On the Study of the Active Tooth Surfaces' Geometry of the Spatial Rack Drives

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***Abstract*—In this study, a spatial rack system, that transforms a rotation motion into rectilinear translation by means of a cylindrical worm and a helical rack is considered. This type of motions' transformation is realized by the conjugated active flanks of the worm and the gear rack. This paper deals with the theoretical approach to their synthesis. It is based on the second principle of T. Olivier. The obtained equations of the surfaces of the spatial rack drive are of importance when the algorithms for the synthesis of this type of mechanism are elaborated.**

***Keywords*—synthesis, mathematical modeling, active tooth surfaces, geometry, spatial rack drives**

I. Introduction

Mechanical transmissions, designed to transform motions by means of a system of high kinematic joints, in accordance with a preliminary defined law and arbitrarily oriented in space vectors of the motions' velocities, are gear mechanisms. These types of three-link mechanisms are characterized by the greatest variety of types and complexity of their research. Among them, spatial rack mechanisms can be treated as a special case of the above-mentioned mechanisms, which realize motions' rotation between fixed crossed axes, when:

a) The number of teeth of one of the movable links is increased „ad infinitium", without increasing the number of the meshed tooth surfaces between the mated links;

b) the rotation axis of the above-mentioned link is displaced in infinity and its motion is transformed into rectilinear translation;

c) number of the teeth and the type of motion of the second movable link remain unchanged;

d) A rotating link with a finite number of teeth will be called a pinion, and the link with an endless number of teeth, realizing rectilinear translation, will be called a gear rack.

This study presents a mathematical model for the study of the active tooth surfaces' geometry of the spatial rack drives, in which the active tooth surfaces of the pinion are parts of cylindrical linear helicoids and the tooth surfaces of the gear rack are theoretically conjugated with active teeth surfaces of the pinion. The rotation axis of the pinion is non-orthogonal crossed with the direction of the rectilinear translation of the gear rack. When the analytical relations for the active tooth surfaces are obtained, they are of great importance for the geometrical and technological synthesis of the spatial rack set. They are essential for the design of the gears' cutting instruments (tools), and for the construction of control equipment, in the generation process of the tooth surfaces.

II. Literature Review

The existing publications in specialized literature, dedicated to the science of gearing theory [1–6] and those—that cover the science of geometric synthesis of gear mechanisms [7–12] consider the processes and mechanisms, which are related to the rotation transformations between crossed, parallel and intersected shafts. The spatial rack drive belongs to the transformers designed for a transformation of rotation into translation (and vice versa) with a preliminary defined transformation law. And the amount of publications considering such gear mechanisms that are called rack drives is negligible.

In most scientific sources, rack gearing is considered instrumental gearing when cylindrical involute gears (with straight or helical teeth) are elaborated. In this case, the movable link, which designation is to realize translation of the gear rack, is the cutting tool, and it is called an "instrumental tool rack" [13–15].

The kinematics of the plane rack mechanisms, which is studied in [16] and is titled "rack cylindrical mechanisms" is summarily described. There, the technological structure and geometrical parameters of the non-spatial rack mechanism and the cylindrical straight-teeth gears are examined. The algorithm of geometrical synthesis and technological limitations on the geometric parameters of these mechanical transmissions is defined. Another special type gear mechanism, realizing a variable function of motions transformation of type $R \leftrightarrow T$ (rotation into translation) is elaborated in [17]. The offered there scientific concept is illustrated by an introduced calculation, design, and manufacture of an experimental model. In publication [18] analysis of "Wildhaber - Novikov" is realized. In this work, a study of the tooth contact and calculation of Hertz contact strength is accomplished. Another publication, researching rack drives [19], as pure rolling rack and pinion mechanisms, is devoted to their geometric design, meshing simulation, and stress analysis.

In the publication [20] the main focus in studying the spatial rack mechanism is put on the design and analysis of the gear rack and pinion to reduce the weight of the components. There, the realized studies are realized in the SolidWorks CAD software application and detailed analysis is done in the simulation integration platform Ansys Workbench.

The performed studies in [21] are focused on the mechanism of a slotting spindle head, which motion is based on the rack-pinion gear drive. There, 3D models of gear rack and pinion are elaborated in the CAD KOMPAS, and analysis of the stress-strain state in the region of mesh is performed based on the finite element method.

In studies [22, 23] spatial rack set is considered a special case of spatial gear mechanism that transforms rotations between crossed axes. In other words, it can be considered that the spatial rack drive is obtained from the spatial gear drive by increasing the teeth' number of one gear to infinity without increasing the number of meshed teeth.

The gear that has a theoretically infinite number of teeth is called a *helical rack*, and the other gear with a limited number of teeth is called a *helical gear*. In general, the helical gear has a conic shape and the number of teeth is greater than six (see Fig. 1). If the number of teeth of the helical gear is less than six, then it transforms into a conic worm. As a result of this change in the spatial gear mechanism, the rotation axis of a helical rack is placed to infinity, and its rotation motion is transformed into rectilinear translation.

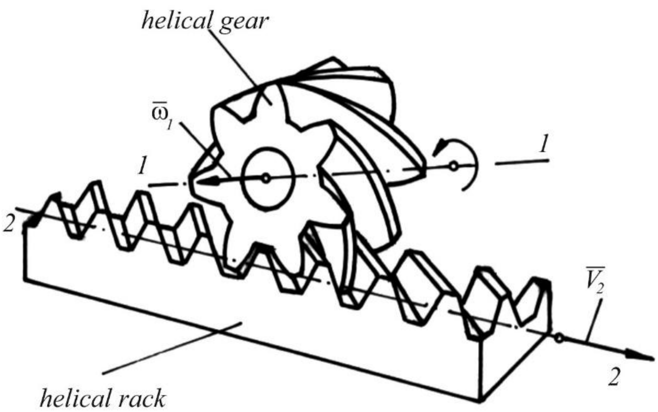

Fig. 1. Spatial rack drive.

A good knowledge of their specific kinematic and geometric characteristics should precede the process of mathematical modeling. This is the condition for the creation of adequate mathematical models for synthesis. Hence, the specific characteristics, which are at the base of the author's concepts for the synthesis of the spatial rack drive are presented.

The object of the study is spatial rack drive, which has a helical gear of a type of cylindrical worm (the angle determined by the conic form of the helical wheel is $\delta_1 = 0$). The active flanks Σ_1 of the cylindrical worm are linear helicoids [23] and the tooth surfaces Σ_2 of the helical rack are envelopes of Σ_1.

III. Materials and Methods

A. Basic Principles for the Generation of the Tooth Surfaces of Spatial Gear Mechanisms

Spatial gear mechanisms, including rack drives, obtain complex kinematic characteristics [1]. Hence, they determine the kinematic approach to their synthesis, and the kinematic character of the elaborated mathematical models, that describe the process of spatial motions transformation.

This approach should answer to two groups of questions:

- the geometry and dimensions of the gear blanks, composing the movable links of the high kinematic joints;
- the geometry and dimensions of the active tooth surfaces, representing the geometric elements of the kinematic joints (tooth meshing).

For this reason, the general structure of any mathematical model depends on:

a) the purpose (designation) of the gear set from a viewpoint of the motions transformation law;
b) the geometry and character of the conjugation of the surfaces (point or linear contact) by means of which the motions transformation is realized (these surfaces are active flanks);
c) the technological devices when the geometry of the tooth surfaces of the tool and the kinematics of the technological process for the generation of the active flanks are chosen.

The desired strength characteristics of the synthesized gear mechanisms depend on their design and are controlled by including adequate analytical quality criteria.

The realization of a given law of motions' transformation with the corresponding accuracy is specifically dependent on the geometry of the kinematically conjugate discrete surfaces which go in and out of contact. Achieving optimal kinematic accuracy in a transformation of motions directly corresponds to the geometric accuracy of the contacting tooth surfaces.

The method of enveloping is suitable when the task for conjugate tooth surfaces generation is treated. On its basis two basic principles are defined by the French geometer T. Olivier [3, 7]:

When forming tooth surfaces Σ_1 and Σ_2 firmly connected with bodies B_1 and B_2 by means of the generating (instrumental) surface Σ_J connected with the body B_J, the law of relative motions $|(\Omega_1)|/|(\Omega_J)|$ and $|(\Omega_2)|/|(\Omega_J)|$ could be arbitrary, respectively. Let's pay attention to the following two possibilities :

- Σ_1 and Σ_2 are one-parameter envelopes of Σ_J, where $\Sigma_i \neq \Sigma_J$, and $B_i \neq B_J$ ($i = 1,\ 2$). If B_1 and B_2 perform given motions and B_J moves in the fixed space according to a defined law, then Σ_J will generate Σ_1 and Σ_2 in the corresponding coordinate systems firmly connected with the bodies B_1 and B_2 as one-parameter envelopes. Thus, at every moment a linear contact between Σ_J and Σ_i ($i = 1,\ 2$) is present and the tooth surfaces Σ_1 and Σ_2 can have linear contact or point contact. (It is possible Σ_1 and Σ_2 to have no contact.) This corresponds to first Olivier's principle.
- Σ_2 is a one-parameter envelope of the instrumental surface Σ_J and the conditions $\Sigma_J = \Sigma_1$, $B_J = B_1$, $|(\Omega_J)|/|(\Omega_2)| = |(\Omega_1)|/|(\Omega_2)|$ are fulfilled. A linear contact between Σ_1 and Σ_2 is always present and this approach to the generation of the active tooth surfaces (flanks) of the gears is in accordance with Olivier's second principle.

The studied spatial rack set is generated in accordance with the second Olivier's principle. Two cases of instrumental gearing are possible for the studied spatial rack drive: $\Sigma_J = \Sigma_1$ or $\Sigma_J = \Sigma_2$. The active tooth surfaces Σ_1 of the helical racks are generated upon the first case, and based on the second—the surfaces Σ_2 of the helical gear.

B. Mathematical Model Elaboration, Oriented to the Synthesis of Spatial Rack Drive

Two approaches for mathematical modelling of the spatial tooth surfaces' synthesis are known.

- pitch contact point approach;
- mesh region approach.

The study is oriented to the application of the second approach for mathematical modeling for synthesis.

When synthesizing spatial gear mechanisms with linear contact, the necessity to control their quality characteristics throughout the mesh region is obvious. Such an approach to the synthesis task requires an adequate mathematical model, such as the mesh region mathematical model. The mathematical model based on the mesh region is not a universal one. The reasons are that the specific geometric and kinematic characteristics of the spatial rack drive mesh region depend on its position in the fixed space and the geometric characteristics of the tool (instrumental) surface $\Sigma_J = \Sigma_i$ (i =1 or 2), which generate the tooth flanks Σ_i (i =2 or 1). In Fig. 2, a kinematic scheme of a spatial rack set realizing a definite law of transformation of type rotation into translation ($R \leftrightarrow T$) by contacting in a line D_{12} tooth surfaces Σ_1 and Σ_2 is illustrated [7].

The tooth surface Σ_1, which belongs to link 1, rotates around the axis $1-1$ with an angular velocity $\overline{\omega}_1$, and Σ_2, which belongs to link 2, translates into direction $2-2$ with velocity $\overline{V}_2$. The axis of rotation motion $1-1$ and the translation direction $2-2$ are fixed, i.e., $\delta = \angle(\overline{\omega}_1, \overline{V}_2) = const$. The realized motions' transformation of this mechanism is characterized by the following kinematic (velocity) ratio:

$$j_{12} = \frac{\omega_1}{V_2} = const. \qquad (1)$$

where ω_1 is the magnitude of the angular velocity vector $\overline{\omega}_1$; V_2 - the magnitude of the translation velocity vector $\overline{V}_2$.

The kinematic scheme shown in Fig. 2 of the spatial rack drive is related to the treated mathematical model. The equations of tooth surfaces Σ_1 and Σ_2 are written by means of following the right-hand orthogonal coordinate systems: $S(O,x,y,z)$—firmly connected with mechanism posture; $S_1(O_1,x_1,y_1,z_1)$—firmly connected with the worm and $S_2(O_2,x_2,y_2,z_2)$—firmly connected with the helical rack. The application of the kinematic method for synthesis of the conjugate tooth surfaces Σ_1 and Σ_2 requires to define the meaning of *geometric* and *kinematic conjugation* of high kinematic joints applied as meshed tooth surfaces [1].

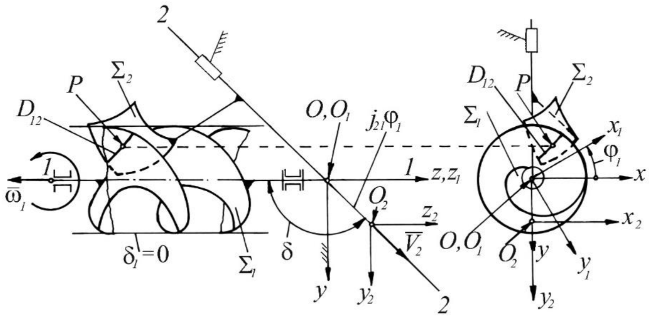

Fig. 2. Kinematic scheme of spatial rack set, with a cylindrical worm.

Geometric conjugation. We connect with this term the geometric elements of the kinematic joint, namely surfaces, lines, and points. We call a conjugate contact point (a point of tangent contact) a common point between two surfaces, which have a common tangent plane in this point. A conjugate contact line (a line of tangent contact) is the set of conjugate contact points of two surfaces. Conjugate surfaces (surfaces with a tangent contact) are two surfaces, which are the locus of conjugate contact points and at least one common point.

Kinematic conjugation. This conjugation is connected with kinematic joints that are presented in the mechanism. Kinematically conjugate joints ($\Sigma_1 : \Sigma_2$) are such joints, in which geometric elements (Σ_1 and Σ_2) are geometrically conjugated in a definite interval of time, i.e. for an arbitrary value of a generalized coordinate belonging to a given definition set.

The kinematic conjugation of the joint $\Sigma_1 : \Sigma_2$ requires their geometric elements (i.e., the tooth surfaces Σ_1 and Σ_2) to be analytically described by continuous functions and to have continuous derivatives in first order [1]. Additionally, it is required Σ_1 and Σ_2 to have one conjugate point at least. The kinematic conjugation of the joints $\Sigma_1 : \Sigma_2$ suppose the transformation of motions from the geometric element Σ_1 to the geometric element Σ_2. In this case of spatial rack drive, the following specific limitations concerning tooth surfaces Σ_1 and Σ_2 are put: to present the contact at one conjugate point P at a given moment as a contract between two infinitely small areas of Σ_1 and Σ_2. The point P lies in a common tangent plane to the Σ_1 and Σ_2. It is obvious, that these areas have a relative motion which is realized by the velocity vector $\overline{V}_{12} = \overline{V}_1 - \overline{V}_2$. $\overline{V}_{12}$ is placed in the tangent plane, i.e.:

$$\overline{n}_i \overline{V}_{12} = 0, \qquad (2)$$

where $\overline{n}_i$ is a normal vector to the Σ_i ($i = 1,\ 2$). The relation Eq. (2) is known as a *basic equation of meshing* [7].

The relative velocity $\overline{V}_{12}$ in every common contact point P from the contact line D_{12} of Σ_1 and Σ_2 in the coordinate system $S(O,x,y,z)$ is

$$\overline{V}_{12} = y\bar{i} - (j_{21}\sin\delta + x)\bar{j} + j_{21}\cos\delta\bar{k} \qquad (3)$$

The vector Eq. (3) is written when $\omega_1 = 1$ rad/sec and $V_2 = 1/j_{12} = j_{21}$, respectively.

C. Synthesis of the Active Flanks Gear-Worm and Helical Rack

1) Geometry of a linear helicoid

In Fig. 3 a case of the generation of right-hand cylindrical linear convolute helicoids $\Sigma^{(j)}$ ($j = 1,\ 2$) is illustrated. The generation process of these helical surfaces is examined in the coordinate system $S_1^{(j)}(O_1^{(j)}, x_1^{(j)}, y_1^{(j)}, z_1^{(j)})$. The generatrix line $L^{(j)}$ doesn't intersect the axis $O_1^{(j)}z_1^{(j)}$, which coincides with the geometric axis of the worm. The angle $0.5 < \xi^{(j)} < \pi$ is between $L^{(j)}$ and the direction of the axis $O_1^{(j)}z_1^{(j)}$ (geometric axis of the worm). Also $L^{(j)}$ belongs to the plane $T^{(j)}$, which is tangential to the directed circled cylinder $C^{(j)}$.

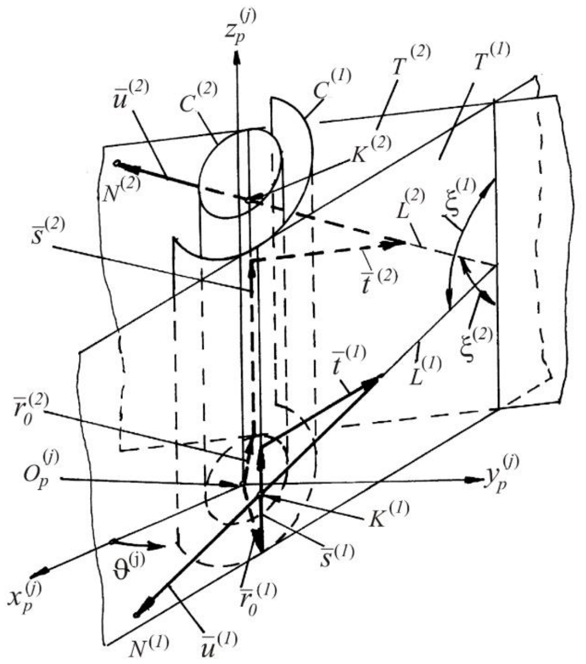

Fig. 3. Scheme of linear helicoid generation.

The cylindrical helicoid $\Sigma^{(j)}$ ($j = 1,\ 2$) is generated by $L^{(j)}$, that performs a helical motion along the axis $O_1^{(j)}z_1^{(j)}$ with a parameter $p_s^{(j)} = const.$ [7, 24]. $\Sigma_1^{(j)}$ $(j = 1)$ is a cylindrical linear helical surface, which is oriented to the positive direction of the axis $O_1^{(1)}z_1^{(1)}$, and $\Sigma_1^{(j)}$ $(j = 2)$ is a helicoid, which is turned to the negative direction of the $O_1^{(2)}z_1^{(2)}$.

Areas(particles) of $\Sigma_1^{(1)}$ and $\Sigma_1^{(2)}$ are used as active surfaces of the teeth of a cylindrical worm.

The vector equation of the linear helical surface $\Sigma_1^{(j)}$, according to Fig. 3 is

$$\overline{\rho}_1^{(j)} = \overline{r}_0^{(j)} + \overline{s}^{(j)} + \overline{u}^{(j)}, \qquad (4)$$

where $\overline{\rho}_1^{(j)}$ is a radius - vector of point $N^{(j)}$ that belongs to the linear helicoid; $\overline{r}_0^{(j)}$ - radius-vector of the directed cylinder; $u^{(j)}$, $\vartheta^{(j)}$ - curvilinear coordinates of the helicoids; $s^{(j)} = p_s^{(j)}\vartheta^{(j)}$ - axial location displacement of the generatrix $L^{(j)}$.

In the coordinate system $S_1^{(j)}$ for Eq. (4) it is obtained:

$$\begin{aligned} x_1^{(j)} &= r_0^{(j)}\cos\vartheta^{(j)} \pm u^{(j)}\sin\xi^{(j)}\sin\vartheta^{(j)}, \\ y_1^{(j)} &= r_0^{(j)}\sin\vartheta^{(j)} \mp u^{(j)}\sin\xi^{(j)}\cos\vartheta^{(j)}, \\ z_1^{(j)} &= p_s^{(j)}\vartheta^{(j)} \pm u^{(j)}\cos\xi^{(j)}. \end{aligned} \qquad (5)$$

After the substitution of expressions Eq. (6) into Eq. (5),

$$U^{(j)} = u^{(j)}\sin\xi^{(j)}. \qquad (6)$$

the following equation systems are written:

$$\begin{aligned} x_1^{(j)} &= r_0^{(j)}\cos\vartheta^{(j)} \pm U^{(j)}\sin\vartheta^{(j)}, \\ y_1^{(j)} &= r_0^{(j)}\sin\vartheta^{(j)} \mp U^{(j)}\cos\vartheta^{(j)}, \\ z_1^{(j)} &= p_s^{(j)}\vartheta^{(j)} \pm U^{(j)}\cot\xi^{(j)}. \end{aligned} \qquad (7)$$

Eqs. (5) and (7) represent *cylindrical linear surfaces* $\Sigma_1^{(j)}$, with helical parameter $p_s^{(j)}$ and curvilinear coordinates $u^{(j)}$, $\vartheta^{(j)}$ and $U^{(j)}$ and $\vartheta^{(j)}$, respectively.

Let, Eq. (4) is written in the following type [24]:

$$\overline{\rho}_1^{(j)} = \overline{\rho}_0^{(j)}(\overline{\vartheta}^{(j)}) + U^{(j)}\bar{l}^{(j)},$$

$$\begin{Vmatrix} \rho_{0,x_1}^{(j)} \\ \rho_{0,y_1}^{(j)} \\ \rho_{0,z_1}^{(j)} \end{Vmatrix} = \begin{Vmatrix} r_0^{(j)}\cos\vartheta^{(j)} \\ r_0^{(j)}\sin\vartheta^{(j)} \\ p_s^{(j)}\vartheta^{(j)} \end{Vmatrix}, \qquad (8)$$

$$\begin{Vmatrix} l_{x_1}^{(j)} \\ l_{y_1}^{(j)} \\ l_{z_1}^{(j)} \end{Vmatrix} = \begin{Vmatrix} \pm\sin\xi^{(j)}\cos\vartheta^{(j)} \\ \mp\cos\xi^{(j)}\cos\vartheta^{(j)} \\ \pm\cos\xi^{(j)} \end{Vmatrix},$$

where $\bar{l}^{(j)}$ is a direction vector of $L^{(j)}$.

Then for the distribution parameter of cylindrical convolute helicoid $\Sigma_1^{(j)}$ it can be written:

$$\bar{h}_1^{(j)} = \frac{[d\overline{p}_0^{(j)},\ \bar{l}^{(j)},\ d\bar{l}^{(j)}]}{d\bar{l}^{(j)^2}}. \qquad (9)$$

After a transformation from Eqs. (9) and (8), it is obtained:

$$h_1^{(j)} = p_s^{(j)} + r_0^{(j)} \cot\xi^{(j)}. \quad (10)$$

When Eqs. (5) and (7) describe the geometry of the cylindrical convolute helicoid, then $h_1^{(j)} \neq 0$.

If $h_1^{(j)} = 0$, i.e.

$$\cot\xi^{(j)} = -\frac{p_s^{(j)}}{r_0^{(j)}}. \quad (11)$$

then $\Sigma_1^{(j)}$ transforms into an *involute helicoid*.

When $\bar{r}_0^{(j)} = \bar{0}$ is substituted in Eq. (4), then the systems of Eqs. (5) and (7) are of the form:

$$\begin{aligned} x_1^{(j)} &= \pm u^{(j)} \sin\xi^{(j)} \sin\vartheta^{(j)}, \\ y_1^{(j)} &= \mp u^{(j)} \sin\xi^{(j)} \cos\vartheta^{(j)}, \\ z_1^{(j)} &= p_s^{(j)}\vartheta^{(j)} \pm u^{(j)} \cos\xi^{(j)} \end{aligned} \quad (12)$$

$$\begin{aligned} x_1^{(j)} &= \pm U^{(j)} \sin\vartheta^{(j)}, \\ y_1^{(j)} &= \mp U^{(j)} \cos\vartheta^{(j)}, \\ z_1^{(j)} &= p_s^{(j)}\vartheta^{(j)} \pm U^{(j)} \cot\xi^{(j)}. \end{aligned} \quad (13)$$

Eqs. (12) and (13) are equations *of an Archimedean helicoid*. For the Archimedean helicoid is fulfilled:

$$h_1^{(j)} = p_s^{(j)}. \quad (14)$$

$j = 1$, and upper signs in all equations and in further equations are referred to the active surface $\Sigma_1^{(1)}$ which rotates with the angular velocity $\bar{\omega}_1$ and the corresponding translation velocity of $\Sigma_2^{(1)}$ is $\bar{V}_2$ (see Fig. 2); $j = 2$ and the lower signs are referred to those meshed surfaces $\Sigma_1^{(2)}$ and $\Sigma_2^{(2)}$ which velocities are ($-\bar{\omega}_1$) and ($-\bar{V}_2$).

2) Geometry of a helical rack flanks

In the case of spatial rack drive, the equations of the helical rack tooth surfaces $\Sigma_2^{(j)}$ ($j = 1,\ 2$) are written by using the equations of their kinematic conjugate linear helicoids $\Sigma_1^{(j)}$ ($j = 1,\ 2$) that belong to the cylindrical worm. In this case, $\Sigma_2^{(j)}$ ($j = 1,\ 2$) are envelopes of the defined helicoids $\Sigma_1^{(j)}$ ($j = 1,\ 2$). The analytical description of this statement is defined as Eq. (2), i.e.:

$$\begin{aligned} &\bar{n}_1^{(j)} \bar{V}_{12} = n_{1,x}^{(j)} V_{12,x} + n_{1,y}^{(j)} V_{12,y} + n_{1,z}^{(j)} V_{12,z} = \\ &= f(u^{(j)}, \vartheta^{(j)}, \varphi_1), \\ &\bar{n}_1^{(j)} = \frac{\partial \bar{\rho}_1^{(j)}}{\partial u_1^{(j)}} \times \frac{\partial \bar{\rho}_1^{(j)}}{\partial \vartheta_1^{(j)}}. \end{aligned} \quad (15)$$

Here, $\bar{n}_1^{(j)}$ is the normal vector to the $\Sigma_1^{(j)}$; $\bar{V}_{12}$ is the defined with equation (3) relative velocity vector between $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in arbitrary contact point P (see Fig. 2); φ_1 is the parameter of meshing.

By using the kinematic method [7], the equations of the mesh region are determined, as a locus of the contact lines of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ in the fixed space $S(O,x,y,z)$. Let for cases of the linear helicoid, $\bar{n}_1^{(j)}$ is written in the coordinate system $S_1^{(j)}$. Then from the equations systems Eqs. (15) and (7), the normal vectors of these surfaces for the linear helicoid are obtained as:

$$\begin{aligned} n_{1,x_1}^{(j)} &= \pm h_1^{(j)} \sin\xi^{(j)} \cos\vartheta^{(j)} - \\ &- U^{(j)} \cos\xi^{(j)} \sin\vartheta^{(j)}, \\ n_{1,y_1}^{(j)} &= \mp h_1^{(j)} \sin\xi^{(j)} \sin\vartheta^{(j)} + \\ &+ U^{(j)} \cos\xi^{(j)} \cos\vartheta^{(j)}, \\ n_{1,z_1}^{(j)} &= U^{(j)} \sin\xi^{(j)}. \end{aligned} \quad (16)$$

When using the transition matrices from a system $S_1^{(j)}$ to S:

$$M_{SS_1} = \begin{Vmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix}, \quad L_{SS_1} = \begin{Vmatrix} \cos\varphi_1 & \sin\varphi_1 & 0 \\ -\sin\varphi_1 & \cos\varphi_1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}. \quad (17)$$

For the equations of the mesh region of the spatial rack set is received:

$$\begin{aligned} &y^{(j)} n_{1,x}^{(j)} - (x + j_{21}\sin\delta) n_{1,y}^{(j)} + j_{21}\cos\delta n_{1,z}^{(j)} = 0, \\ &x^{(j)} = r_0^{(j)} \cos(A) \pm U^{(j)} \sin(A), \\ &y^{(j)} = r_0^{(j)} \sin(A) \mp U^{(j)} \cos(A), \\ &z^{(j)} = p_s^{(j)}\vartheta^{(j)} \pm U^{(j)} \cot\vartheta^{(j)}, \end{aligned} \quad (18)$$

$$\begin{aligned} n_{1,x}^{(j)} &= \mp h_1^{(j)} \sin\xi^{(j)} \cos(A) - U^{(j)} \cos\xi^{(j)} \sin(A), \\ n_{1,y}^{(j)} &= \mp h_1^{(j)} \sin\xi^{(j)} \sin(A) + U^{(j)} \cos\xi^{(j)} \cos(A), \\ n_{1,z}^{(j)} &= U^{(j)} \sin\xi^{(j)}, \end{aligned}$$

$$A = \vartheta^{(j)} - \varphi_1.$$

When $h_1^{(j)} \neq 0$, the Eq. (18) define the mesh region of the spatial convolute rack set. If $h_1^{(j)} = 0$, then Eq. (18) define the mesh region of the spatial involute rack set, and if $h_1^{(j)} = p_s^{(j)}$.

IV. Result and Discussion

For the complete and full study of every gear mechanism (including spatial rack drives), it is necessary for the process of synthesis and design of these mechanisms to be accompanied by corresponding software assurance. It will ensure the correctness of the elaborated mathematical models (like eliminating the undercutting points from the mesh surfaces of the synthesis gears, visualization of the active tooth surfaces and mesh region), and improvement of the algorithmic and computational approaches as well. The creation of such programs contributes to the process of the design of new and/or improved gear mechanisms.

For this reason, computer programs with similar structures and organization of the calculation process are written. One big part of the elaborated algorithms and computer programs is oriented to examine the nature of the active tooth surfaces and especially their geometric features, and their visualization as well. In this concrete study, the algorithmic flow of the created programs is related to the:

-calculation process of the basic parameters of the directed cylinder:

-selecting the proper coordinates of the helicoid;

-proper determination of the variation of the parameter of meshing;

-elimination of the undercutting points.

-software illustration of the active tooth surfaces $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$, which are connected with the gear and the rack correspondingly, and visualization of the mesh region as well.

In this study, the graphics in Figs. 4 and 5 show the results of the created programs for the synthesis and analysis of spatial rack drives with cylindrical linear rotating helicoids (cylindrical convolute and Archimedean drive). The results are obtained for the velocity ratio $j_{21} = 3$.

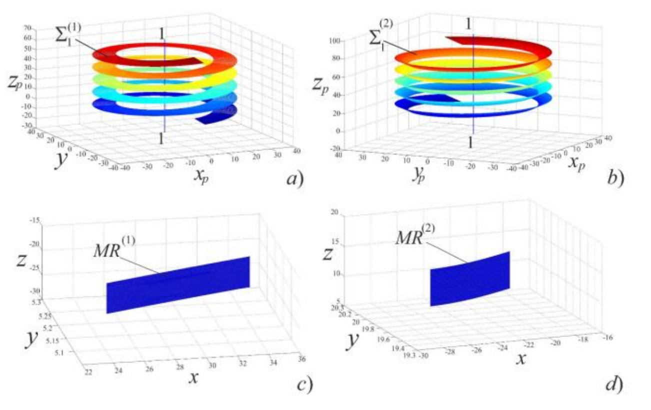

Fig. 4. Spatial cylindrical convolute rack drive with velocity ratio $j_{21} = 3$; number of the teeth $z_1 = 1$: a) cylindrical convolute right-handed helicoid $\Sigma_1^{(1)} \Rightarrow \xi^{(1)} = 95^\circ$; $r_0^{(1)} = 4{,}27$; b) cylindrical convolute right-handed helicoid $\Sigma_1^{(2)} \Rightarrow \xi^{(2)} = 125^\circ$; $r_0^{(2)} = 21.64$ c) mesh region $MR^{(1)}$; d) mesh region $MR^{(2)}$.

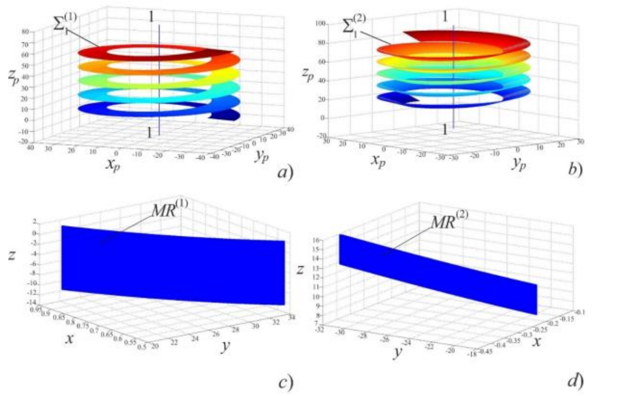

Fig. 5. Spatial cylindrical Archimedean drive with velocity ratio $j_{21} = 3$; number of the teeth $z_1 = 1$: a) cylindrical Archimedean right-handed helicoid $\Sigma_1^{(1)} \Rightarrow \xi^{(1)} = 95^\circ$; b) cylindrical Archimedean right-handed helicoid $\Sigma_1^{(2)} \Rightarrow \xi^{(2)} = 125^\circ$, c) mesh region $MR^{(1)}$; d) mesh region $MR^{(2)}$.

The aim of the author is to present a progressive class of spatial mechanical transmissions, little known and applied to realize the spatial transformation of "rotation and translation" movements and vice versa. The geometrical and kinematical similarity of these types of transmissions with hyperboloid gears is specifically defined and as a result, the mathematical modeling approach for mesh field synthesis according to Olivier's second method applicable to cross-axis gears is justified. This study illustrates the mathematical model for the synthesis of the active flank gear-worm and helical rack of the Archimedean cylindrical rack mechanism. As a result of the obtained mathematical relations and developed program here is illustrated two types of cylindrical linear helicoids. This study gives a premise that these types of special mechanisms, which have specially studied the geometry of the active flanks are suitable for implementation as actuators in various fields of techniques. Of particular interest is their incorporation into the constructions of bio-robots [23], as an alternative to spatial hyperboloid gears [24].

V. Conclusion

The equations of the active tooth surfaces of spatial convolute, involute, and Archimedean rack sets obtained in the article represent the base of the algorithm for synthesis and analysis of the spatial rack sets when a helical gear is a linear cylindrical worm. The obtained mathematical relations are on the basis of the elaborated programs for the visualization of spatial rack drives with cylindrical linear rotating helicoids. These analytical relations are very important for the geometrical and technological synthesis of the mechanisms. They have an essential place in the design of the tools for gear generation (cutting), and for the construction of control equipment, in the generation process of the tooth surfaces. When synthesizing gear mechanisms of this type it is extremely important to eliminate the singular points in their mesh region. Besides, small parts of $\Sigma_1^{(j)}$ and $\Sigma_2^{(j)}$ are constrained in the process of design, so the singular points should be eliminated. The written relations in this study are also oriented toward solving the problem for the elimination of undercutting points in the mesh region of the spatial rack drives.

CONFLICT OF INTEREST

The author declares no conflict of interest.

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