Vibration Suppression of a Beam of Structures

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Abstract—In this issue we try to control the free vibration response of aluminum or iron or other substance cantilever beam with 2 Piezoelectric patches (Actuator & Sensor) attached on the top and bottom surfaces of the beam. On the other hand we also want to reduce the amplitude of the vibrating beam in a controlled manner. In order to achieve these tasks we model the smart structure by using the Euler-Bernoulli beam theory. The concept of finite element (FE) theory is used in this issue. In this method the beam is divided into several equal segments (for example in this paper the beam is divided into 4 equal part), which is also equal to the length of the actuator and sensor or not equal to them and Piezoelectric patches are attached to the second element. In this method the beam is divided into several equal segments (for example in this paper the beam is divided into 4 equal part), which is also equal to the length of the actuator and sensor or not equal to them and Piezoelectric patches are attached to the second element.

Index Terms — Beam, Vibration, suppression, Piezoelectric

I. INTRODUCTION

From the technical point of view, recently most of the structures such as helicopter blades, space mirrors and wings of the aircrafts are consist of beams. Most of the time, undesirable vibrations result in poor functionality and inadequate performance of the system. In most of these applications, particularly in aerospace applications it is very vital for us to have the ability of damping these vibrations in a controlled manner, even for a short period of time. Due to this fact, the "Smart Structures" are introduced by the researchers. "Smart Structures" are consisted of host structure, smart material and the controlling system. Smart materials are able to act as sensor and actuator.

II. METHODOLOGY

In the table you can see the geometric and material properties for the smart beam; also we can change the feature

TABLE 1 : GEOMETRIC AND MATERIAL PROPERTIES FOR THE SMART BEAM

Physical Parameters	Cantilever Beam	Piezoelecteric(PZT) (sensor/actuator)
Length	L_b=0.3 (m)	l_p=0.075 (m)
Width	b=0.03 (m)	b=0.03 (m)
Thickness	t_b=0.5 (m)	$t_a=t_s$=0.35(m)
Density	$\rho_{b=8030}\,{}^{Kg}/_{m^3}$	$\rho_{p=7700}\,{}^{Kg}/_{m^3}$
Young's Modulus	E_b =193.06 Gpa	E_p =68 Gpa
Damping Constants used in C*	$\propto$ = 0.001 β = 0.0001	———
PZT Strain Constant	———	d_{31} = 125× 10^{-12} (m/v)
PZT Strain Constant	———	g_{31} = 10.5× 10^{-3}(VmN^{-1})

and value of them.

(1) Equivalent Young's Module:

$$(E)_{eq} = \frac{E_b \cdot E_p}{V_p \cdot E_b + V_b \cdot E_p}$$

Where V_b and V_p are the volume fractions of the base material and the PZT respectively.

(2) Equivalent (EI):

$$(EI)_{eq} = \frac{1}{3} \cdot \sum E_{eq} \cdot (z_{k+1}^3 - z_k^3)$$

(3) Moment of inertia of Piezoelectric layer:

$$I_p = \frac{1}{12} \times bt_a^3 + bt_a\left(\frac{t_b + t_a}{2}\right)^2$$

(4) Stiffness matrix of beam element:

$$[K_b] = \frac{2E_b I_b}{l_b^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix}$$

(5) Stiffness matrix of Piezoelectric beam element :

$$[K_p] = \frac{2EI_{eq}}{l_p^3}\begin{bmatrix} 6 & 3l & -6 & 3l \\ 3l & 2l^2 & -3l & l^2 \\ -6 & -3l & 6 & -3l \\ 3l & l^2 & -3l & 2l^2 \end{bmatrix}$$

(6) Mass matrix of beam element:

$$[M_b] = \frac{M_b}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

(7) Stiffness matrix of Piezoelectric beam element:

$$[M_p] = \frac{M_p}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix}$$

(8) Mass of the piezoelectric element:

$$M_p = \rho_p t_p \ell \, b$$

(9) Mass of the beam element:

$$M_b = \rho_b t_b \ell \, b$$

(10) Mass of the piezoelectric beam element:

$$M_p = M_b + 2 \times M_{pzt}$$

Assembling will result in:

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$$(11)\ M = \begin{bmatrix} a_1 & b_1 & 0 & 0 & 0 \\ c_1 & d_1+a_2 & b_2 & 0 & 0 \\ 0 & c_2 & d_2+a_3 & b_3 & 0 \\ 0 & 0 & c_3 & d_3+a_4 & b_4 \\ 0 & 0 & 0 & c_4 & d_4 \end{bmatrix}$$

By applying boundary conditions these matrixes will become:

$$(12)\ M = \begin{bmatrix} d_1+a_2 & b_2 & 0 & 0 \\ c_2 & d_2+a_3 & b_3 & 0 \\ 0 & c_3 & d_3+a_4 & b_4 \\ 0 & 0 & c^4 & d_4 \end{bmatrix}$$

Natural frequency is the square of the Eigen vectors of these 2 matrixes($\sqrt{Q_i}$) and the Modal matrix V is the Eigen vectors of these 2 matrixes.

(13)Sensor constant:

$$S_c = G_c e_{31} z b$$

Where G_c=40000 and e_{31} is equal to $g_{31}$and $z = \frac{t_b}{2} + t_a$

(14)Actuator constant:

$$a_c = E_p d_{31} b \bar{z}$$

Where $\bar{z} = \frac{t_b + t_a}{2}$

(15) $h^T = a_c[-1 \quad 0 \quad -1 \quad 0 \quad 0 \quad 0 \quad 0]$

(16) $P^T = S_c[0 \quad -1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

(17) $\dot{x} = A\,x(t) + B\,u(t)$ and $y = C^T x(t) + D u(t)$

$$(18)\ A = \begin{bmatrix} 0 & I \\ -M*^{-1}K* & -M*^{-1}C* \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ M*^{-1}Th \end{bmatrix}$$

$C = [0 \quad p T]$,   D=0

$$(19)\ \begin{bmatrix} \dot{x}1 \\ \dot{x}2 \\ \dot{x}3 \\ \dot{x}4 \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M*^{-1}K* & -M*^{-1}C* \end{bmatrix} \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{bmatrix} + \begin{bmatrix} 0 \\ -M*^{1}Th \end{bmatrix} u(t) + \begin{bmatrix} 0 \\ -M*^{1}Tf \end{bmatrix} r(t)$$

(20) $M^* = V^T \times M \times V$

(21) $K^* = V^T \times K \times V$

(22) $C^* = \propto \times M^* + \beta \times K^*$

(23) Changing the Modal coordinate to Physical coordinate:
$d = \sum \varphi_1^T \zeta$

III. NUMERICAL ANALYSIS

First of all we have divided the beam into 4 equal elements. The sensor and actuator are integrated on the top and bottom surfaces of the second element of the beam.
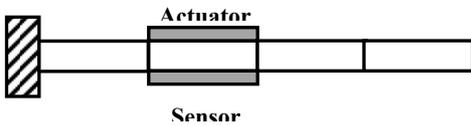
Figure 1 – Beam with one actuator and one sensor

In this step we use the equation (1) to calculate (EI)$_{eq}$. Then by using equations (4), (5), (6) and (7) K_b, K_p, M_b, M_p are calculated respectively. We have assembled the stiffness matrixes of those 4 elements by equation (11).The same procedure has carried out for the mass matrix. Due to the fact that $w_{1\ and}\ \Theta_1$ are equal to zero (in the fixed end of the beam), we can eliminate the 2 first rows and columns of the M and K matrixes.

M = [0.0112   0.0000   0.0027   -0.0000   0
0   0   0;
0.0000   0.0000   0.0000   -0.0000   0
0   0   0;
0.0027   0.0000   0.0112   -0.0000   0.0012
-0.0000   0   0;
-0.0000   -0.0000   -0.0000   0.0000   0.0000
-0.0000   0   0;
0   0   0.0012   0.0000   0.0067
0   0.0012   -0.0000;
0   0   0   -0.0000   -0.0000   0
0.0000   0.0000   -0.0000;
0   0   0   0   0.0012
0.0000   0.0034   -0.0000;
0   0   0   0   -0.0000
-0.0000   -0.0000   0.0000].

K = 1.0e+004 *
[1.2914   0.0226   -0.9472   0.0355   0
0   0   0;
0.0226   0.0024   -0.0355   0.0009   0
0   0   0;
-0.9472   -0.0355   1.2914   -0.0226   -0.3442
0.0129   0   0;
0.0355   0.0009   -0.0226   0.0024   -0.0129
0.0003   0   0;
0   0   -0.3442   -0.0129   0.6884
0   -0.3442   0.0129;
0   0   0.0129   0.0003   0
0.0013   -0.0129   0.0003;
0   0   0   0   -0.3442
-0.0129   0.3442   -0.0129;
0   0   0   0   0.0129
0.0003   -0.0129   0.0006].

By applying the following equation we can easily determine the Eigen vectors and Eigen values of the system. The square of these Eigen values are the Natural frequencies of the system.

[T,Q]=eig(K,M)
Natural_Frequency = sqrt (Q)

T is the Modal matrix. The first and second columns of the T matrix give us the 2 first modes of the beam respectively. We can see these 2 modes by using the following command:
plot (V(1:2:end,1))
hold on
plot (V(1:2:end,2))

By using the equations (20), (21) and (22) we can calculate the M*, K* and C*.

M* = [1.0000   0.0000; 0.0000   1.0000]

K* = 1.0e+004 * [0.1690   0.0000; 0.0000   5.3039]

C* = [0.1700   0.0000; 0.0000   5.3049]

In this stage we are able to form the state space matrixes of the system. h^T and P^T must be calculated with equation (15) and (16), and then substituted in the B and C matrixes.

A = 1.0e+004 * [0   0   0.0001   0
B = 1.0e-003 * [0   0   0.2678   -0.2331]
0   0   0   0.0001
C = [0   0   0.0454   0.3724]
-0.1690   -0.0000   -0.0000   -0.0000
D = [0]
-0.0000   -5.3039   -0.0000
-0.0005]

Now we can form our system by using these matrixes and

apply an initial condition (initial displacement) with "lsim" function to draw the behavior of the vibrating beam.

Model=ss(A,B,C,D)
x0=zeros(4,1);
x0(1,1)=0.01/V(end-1,1);
[y,t,x]=lsim(Model,u,t,x0);

By using the following MATLAB command we can change the Modal coordinate to the Physical coordinate. This is the displacement of the tip of the beam.

Tipdis = (V(end-1,1)*x(:,1)+V(end-1,2)*x(:,2))*1000;

Here output of the system (y) is the voltage of the sensor and we can draw it as follow:

plot(t(1:4:end),y(1:4:end,1),'r')

In this step we are going to design the controlling system in order to suppress the free vibration response and amplitude in a controlled manner. To achieve this task we must change the poles of the system in order to stabilize it. Due to this fact, we can implement the State Feedback method. The new model would become as follow:

Ac = (A-B*Gc*C)
Model_New=ss(Ac,B,C,D)

Again we plot the behavior of the system with "lsim" and compare the results with the uncontrolled system.

In addition, some other kinds of controllers can also be used, for instance, PID, PI, ….

In PID controller we have 3 different gains (K_d, K_I, K_p). We are able to set these gains in order to control both the damping time and vibration amplitude.

Kp1 = 0.2;
Ki1 = 0.001;
Kd1 = 0.01;
num_PID = [Kd Kp Ki];
den_PID = [1 0];
contr_PID = tf (num_PID,den_PID);
sys_PID = feedback(TF*contr_PID,-1)
step(sys_PID,t)

If we want to compare the results which are obtained from the PID, PI controller and the initial system with each other, we have to plot them with a same MATLAB command, for example, "step" function.

IV. RESULTS

Here in **Figure 1** we can clearly see the first and second vibrating modes of the beam. The red line indicates the first mode, while, the black one shows the second mode. Both these 2 figures begin from the origin but due to the fact that we eliminate w1 and Θ1, the first points have cancelled out.
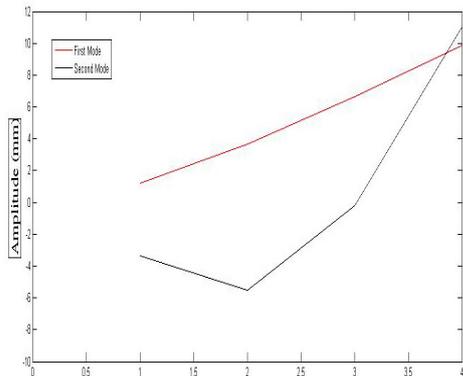

Figure 2 - First & second modes of the beam

In **Figure 3** we have compared the vibration of the cantilever beam with and without the controller. In this figure the red and the cyan lines include the controllers. We use 2 different types of Pole Placement method to control the vibration. In the first method we set a Gain for the controller, while, in the second one we enter the desired poles personally. The damping time and amplitude can be changed by choosing different gains and poles. So this method enables us to damp our system within the desired time.
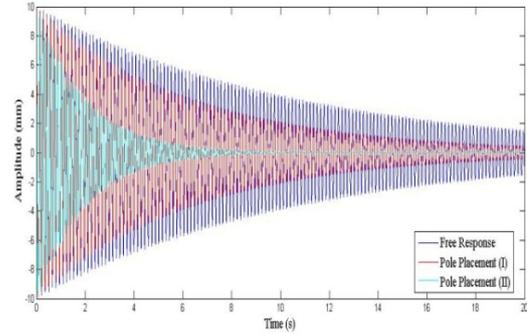

Figure 3 –Vibration of the cantilever beam with and without the controller

As you can see in Figure 4, we compare the influences of PID and PI controllers on the vibration of the beam. It is obvious that PI controller (the green line) has the most effective role. We can reduce the damping time by changing the gains of the controllers.
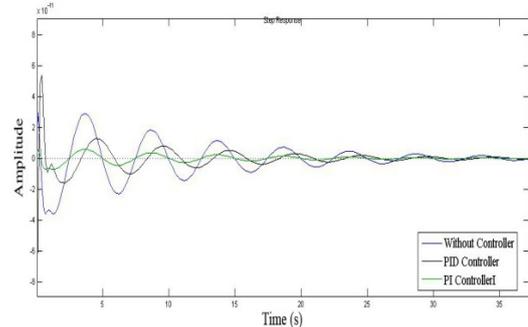

Figure 4 –Comparison between the influences of the PID and PI controllers on the system

V. CONCLUSION

In conclusion we can say that we are able to successfully suppress the vibration of the cantilever beam using 2 piezoelectric patches as a sensor and actuator. We found out the Pole Placement method is an applicable way to eliminate the vibration of the beam in a controlled manner. On the other hand it is possible to implement Classical controllers, such as PID and PI to control the amplitude and time of the vibration.

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