Comparison of GUM and Monte Carlo Methods for the Measurement Uncertainty Circular Runout Error of Shafts

Pornpawit Ounjutturaporn, Ramil Kesvarakul, Pipitanon Poonsawat, and Khompee Limpadapun

***Abstract*—Measurement uncertainty is one of the most important concepts. The ISO IEC 17025:2005 standard: describes harmonized policies and procedures for testing and calibration laboratories. Guide to the expression of uncertainty in measurement (GUM) is a direct uncertainty analysis method, which calculates the combined standard uncertainty and expanded uncertainty by law of propagation of uncertainty. Monte Carlo Method (MCM) as presented by the (GUM S1) involves the propagation of the distributions of the input sources of uncertainty by using a model to provide the distribution of the output. By random sampling, the probability density function of the input quantities. In this paper, present measurement uncertainty to circular runout error. By use shaft standard with a diameter of 32 mm., length 100 mm. From the experiment results, Comparison of GUM and MCM showed no differences. The cases the estimated uncertainty using the GUM approach slightly overestimated the results obtained with the MCM. However, the use of numerical methods such MCM as a valuable alternative to the GUM approach. The practical use of MCM it has proven to be a fundamental tool, being able to address more complex measurement problems that were limited by the GUM approximations.**

***Index Terms*—Circular runout error, Guide to the expression of Uncertainty in Measurement (GUM), Monte Carlo Method (MCM), measurement uncertainty.**

I. Introduction

Metrology is the science that covers theoretical and practical concepts involved in a measurement, which when applied are able to provide results with appropriate accuracy and metrological reliability to a given measurement process.

The ISO IEC 17025:2005 standard [1], describes harmonized policies and procedures for testing and calibration laboratories. The GUM (JCGM 100) provides guidelines on the estimation of uncertainty in measurement [2], [3]. The GUM S1 (JCGM 101) is responsible to give practical guidance on the application of MCM to the estimation of uncertainty [4].

Measurement uncertainty is a quantitative indication of the quality of measurement results, without which they could not be compared between themselves, with specified reference values or to a standard. This document provides a full set of tools to treat different situations and processes of measurement. Estimation of uncertainty, as presented by the GUM, is based on the law of propagation of uncertainty.

Due to these limitations of the GUM, the use of MCM for the propagation of the full probability distributions has been addressed in the GUM S1 provides on the application of MCM to metrological situations [5].

There are many researches on the circular runout, roundness error, error of spindle rotation and tilt error of the installation axis. Mao *et al.* [6], use uncertainty evaluation method based on GUM. Although GUM is the standard for the evaluation of the measurement uncertainty in metrology, it is mainly applicable to the linear models. GUM S1 provides a general numerical approach implemented by MCM, and it is consistent with the broad principles of GUM. Cox *et al.* [7], define the GUM as an approximation method for the evaluation of uncertainty and explain that an MCM is an effective. Matus [8], use MCM to evaluate the measurement uncertainty of gauge blocks, whereas GUM cannot be applied to this problem. Moschioni *et al.* [9], proposed a method that combined the factorial design of experiments and MCM to guide the instrument designer in the instrument configuration optimization. Kruth *et al.* [10], presented a method to determine measurement uncertainties for feature measurements on CMM based on MCM and a profile database of realistic form profiles. Lian and Chen [11], proposed the uncertainty evaluation of roundness measurement based on MCM, but the simulation trials need to be set in advance. Couto *et al.* [5], described four cases with this method and recommended it for more complex measurement problems that could not be solved by GUM. Chew and Walczyk [12], demonstrated that a standard spreadsheet software program, such as Microsoft Excel, could be used to estimate measurement uncertainty by the MCM. Yang *et al.* [13], estimate the uncertainty of task-specific laser tracker measurements by using the GUM and MCM, a case study involving the uncertainty estimation of a cylindrical measurement process was illustrated. The results indicate that the MCM is a practical tool for applying the principle of propagation of distributions and does not depend on the assumptions and limitations required by the law of propagation of uncertainty [14].

In this paper, we present a comparative study of measurement uncertainty to circular runout error. The first method uses the universal GUM approach. That has been developed according to the guidelines which takes into account information of uncertainty, while the second method uses MCM numerical data to examine the factors affecting measurement uncertainty.

II. Circular Runout Error Methodology

Circular runout value is the number of attributes or

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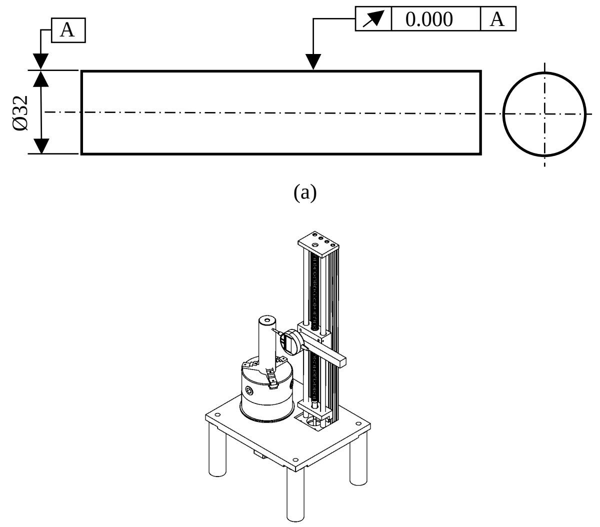
attributes referenced as shown in Fig. 1(a), which vary to each datum when the part rotated 360° around the datum axis. It is a basically controlling the circular feature and spindle variation. Measurement Circular runout is measured by using a simple dial gauge on the reference surfaces. The datum axis is controlled by all fixing datum points and rotating the central datum axis. The part is usually constrained with V-blocks, or spindle, on each datum that required to be controlled. The part is then rotated around this axis, and the variation measured by using the dial gauge perpendicular to the part surface as shown in Fig. 1(b).



(a)

(b)

Fig. 1. (a) circular runout; (b) A circular runout in measurement system.

III. The Methods of GUM and MCM

A Uncertainty Evaluation by GUM

GUM is a direct uncertainty analysis method, which calculates the combined standard uncertainty and expanded uncertainty by law of propagation of uncertainty. It is a reasoning-based method. It is firstly analyzing the source of measurement uncertainty, which depends on the measurement method, condition, equipment and the understanding of the measured quantity value, and then evaluates the standard uncertainty components [5]. The steps are as follows:

1) Definition of the measured and input sources

Measured y as a function of four different input sources: x_1, x_2 and x_3 equation shown in (1).

$$y = f(x_1, x_2, x_3) \tag{1}$$

2) Modeling

The measurement procedure should be modeled in order to have the measured as a result of all the input sources equation shown in (2).

$$y = \frac{x_1 + x_2 + x_3}{n} \tag{2}$$

3) Estimation of the uncertainties of input sources

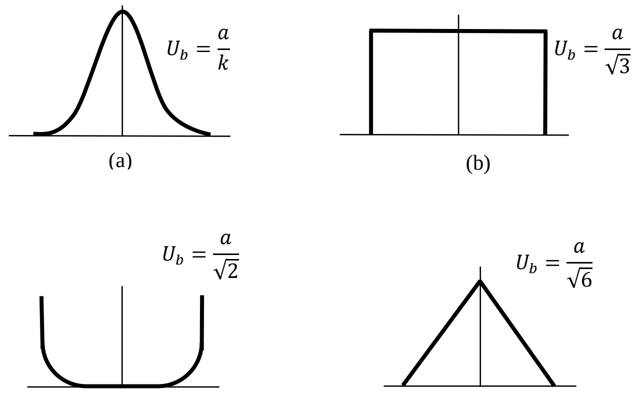
Type A uncertainties from repeatability studies are estimated by the GUM as the standard deviation of the mean obtained from the repeated measurements equation shown in (3 and 4).

$$s(x) = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}} \tag{3}$$

$$U_a = s(\bar{x}) = \frac{s(x)}{\sqrt{n}} \tag{4}$$

where $(\bar{x})$ is the mean value of the repeated measurements, $s(x)$ is its standard deviation, and $s(\bar{x})$ is the standard deviation of the mean.

Type B which are determined from any other source of information, such as instrument specification, calibration certificate, material certificate, accuracy, etc. In the uncertainty form Type B evaluations, it is required to choose a probability distribution that models each source of variability. The most common probability distributions used in uncertainty analysis. You should be able to identify which probability distributions you should use and how to reduce your uncertainty contributors to standard deviation equivalents. The most commonly used probability distributions for estimating measurement uncertainty are [15]; The Normal distribution is a function that represents the distribution of many random variables as a symmetrical bell-shaped graph where the peak is centered about the mean and is symmetrically distributed in accordance with the standard deviation, such as: Repeatability or Calibration Certificate as shown in Fig. 2(a). The Rectangular Distribution is a function that represents a continuous uniform distribution and constant probability. In a rectangular distribution, all outcomes are equally likely to occur, such as: Drift, Resolution or Accuracy as shown in Fig. 2 (b). The U-shaped Distribution is a function that represents outcomes that are most likely to occur at the extremes of the range. The distribution forms the shape of the letter 'U,' but does not necessarily have to be symmetrical. The U-shaped distribution is helpful where events frequently occur at the extremes of the range, such as: Electricity, Energy or Frequency as shown in Fig. 2 (c). The Triangle Distribution is a function that represents a known minimum, maximum, and estimated central value such as Temperature as shown in as shown in Fig. 2 (d).



(a) (b)

(c) (d)

Fig. 2. Sources of variability (a) Normal Distribution, (b) Rectangular Distribution, (c) U-Shaped Distribution and (d) Triangle Distribution.

4) Combined standard uncertainty

The GUM uncertainty is based on the law of propagation of uncertainties. This methodology is derived from a set of approximations to simplify the calculations and is valid for a wide range of model equation shown in (5).

$$U_c = \sqrt{U_a^2 + U_{b1}^2 + U_{b2}^2 + \cdots + U_{bn}^2} \quad (5)$$

where U_c is the combined standard uncertainty for the measured U_a and U_{bi} is the uncertainty for the i input quantity.

5) Expanded uncertainty (U_E)

The result provided by Equation 4 corresponds to an interval that contains only one standard deviation. In order to have a better level of confidence for the result, the GUM approach expands this interval by assuming t-distribution for the measured. The effective degrees of freedom v_{eff} for the t-distribution can be estimated by using the Welch-Satterthwaite equation shown in (6).

$$v_{eff} = \frac{U_c^4}{\sum_{i=1}^{N} \frac{U_{x_i}^4}{v_{x_i}}} \quad (6)$$

The expanded uncertainty is then evaluated by multiplying the combined standard uncertainty by a coverage factor k that expands it to a coverage interval delimited by a t-distribution with a chosen level of confidence equation shown in (7).

$$U_E = k * U_c \quad (7)$$

Report of measurement results. The circular runout, measurement quantities and extended uncertainty are reported in $y \pm U_E$ terms, followed by the confidence level.

B Uncertainty Evaluation by Monte Carlo

The Monte Carlo methodology as presented by the GUM S1 involves the propagation of the distributions of the input sources of uncertainty by using the model to provide the distribution of the output. By random sampling, the probability density function of the input quantities [5]. MCM is applicable to a measurement model with multiple input quantities or single output quantity. In order to achieve random sampling of the probability density function. This method can be used as a guide to verify the distribution of uncertainty by comparing the results. The step for MCM to evaluate uncertainty are as follows:

Step 1: Definition of the measured and input quantities.

Step 2: Modeling.

Step 3: The selection of the most appropriate probability density functions for each of the input quantities.

Step 4: Setup and run the MCM the greater the number of simulation trials, the greater the MCM Applied to Uncertainty in Measurement convergence of the results. This number can be chosen a priori or by using an adaptive methodology. When choosing a priori trials, the GUM S1 recommends the selection of a number M of trials, according to the following general rule, in order to provide a reasonable representation of the expected result equation shown in (8).

$$M > \frac{10^4}{1-p} \quad (8)$$

The numerical tolerance of an uncertainty, or standard deviation, can be obtained by expressing the standard uncertainty as $c \times 10^l$, where c is an integer with a number of digits equal to the number of significant digits of the standard uncertainty and l is an integer. Then the numerical tolerance δ is expressed equation shown in (9).

$$\delta = \frac{1}{2} 10^l \quad (9)$$

Step 5. The last stage is to summarize and express the results. According to the GUM S1, the following parameters should be reported as results:

- An estimate of the output quantity, taken as the average of the values generated for it
- The standard uncertainty, taken as the standard deviation of these generated values
- The chosen coverage probability (usually 95%)
- The endpoints corresponding to the selected coverage interval

C Validation of the GUM by MCM

GUM S1 put forward the MCM to give a method to validate the applicability of GUM method, that is, when MCM and GUM use the confidence interval at the same coverage probability [14]. This indicates that GUM has passed validation. Its execution steps are as follows:

Step 1: The left and right endpoints values y - U_E and y + U_E of the confidence interval (coverage probability p) by GUM.

Step 2: The left and right endpoints values d_{low} and d_{high} of the confidence interval (coverage probability p) by MCM.

Step 3: Calculate the deviations d_{low} and d_{high} at the endpoints of the confidence interval. d_{low} is the absolute value of the difference between the left endpoints of the coverage intervals provided. And d_{high} is the absolute value of the difference between the right endpoints of the coverage intervals provided. By the GUM uncertainty and an MCM.

Step 4: Determine the values of d_{low} and d_{high} with the numerical appropriate tolerance value, if the value is less, the validation is respected equation shown in (10) ~ (11).

$$d_{low} = |y - U_E - y_{low}| \quad (10)$$

$$d_{high} = |y + U_E - y_{high}| \quad (11)$$

where y is the measured estimate, U_E is the expanded uncertainty obtained by the GUM approach and y_{low} and y_{high} are the low and high endpoints obtained by the MCM for a given coverage probability.

IV. RESULTS AND DISCUSSION

In experiment, Shaft standard with a diameter of 32 mm., length 100 mm. Holding the shaft to the reference plane, then measured many times with dial gauge in the same environment condition. Rotate the shaft to find out the runout of the different errors. As shown in Table I. (testing runout 40 times).

TABLE I. EVALUATION RESULTS OF FORTY TIMES MEASUREMENT DATA

number	rotation angles	measurement error [mm]	number	rotation angles	measurement error [mm]
1	0	0.000	21	180	-0.014
2	9	0.001	22	189	-0.003
3	18	0.007	23	198	0.000
4	27	0.007	24	207	0.001
5	36	0.005	25	216	0.001
6	45	0.003	26	225	-0.001
7	54	0.001	27	234	-0.006
8	63	-0.001	28	243	-0.006
9	72	0.005	29	252	0.005
10	81	0.007	30	261	0.003
11	90	0.009	31	270	0.005
12	99	0.009	32	279	0.005
13	108	0.007	33	288	-0.005

14	117	-0.001	34	297	0.003
15	126	-0.002	35	306	0.007
16	135	0.001	36	315	0.011
17	144	0.005	37	324	0.013
18	153	0.005	38	333	0.015
19	162	0.005	39	342	0.017
20	171	0.002	40	351	0.009

A. Uncertainty of GUM

- Results analysis of measurements
  Average deviation Eq. (2)

$$\bar{x} = 0.0034\ mm.$$

Standard deviation Eq. (3)

$$sd = 0.005952\ mm.$$

- Contributions from Type A Evaluations Eq. (4)

$$U_1 = 0.000941 \text{ mm.}$$

- Contributions from Type B Evaluations Eq. (4)

Dial gauge Accuracy ±0.001 mm

$$U_2 = 0.000577 \text{ mm.}$$

Dial gauge Resolution ± 0.001 mm.

$$U_3 = 0.000866\ mm.$$

- The Combined Standard Uncertainty Eq. (5)

$$U_c = 0.001403\ mm.$$

- The Expanded Uncertainty Eq. (7)

$$U_E = 1.96 * 0.001403 = 0.002739\ mm.$$

- Report of measurement results.

The uncertainty of the circular runout a shaft is 32.0034±0.002749 mm. at Confidence 95%.

B. Uncertainty of MCM

MCM was set to run 200,000 trials of the proposed model, using the described input sources. The final histogram representing the possible values for the real efficiency of the cell, as shown on Fig. 3. Table II shows the statistical parameters obtained corresponding to the histogram. The low and high endpoints represent the 95% coverage interval for the final efficiency result of 32.0001 mm.

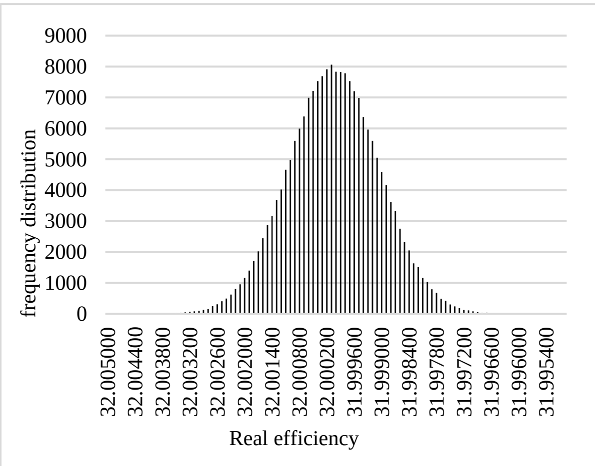


Fig. 3. The frequency distribution of circular runout error from the MCM.

TABLE II: STATISTICAL PARAMETERS OBTAINED FOR THE MONTE CARLO SIMULATION OF THE CIRCULAR RUNOUT MEASUREMENT MODEL

Parameter	Value
Mean	32.000100
Standard deviation	0.001402
d_{Low}	31.998698
d_{High}	32.001502

Once more a comparison with the GUM approach is done and the results obtained by this methodology are shown on Table III, for a coverage probability of 95%.

TABLE III: RESULTS OBTAINED FOR THE CIRCULAR RUNOUT MODEL USING THE GUM UNCERTAINTY APPROACH, WITH A COVERAGE PROBABILITY OF 95%

Parameter	Value
Combined standard uncertainty	0.001403
Effective degrees of freedom (V_{eff})	∞
Coverage factor (k)	1.96
Expanded uncertainty	0.002739

In this situation, d_{low} = 1.3 × 10^{-3} mm. and d_{high} = 1.3 × 10^{-3} mm., and the standard uncertainty 0.001403 mm. can be written as 1.4 × 10^{-3} mm., considering two significant digits, then δ = 1 / 2 × 10-3 mm. = 5 × 10^{-4} mm. As both d_{low} and d_{high} are higher than δ, the GUM approach is not validated. But considering two significant digits, using a less rigid criterion, $\delta=1/2\times10^{-2}$ mm. $=5\times10^{-3}$ mm. and the GUM approach is validated.

V. CONCLUSION

The entire measurement process has a certain degree of uncertainty, and it is imperative to report the uncertainty associated with the measurement. From this research article, it was concluded that:

- The GUM use combined standard uncertainty is 0.001403 mm. The MCM use standard deviation is 0.001402. From the results by the two methods are then compared. The circular runout error found by the two methods is nearly identical and produced no significant differences.

- The result analysis shows that the MCM has many advantages over the GUM in the estimation of uncertainty, especially that of complex systems of measurements. There is no need for complex mathematics to calculating coefficient.

The GUM uncertainty is still the most often used method in metrology for estimating measurement uncertainty. It works well on a wide range of measurement systems. The GUM uncertainty is relatively compatible with the MCM in a conventional uncertainty estimation method of linear systems and systems that have small uncertainties.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

Pornpawit Ounjutturaporn conducted the research, analyzed the data and wrote the manuscript and presentation; Ramil Kesvarakul developed all the procedures and draft manuscript preparation; Pipitanon Poonsawat and Khompee Limpadpun study conception and design, analysis and interpretation of results; All authors reviewed the results and approved the final version of the manuscript.

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