiment results are fitted by polynomials, the results provided basic data for protective packaging design. Wang[2] investigated the cushioning properties of honeycomb paperboard by experimental analysis, the effects of the honeycomb paperboard structures, relative density of paper honeycomb cores, liners and layouts on cushioning properties have been studied. The impact behavior and energy absorption properties of honeycomb paperboard were presented in [3], the experimental results indicated that the increase of the relative density of paper honeycomb cores can efficiently improve the dynamic cushioning properties, the experimental results also indicated that the thickness of paper honeycomb cores has a fluctuant effect on the cushion properties. Hidetoshi Kobayashi et al. [4] studied the effect of loading rate on the strength and absorbed energy of paper honeycomb cores through the quasi-static and dynamic compression experiments. The critical buckling load of honeycomb paperboard under out-of-plane pressure was investigated by analyzing the structure and the collapse mechanism [5], the models and the calculation method in the paper can be used to predict the static critic buckling load. Because honeycomb paperboard is sensitive to the ambient humidity, in [6,7], the mathematical models are developed to predict the plateau stress of axially loaded paper honeycombs in various humidity environments, and describe the relationship between the energy absorption properties of paper honeycombs and ambient humidity.

In this paper, the viscoelastic property of honeycomb paperboard is taken into account, the constitutive law of the viscoelastic material is modeled as a linear differential equation. The mass loaded honeycomb paperboard system is used to simulated the packaging system, an experiment system is formulated to record the free response data of mass loaded honeycomb paperboard system, a parameter identification procedure is presented to obtain the dynamic property parameters under different load conditions. The model and the identified parameters presented in this paper provides theoretical and design basis for the proper use of honeycomb paperboard in packaging.

II. HONEYCOMB PAPERBOARD MODEL

Viscoelasticity is an important aspect of the dynamic behavior of honeycomb paperboard. The response of a viscoelastic material can different significantly from linear elasticity. For example, in studies of the dynamics of a viscoelastic beam[8] and the absorption characteristics of a vibrational damper[9]. Physical manifestation of the honeycomb paperboard's viscoelastic properties can be seen in its force relaxation behavior and the creep phenomenon. If

Manuscript received June 28, 2011; revised August 25, 2011. This work is supported in part by the China national science foundation under Grant 61064012 and Gansu provincial science foundation under Grant 1014RJZA047.

Zhu Dapeng and Zhou Shisheng are with the Faculty of Printing and Packaging Engineering, Xi'an University of Technology, Shaanxi, Xi'an, China. (e-mail: dapeng_zhu@163.com; zhoushisheng@xaut.edu.cn).

He Ruichun is with the School of Traffic and Transportation, Lanzhou Jiaotong University, Lanzhou, China (e-mail: heruichun@mail.lzjtu.cn).

the honeycomb paperboard is subjected to a constant compressive strain, the force in the material decreases over time, logarithmically approaching its steady state value. Also, if honeycomb paperboard specimen is loaded with a given mass, the compressive strain increases over time from the initial value. A typical force relaxation measurement for the honeycomb paperboard specimen is shown in Fig. 1. A constant deformation of 0.95mm was applied to the honeycomb paperboard specimen, the resulting force was measured over a period 1000 seconds. Although this relaxation behavior is similar to plasticity, a viscoelastic material typically recovers to its undeformed state once the loading is removed.

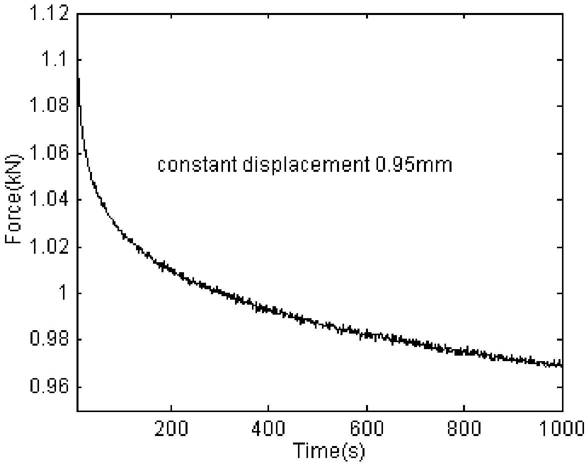

Fig. 1. Typical force relaxation under constant displacement

In addition to the static effects, viscoelasticity also influences the behavior of honeycomb paperboard. When subjected to a dynamic excitation, a mass loaded honeycomb paperboard system is seen to display additional dynamic creep beyond its static equilibrium. It has been shown that the stiffness and damping characteristics of honeycomb paperboard undergoing forced vibration depend on the length of time that the length of time that the honeycomb paperboard specimen has been exercised[10]. The linear stiffness is seen to increase, whereas the linear viscous damping coefficient is seen to decrease, both approaching their final steady state values logarithmically with increasing excitation time. The behavior of the viscoelastic material under dynamic load conditions is also called dynamic creep.

There are several approaches to modeling the viscoelastic nature of honeycomb paperboard. One model that has received much attention involves the use of fractional derivatives[11,12]. This approach has been shown to predict the frequency dependence of linear viscoelastic dampers fairly well. The fractional model requires fewer parameters for comparable prediction compared with models using derivatives of integer order. A related approach involving fractional integrals has also been propose[13]. A different type of model involving a nonlinear spring in series with an in-parallel linear spring and damper combination has also been investigated[14]. To account for nonlinear elastic and creep properties, and a multiplicative decomposition model has been developed[15].

In this work, In the theory of viscoelasticity, one of the models is a constitutive law of the form[16]:

$$a_0\sigma + a_1\frac{d\sigma}{dt} + \cdots + a_m\frac{d^m\sigma}{dt^m} = b_0\varepsilon + b_1\frac{d\varepsilon}{dt} + \cdots + b_n\frac{d^n\varepsilon}{dt^n} \quad (1)$$

ε is the strain in the viscoelastic material and σ is the resulting stress response. It can be assumed that at t=0, the material was unstressed and undeformed. Taking the Laplace transformation of the equation above, one can obtain:

$$\sigma(s) = \frac{b_0 + b_1 s + \cdots + b_n s^n}{a_0 + a_1 s + \cdots + a_m s^m}\varepsilon(s) \quad (2)$$

$\sigma(s)$ and $\varepsilon(s)$ are the counterparts of σ, ε in Laplace domain respectively. Because of the considerably big stiffness coefficient[1,2] of honeycomb paperboard, the deformation of the honeycomb paperboard specimen is very small if excited by a shock, one can obtain the basic relationship between the stress σ and strain ε

$\sigma = E\varepsilon$

where E is the instantaneous Young's modulus. Thus, one can assume that in (1), m=n. then, (2) can be represented as a sum of fractions. i.e.,

$$\sigma(s) = E[\varepsilon(s) - \varepsilon(s)\sum_{i=1}^{n}\frac{a_i}{s+\alpha_i}] \quad (3)$$

Using the inverse Laplace transformation and the convolution theorem, one obtains from (3)

$$\sigma = E[\varepsilon - \int_0^t \Gamma(t-\tau)\varepsilon(\tau)d\tau] \quad (4)$$

The weight function $\Gamma(t\text{-}\tau)$ is the sum of exponential terms:

$$\Gamma(t-\tau) = \sum_{i=1}^{N} a_i \exp[-\alpha_i(t-\tau)] \quad (5)$$

$\Gamma(t\text{-}\tau)$ is called the relaxation kernel. (4) and (5) are the linear governing equation for stress and strain under uniaxial loading of honeycomb paperboard.

III. Free Response of Mass Loaded Honeycomb Paperboard System

In the packages, the product and the honeycomb paperboard in the package formulate a mass loaded honeycomb paperboard system, the honeycomb paperboard serves as the elastic and dissipative element in the system and its mass is negligible compared to that of the product. When the mass loaded honeycomb paperboard system is exerted the force $f(t)$,the summation of elastic force and the viscoelastic force can be written in the following form according to (4) and (5)

$$F = k[x - \int_0^t \Gamma(t-\tau)x(\tau)d\tau]$$

where k is the stiffness coefficient, x is the deformation of the honeycomb paperboard. A linear viscous damping term is included in this model to account for the viscous losses in the material, thus, the equation of motion for the dynamic response of the mass in the mass loaded honeycomb paperboard system can be expressed as

$$m\ddot{x} + c\dot{x} + kx - k\int_0^t \sum_{i=1}^{n} a_i e^{-\alpha_i(t-\tau)}x(\tau)d\tau = f(t)$$

where m is the mass of the system, c is the viscous damping coefficient, a_i and α_i, i=1,2,…n are the n viscoelastic parameters, $f(t)$ is the force exerted on the mass. In the free response phase of the material-mass system, $f(t)$=0, the equation of motion of the system is

$$m\ddot{x}+c\dot{x}+kx-k\int_0^t\sum_{i=1}^{n}a_ie^{-\alpha_i(t-\tau)}x(\tau)d\tau=0 \qquad (6)$$

The free responses are generated by means of suitable initial conditions $x(0)$ and $\dot{x}(0)$.

Applying the Laplace transformation to (6), one obtains:

$$m[s^2x(s)-\dot{x}(0)-sx(0)]+c[sx(s)-x(0)]+kx(s)$$
$$-kx(s)\sum_{i=1}^{n}\frac{A_i}{s-\alpha_i}=0$$

where $x(s)$ is the counterpart of $x(t)$ in Laplace domain. Thus,

$$x(s)=\frac{m[\dot{x}(0)+sx(0)]+cx(0)}{s^2m+sc+k\left(1-\sum_{i=1}^{n}\frac{A_i}{s-\alpha_i}\right)}=\frac{R(s)}{T(s)} \qquad (7)$$

where $R(s)$ is a polynomial of order n+1, and $T(s)$ of order n+2. Knowing the roots p_i(i=1,2,…,n+2) of polynomial $T(s)$, and assuming p_i are simple(multiplicity of 1), one can express (7) as the sum of fractions:

$$x(s)=\frac{c_1}{s-p_1}+\frac{c_2}{s-p_2}+\cdots\cdots+\frac{c_{n+2}}{s-p_{n+2}} \qquad (8)$$

Taking inverse Laplace transform of (8), one obtains:

$$x(t)=\sum_{i=1}^{n+2}c_ie^{p_it} \qquad (9)$$

where c_i and p_i are real or complex conjugate, the coefficient c_i are called residues, p_i are the system eigenvalues, n is the number of terms in the relaxation kernel. Thus, the free response of honeycomb paperboard system can be expressed as the sum of n+2 complex exponential terms.

IV. Modified Prony Method

Prony method is a technique for modeling data of equally spaced samples by a linear combination of exponentials. Assuming one has the N data samples $x[1]$,$x[2]$,…$x[N]$, the Prony method estimates the x with a n+2-term complex exponential model

$$\hat{x}_i=\sum_{m=1}^{n+2}c_mz_m^i \qquad i=1,2\cdots,N \qquad (10)$$

where the caret denotes an estimate, z_m is called system poles and

z_m=exp$(p_m\Delta t)$ (11)

Δt denotes the time interval between successive samples. The Prony algorithm can be summarized as follows[17]:

Step 1: Record the free response data of mass loaded honeycomb paperboard $x[1]$,$x[2]$,…$x[N]$, let p_e>>n+2, compute matrix $\mathbf{R}$ given by

$$\mathbf{R}=\begin{bmatrix} r(1,0) & r(1,1) & \cdots & r(1,p_e) \\ r(2,0) & r(2,1) & \cdots & r(2,p_e) \\ \vdots & \vdots & \vdots & \vdots \\ r(p_e,0) & r(p_e,1) & \cdots & r(p_e,p_e) \end{bmatrix} \qquad (11)$$

where

$$r(i,j)=\sum_{k=p+1}^{N}x(k-j)x(k-i)$$

Determine the effective rank n+2 of matrix $\mathbf{R}$ and the AR coefficients a_1,a_2,…,a_{n+2} by use of SVD-TLS method[20];

Step 2: Form the polynomial

$$1+a_1z^{-1}+\cdots+a_{n+2}z^{n+2}=0 \qquad (12)$$

and solve to find the roots which are the system poles z_i in the series of complex exponentials in (10), system eigenvalues p_i can be obtain by (11);

Step 3: Rewrite (10) as matrix form:

$$\mathbf{ZC}=\hat{\mathbf{x}}$$

where

$$\mathbf{Z}=\begin{bmatrix} z_1 & z_2 & \cdots & z_{n+2} \\ z_1^2 & z_2^2 & \cdots & z_{n+2}^2 \\ \vdots & \vdots & \vdots & \vdots \\ z_1^N & z_2^N & \cdots & z_{n+2}^N \end{bmatrix},\mathbf{C}=\begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_{n+2} \end{bmatrix},\hat{\mathbf{x}}=\begin{bmatrix} x(1) \\ x(2) \\ \vdots \\ x(N) \end{bmatrix}$$

Because N>n+2, then, vector $\mathbf{C}$ can be obtained by

$$\mathbf{C}=[\mathbf{Z}^T\mathbf{Z}]^{-1}\mathbf{Z}^T\hat{\mathbf{x}} \qquad (13)$$

where the superscript T and -1 denote the transpose and inverse operation of the matrix respectively.

Using the steps outlined above, one can obtain the order of viscoelastic terms n, the system poles p_i and residues c_i, but the measurement noise makes the estimates of the system residues and system poles inaccurate, both in terms of variance and bias. Therefore, modifications to the methodology that improve the accuracy of estimation of system poles and residues should be made. Several modifications to the Prony method have been suggested over the past few years to overcome the problems arising due to noise. Significant among these techniques are: use of high prediction orders[18], use of both forward and backward linear prediction polynomial zeros, singular value decomposition(SVD) based methods, and some non-linear schemes. In this work, the high prediction orders method, as well as the use of both forward and backward linear prediction methods are adopted to obtain system poles and residues accurately.

The estimation bias can be reduced by choosing a linear prediction order much higher than the number of exponentials actually present in the signal. It has been observed the if models of several different order are fitted to the data, the signal poles change very little at high model orders. However, the extraneous poles, which in effect attempt to model the noise, change significantly as the model order is changed. Thus, as the prediction order p_e in (11) changes, the system poles calculated from (12) will cluster around the positions correspond to the signal poles in the z-domain. The average of the clustered system poles can be regarded as the estimate of the system poles.

However, the selection of higher linear prediction order has a negative effect, making it difficult to separate the zeros due to the actual exponential signals from the zeros due to noise. The use of both forward and backward linear prediction can be utilized to separate the two kinds of zero clusters[19]. When one identifies the signal zeros with the high linear prediction order, the forward linear prediction characteristic polynomial has roots at z=exp$(p_i\Delta t)$, i=1,2,…,n, while the backward linear prediction characteristic polynomial has roots at z=exp$(-p_i^*\Delta t)$, i=1,2,…,n, the asterisk denotes conjugation, thus, the roots of the forward linear prediction characteristic polynomial fall inside the unit z-plane circle, while the roots of the roots of the backward linear prediction characteristic polynomial fall outside the unit circle.

Choose different p_e in (11), estimate the system poles by (12) using the recorded time sequence and the time-reversed data respectively, the predicted forward and backward system poles will cluster inside and outside unit circle in z-plane respectively. The clustered poles inside the unit circle can be used as the estimation of the system poles. System residues can be obtained by (13). A typical predicted forward and backward poles cluster condition in the z-plane is shown in Fig. 2.

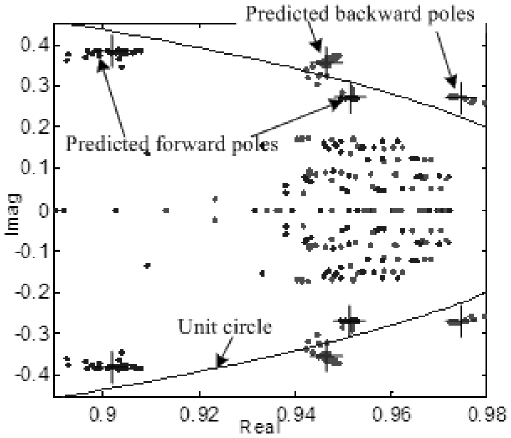

Fig. 2. Forward and backward pole positions in complex z-plane for model orders of 20-80

V. Parameters Identification Procedure

The free response of the mass loaded honeycomb paperboard is express as the sum of n+2 complex exponential terms shown in (9), substituting (9) in (6), one obtains

$$\sum_{j=1}^{n+2}\left(mp_j^2+cp_j+k-k\sum_{i=1}^{n}\frac{a_i}{p_j+\alpha_i}\right)c_j e^{p_j t}+k\sum_{i=1}^{n}\left(\sum_{j=1}^{n+2}\frac{a_i}{p_j+\alpha_i}c_j\right)e^{-\alpha_i t}=0$$

In this equation, there is a sum of exponential functions. To satisfy this equation, one can set all the coefficients of these function to zero, namely:

$$mp_j^2+cp_j+k-k\sum_{i=1}^{n}\frac{a_i}{p_j+\alpha_i}=0 \qquad j=1,2,\cdots\cdots,n+2 \tag{14}$$

and

$$\sum_{j=1}^{n}\frac{a_i}{p_j+\alpha_i}c_j=0 \qquad i=1,2,\cdots\cdots,n \tag{15}$$

For the system identification problem, the task is to use the samples of free response data to obtain the honeycomb paperboard dynamic parameters k, c, α_i, a_i. This can be achieved through the process described below:

Step 1: Record the free response data of the mass loaded honeycomb paperboard system, express the data as a sum of complex exponentials, as given in (9), estimate the system poles and residues by use of the modified Prony method. It is important to note that the samples of displacement response $x(t)$ are not directly available. In actual experiments, the acceleration response $\ddot{x}(t)$ is first expressed as a sum of complex exponentials

$$\ddot{x}(t)=\sum_{i=1}^{n+2}d_i e^{p_i t}$$

Therefore, the system poles and residues of the acceleration data can be obtained by using modified Prony method, the coefficients c_i in (9) can be calculated by $c_i=d_i/p_i^2$.

Step 2: Use the estimates of system poles and residues to calculate α_i. (15) can be rewritten as

$$\sum_{j=1}^{n+2}c_j\prod_{i=1,i\neq j}^{n+2}(p_j+\alpha_i)=0 \tag{16}$$

Finding the roots of (16), one obtains α_i.

Step 3: The remaining honeycomb paperboard parameters can be determined by use of (14), which leads to n+2 simultaneous equations of k, c, a_i. These equations can be written in matrix form as

$$\begin{bmatrix} p_1 & 1 & \frac{-1}{p_1+\alpha_1} & \cdots & \frac{-1}{p_1+\alpha_n} \\ p_2 & 1 & \frac{-1}{p_2+\alpha_1} & \cdots & \frac{-1}{p_2+\alpha_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ p_{n+2} & 1 & \frac{-1}{p_{n+2}+\alpha_1} & \cdots & \frac{-1}{p_{n+2}+\alpha_n} \end{bmatrix}\begin{bmatrix} c \\ k \\ ka_1 \\ \vdots \\ ka_n \end{bmatrix}=-m\begin{bmatrix} p_1^2 \\ p_2^2 \\ p_3^2 \\ \vdots \\ p_{n+2}^2 \end{bmatrix}$$

VI. Experiment System and Results

An experiment system is fabricated to record the free response data of the mass loaded honeycomb paperboard system, as shown in Fig. 3.

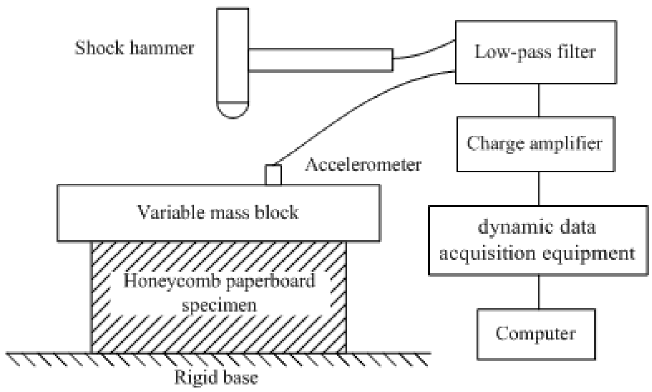

Fig. 3. Schematic diagram of experiment system

The honeycomb paperboard is sandwiched between a variable mass block and rigid base. Before the experiment, the honeycomb paperboard specimens are cut to be 250mm×250mm, and are preprocessed for 24 hours at temperature 21℃, and relatively humidity 64%. The specimen is glued to thin metal plates, which are in turn bolted to the base plate and the top mass block to prevent the top mass block from losing contact with the honeycomb paperboard specimen during the tests. The specimens are provided by Xi’an Hongda packaging material company, in this paper, the thickness of the honeycomb paperboard is 40mm. The upper and lower face sheets are Kraft linerboard paper with substances of 300g/m^2, and the honeycomb core are made of reusable paper whose weight is 100g/m^2. The shape of the honeycomb core unit is regular hexagon, the length of each sides is 5mm. A piezoelectric shock hammer with a rubber tip is used to generate and measure the impulsive input to the mass loaded honeycomb paperboard system. The acceleration response of the mass block is recorded by the piezoelectric accelerometer attached on the mass. The acceleration data from the shock hammer and from the accelerometer attached on the mass is passed through the low pass anti-aliasing filter. The charge amplifier is used to transform the charge signal from the piezoelectric accelerometers into voltage signal. A dynamic data acquisition equipment is used to transform the voltage signal from charge amplifier into digital data, and transmit the data to the computer. All the equipments above are provided by Sinocera Piezotronics company, a YE3760A filter is used, the cut-off frequency is set to be 500Hz, the roll-off is 36dB/OCT, A YE6230B dynamic data acquisition equipment(16 bit) is used, the sampling frequency is set to be 5000 samples/s. All the data collection process in this work was under the control of the YE7600 software package.

In the tests, record the time histories of the shock hammer and accelerometer, the time at which the shock hammer ends can be seen as the beginning of the free response of the mass loaded honeycomb paperboard system. After recording the free response data of the system, one can obtain the residues and poles of the system by use of modified Prony method discussed above, and identify the dynamic parameters using the method presented in Ⅴ. The estimated values of the system parameters under different load conditions, along with the maximum and minimum estimates of these parameters from different tests are shown in Fig. 4.

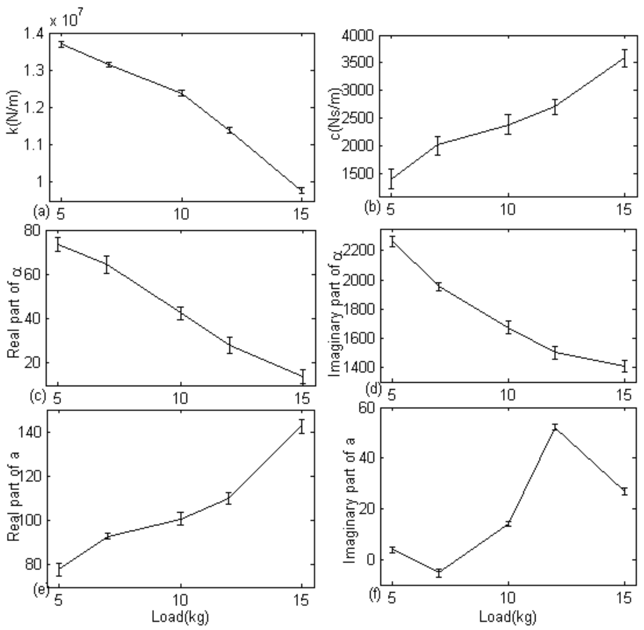

Fig. 4. Estimated values of the system parameters along with error bounds

From the free response data recorded by experimental system shown in Fig. 3 under different load conditions, two pairs of complex conjugate poles could be distinguished, thus, one pair of viscoelastic parameters $\alpha_{1,2}$, $a_{1,2}$ can be used to predict the viscoelastic behavior of honeycomb paperboard.

The estimates of the honeycomb paperboard parameters are based on data from five independent response realizations. With the increase of the static load, the stiffness values, the real and imaginary part of viscoelastic parameter α tend to decrease, while the damping value, the real part of a tend to decrease. No definitive trends could be observed in the imaginary part of a. The error bounds of damping value is bigger comparing with other values, this means the linear viscous damping model is not a perfect model for honeycomb paperboard.

VII. Conclusion

1. The honeycomb paperboard is regarded as linear material with viscoelastic property, the viscoelastic term in the material model is a convolution of the response with a sum of exponentials kernel function.

2. The free response of the mass loaded honeycomb paperboard system is developed, the free response is expressed as the sum of complex exponentials. Prony method is modified to identify the system poles and residues of free response data in noise.

3. A experimental system is fabricated to record the free response of the mass loaded honeycomb paperboard system, a parameter identification procedure based on the substitution strategy is presented to estimate the dynamic parameters in the material model under different load conditions.

The model for the dynamic property of honeycomb paperboard and the identified parameters presented in this paper provide theoretical and design basis for proper use of honeycomb paperboard in protective packaging. This is important for reducing the losses caused by the over-packaging or the insufficient packaging.

References

[1] Y.F. Guo, J.H. Zhang, “Shock Absorbing Characteristics and Vibration Transmissibility of Honeycomb Paperboard”, *Shock and Vibration*, vol. 11, pp. 521-531, 2004.

[2] D.M. Wang, Z.W. Wang, “Experimental investigation into the cushioning properties of honeycomb paperboard”, *Packaging Technology and Science*, vol. 21, no. 1, pp.309-316, 2008.

[3] D.M. Wang, “Impact behavior and energy absorption of paper honeycomb sandwich panels”, *International Journal of Impact Engineering*, vol. 36, no. 1, pp.110-114, 2009.

[4] H. Kobayashi, M. Daimaruya, “Dynamic and static compression tests for paper honeycomb cores and absorbed energy”, *JSME International Journal*, vol. 41, no. 3, pp.338-344, 1998.

[5] L.X. Lu, Z.W. Wang, “Critical buckling load of paper honeycomb under out-of-plane pressure”, Packaging Technology and Science, vol. 18, no. 2, pp. 141-150, 2005.

[6] Z.W. Wang, “Mathematical modeling of energy absorption property for paper honeycomb in various ambient humidities”, *Materials and Design*, vol. 31, no. 9, pp.4321-4328, 2010.

[7] Y.P. E, Z.W. Wang, “Plateau Stress of Paper Honeycomb as Response to Various Relative Humidities”, *Packaging technology and science*, vol. 23, no. 4, pp. 203-216, 2009.

[8] A. Muravyov, S.G. Hutton, “ Closed form solutions and the eigenvalue problem for vibration of discrete viscoelastic systems”, *Journal of Applied Mechanics*, vol. 64, pp. 684-691, 1997.

[9] O.E. Ozbulut, C. Mir, “A fuzzy model of superelastic shape memory alloys for vibration control in civil engineering applications”, *Smart Materials and Structures*, vol. 16, no. 3, pp. 818-829, 2007.

[10] K. Wu, Y.H. Cheng “Simulation on the time dependence of breakdown strength in insulating polymers”, *Journal of Applied Physics*, vol.101, pp. 361-373, 2007.

[11] Y. A. Rossikhin,M. V. Shitikova, “Analysis of damped vibrations of linear viscoelastic plates with damping modeled with fractional derivatives”, *Signal Processing*, vol. 86, no. 10, pp. 2703-2711, 2006.

[12] L.E. Suarez, A. Shokooh, “ An eigenvector expansion method for the solution of motion containing fractional derivatives”, *Journal of Applied Mechanics*, vol. 64, pp. 629-635, 1997.

[13] M. Enelund, A. Fenander and P. Olsson, “Fractional integral formulation of constitutive equations of viscoelasticity”, *AIAA Journal* vol. 38, no. 8, pp. 1356-1362, 1997.
[14] F. Gandhi, I. Chopra, “A time-domain non-linear viscoelastic damper model”, *Smart Materials and Structures*, vol.5, pp. 403-420, 1997.
[15] S. Govindjee, S. Reese, “A presentation and comparison of two large deformation viscoelasticity models”, *Journal of Engineering Materials and Technology*, vol. 119, pp. 251-255, 1997.
[16] T. Atanackovic, S. Pilipovic, “On a class of equations arising in linear viscoelasticity theory”, *Journal of Applied Mathematics and Mechanics*, vol. 85, no. 10, pp. 748-754, 2006
[17] X.D. Zhang, *Modern signal processing*, 1st ed, Beijing: Tsinghua University Press, 1995, ch. 3, pp. 120-125.(in Chinese)
[18] S.L. Marple, *Digital Spectral Analysis with Applications*, 1st ed, Englewood Cliffs, NJ: Prentice-Hall, 1987, ch. 11, pp. 303-331.
[19] V. Slivinskas;V. Simonyte, “Estimation of parameters of impulse responses of mechanical systems by modified Prony Method”, *Diffusion and Defect Data*, vol. 113, pp. 190-194, 2006.
[20] S.V. Huffel, C.L. Cheng, N. Mastronardi, “Total least squares and errors-in-variables modeling”, *Computational Statistics and Data Analysis*, vol. 52, no. 2, pp. 1076-1079, 2007.

Zhu Dapeng was born in Henan, China, in 1977, is a PhD student of the Faculty of Printing and Packaging Engineering, Xi’an University of Technology. He receive his master degree on packaging engineering in Xi’an University of Technology, his research interests include transport packaging, cushion material property testing.
Email address: dapeng_zhu@163.com

Zhou Shisheng, born in 1963, is currently a professor of Xi’an University of Technology, China. He received his PhD degree from X’an Jiaotong University. His research interests include printing engineering technology, the color reproduction theories and technology.
Email address: zhoushisheng@xaut.edu.cn

He Ruichun, born in 1970, is currently a professor of Lanzhou Jiaotong University, China. She received her PhD degree from Lanzhou Jiaotong University. Her research interests include railway transportation security, traffic system simulation technology.
Email address: heruichun@mail.lzjtu.cn