Analysis of Modal Frequencies Estimated from Frequency Domain Decomposition Method

Mohammad Ali Hadianfard and Soroosh Kamali

Abstract—**Frequency Domain Decomposition (FDD) is an operational modal analysis method by which the dynamic parameters of structures are obtained using response signals recorded on several locations of structures. In recent years, many researchers have employed the method to estimate the modal parameters of several Multi-Degree-of-Freedom (MDOF) mechanical and civil systems. The accuracy of the results depends on recording and signal processing parameters such as sampling frequency, windowing, filtering etc. As a result, investigation on the influence of these parameters on the accuracy of the estimates is a good practice. In this research, the uncertainty of modal frequencies obtained from FDD method is analyzed. In order to achieve this goal, the exact values of modal parameters must be available to be compared with the results from FDD method. For this purpose, synthetic signals with random characteristics the same as the ambient vibrations' are produced and structures with known dynamic parameters are simulated and loaded. The effect of several parameters are investigated in the accuracy of results and proper values and settings are proposed to minimize the errors.**

Index Terms—**Frequency Domain Decomposition (FDD), Operational Modal Analysis (OMA), modal identification, uncertainty analysis, ambient vibrations**

I. INTRODUCTION

Seismic methods can be used to assess the dynamic parameters of structures. Experimental Modal Analysis (EMA) was traditionally used to obtain the latter. This method employed both input and output signals to identify the system characteristics. The constraints of using input signal and measurement difficulties made EMA less popular.

At early 90's, more attention was paid to the operational modal analysis. Operational modal analysis is an output-only method which does not require the input signal for assessment of the system characteristics. time- or frequency-domain analyses on the recorded signals are possible for this purpose.

At 1992, James et al. proposed a method which used the system output of natural excitations called Natural Excitation Technique (NExT). This method was directly used in later researches [1].

In 2000, Frequency Domain Decomposition was introduced by Brickner et al. for the first time [2]. This method uses the power spectral density function of response signals to obtain the modal parameters of a multi-degree-of-freedom structure. The original method can

estimate the modal frequencies and the mode shapes; while it doesn't yield the modal damping of the structures.

Damage detection is one of the most important applications of OMA. Teughels and De Roeck used OMA for this purpose in 2004 [3].

Extended efforts about applications and theory of OMA was initiated with the International Operational Modal Analysis Conference in 2005 [4].

Random Decrement Method (RDM) is a process by which the free decay response of a system is separated from the total response. RDM can be used as a signal processing tool for OMA and also for estimation of dynamic parameters of structures.

Hadianfard et al. analyzed the applications of RDM and Floor Spectral Ratio (FSR) such as the effect of retrofit or adding bracing on dynamic parameters of structures [5]. Moreover, they studied the fundamental dynamic parameters and vulnerability indices of historical buildings which are difficult to extract due to complexity and sensitivity of these structures [6].

Modak et al. used RDM to eliminate undesirable harmonics as noises while using OMA [7]. Zhang and Song investigated the effect of window overlaps of RDM in the accuracy of OMA method [8]. Chen et al. assessed the modal parameters of an 11-span concrete bridge using OMA [9], while Compan et al. used it to obtain modal parameters of a historical building in Germany [10].

Many researchers have used OMA for different purposes from modal identification to model updating and damage detection. But it must be noticed that OMA techniques are indeed signal processing methods. Many parameters and signal processing variables can influence the accuracy of the results and these settings must be chosen carefully. Thus, it is a good practice to investigate the effect of these variables in the accuracy of the estimations prior to any other processes such as those mentioned.

II. FREQUENCY DOMAIN DECOMPOSITION METHOD

Ambient vibrations or micro-tremors are low-amplitude signals with several sources (natural or manmade). They are random signals and can be treated as stationary ergodic signals if recorded within a confident distance from transient sources or noises. On the other hand, they can be assumed as zero-mean white Gaussian noise (normal distribution of probability) according to the central limit theorem.

The response of an MDOF structure to the ambient vibrations is the same as its response to any other ground motion. Since the ambient vibrations are low amplitude, the system is expected to behave linear. The response of an MDOF system to such load is as follows:

$$\overrightarrow{u(t)} = \Phi \overrightarrow{q(t)} \tag{1}$$

where Φ is the mode shapes matrix obtained from the eigenvalue problem and $\overrightarrow{q(t)}$ is a vector with components representing the equivalent Single-Degree-of-Freedom (SDOF) displacement response of each mode (generalized coordinates) in the modal superposition method. These components are obtained by solving the differential equation of motion for each of the equivalent SDOF systems. $\overrightarrow{u(t)}$ is a vector referring to the total displacement of DOF's at each time instant t. Similar notations can be used for velocity and acceleration responses.

Since the nature of ambient vibrations is random, correlation functions and power spectral density functions are required to be obtained. In the case of ergodic signals, the correlation functions are independent of time and are only a function of time lag τ:

$$R_{uu}(t, t+\tau) = R_{uu}(\tau) = E[u(t+\tau)u(t)^T] \tag{2}$$

where the operator $E[\ ]$ represents the ensemble average and $R_{uu}(\tau)$ is the matrix of auto- and cross-correlation functions. Using (1), (2) can be rewritten as:

$$E[u(t+\tau)u(t)^T] = E[\Phi\, q(t+\tau)\, q(t)^T\, \Phi^T] = \Phi E[\, q(t+\tau)\, q(t)^T]\Phi^T \tag{3}$$

Defining $R_{qq} = E[\, q(t+\tau)\, q(t)^T]$:

$$R_{uu}(\tau) = \Phi R_{qq} \Phi^T \tag{4}$$

According to the Wienner-Khinchin theorem, correlation and Power Spectral Density (PSD) functions are Fast Fourier Transform (FFT) conjugates. As a result, the power spectral density matrix can be defined as follows:

$$G_{uu}(\omega) = \Phi G_{qq}(\omega)\, \Phi^H \tag{5}$$

where $G_{uu}(\omega) = FFT(R_{uu})$, ω is the circular frequency and Φ^H represents the Hermitian adjoint of the matrix Φ. While recording the signals on several DOF's of a structure, no information is available about the mode shape matrix. The only available parameter is $G_{uu}(\omega)$ which can be calculated directly using recorded signals. If singular value decomposition is employed, any matrix of PSD functions can be decomposed as follows:

$$G_{uu}(\omega) = U\Sigma U^H \tag{6}$$

Using a one-by-one comparison, Σ can be assumed to be the PSD matrix of the generalized coordinates $G_{qq}(\omega)$. The singular value spectra are maximum at the fundamental modal frequencies. This fact can be used to detect the modal frequencies of an MDOF structure.

III. RESEARCH OBJECTIVES

The most important goal of FDD is to estimate the modal frequencies of structures. Code-based structural dynamic analyses, especially those for vulnerability assessment,

require proper estimates of modal frequencies. These values, significantly influence the results of the dynamic analyses of structures. In this paper, the effort concentrates on the effect of several parameters on the accuracy of modal frequencies estimation obtained from FDD method.

For this purpose, some structures are modeled with dynamic parameters introduced in Table I. On the other hand, synthetic signals with statistical characteristics similar to those of ambient vibrations are generated. The structures are loaded with synthetic ambient vibrations and their responses are calculated using numerical methods (Newmark linear acceleration).

TABLE I: MODAL PROPERTIES OF THE STRUCTURES

Number of DOF's	Mode number	Modal Frequency (rad/s)
2	1	6.82
	2	17.86
4	1	3.833
	2	11.03
	3	16.91
	4	20.73
6	1	2.66
	2	7.83
	3	12.54
	4	16.52
	5	19.53
	6	21.42
8	1	2.04
	2	6.04
	3	9.84
	4	13.3
	5	16.3
	6	18.75
	7	20.56
	8	21.67
10	1	1.65
	2	4.91
	3	8.06
	4	11.03
	5	13.75
	6	16.16
	7	18.21
	8	19.85
	9	21.05
	10	21.78

In order to investigate the effect of each parameter, rational values are given to them. These values changes in loops and the modal frequencies are estimated for each case. The other parameters remain constant and equal to an engineering value. Parameters under investigation consist of the number of windows, sampling rate, number of DOF's and the signal nature.

One might wonder why experimental studies on actual cases are not performed. True dynamic parameters of existing structures are unknown and available experiments are approximate themselves. Thus, the results obtained from FDD cannot be compared with the exact values.

As a result, one of the most outstanding features of this research is the complete information about the true values of the modal frequencies which provides a basis for correct comparison of the estimated and exact frequencies.

IV. Effect of Different Variables in the Estimation Accuracy

A. Effect of the Number of Windows while Building PSD Function

Since corresponding signals are stationary and ergodic, auto- and cross-PSD functions can be obtained from a single or a pair of signals respectively. This is possible by breaking them into several time windows or segments and finding their Fourier spectra. Ensemble average in the PSD function formulation will be replaced by mean value calculation. The result would be exact if the number of windows were infinite. In practice, a finite number of windows must be used. Thus, the number of windows in order to build the PSD functions is of importance. To investigate the effect of the mentioned parameter in the accuracy of FFD method, synthetic signals are generated as ground motions and the introduced structures are loaded.

In order to investigate the effect of number of windows, different numbers are employed and the process is repeated to gain different estimates of modal frequencies. At this step, the damping ratio is takes as 0.03 for the first two modes and the other modal damping ratios are calculated using Rayleigh method. For the 4-DOF structure, the damping ratio values are as Table II.

TABLE II: The Modal Damping Ratios of the 4-DOF Structure

MODES	1	2	3	4
Rayleigh Damping Ratio	0.03	0.03	0.0392	0.0460

The sampling frequency is taken as 50 Hz which is more than twice the largest modal frequency of the mentioned structure. The window length is 0.5 sec and the number of rectangular windows alternates between the values 100, 250, 500, 750, 1000, 2000, 5000 and 10000. As an example, the SVD spectrum for the 1000-window case is illustrated at Fig. 1. The obtained results are presented in Fig. 2 representing the error of the estimated frequencies vs. the number of windows.
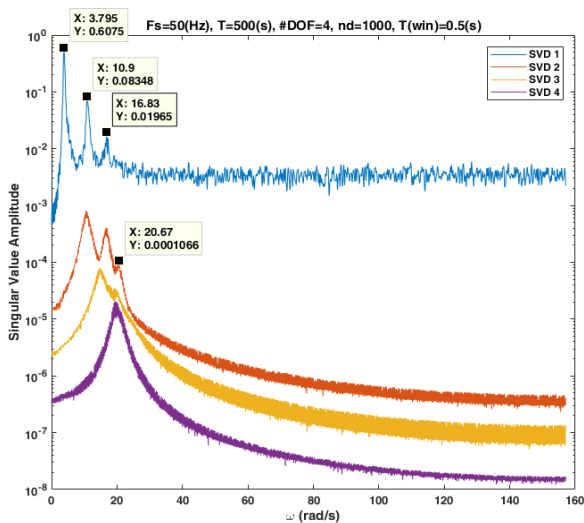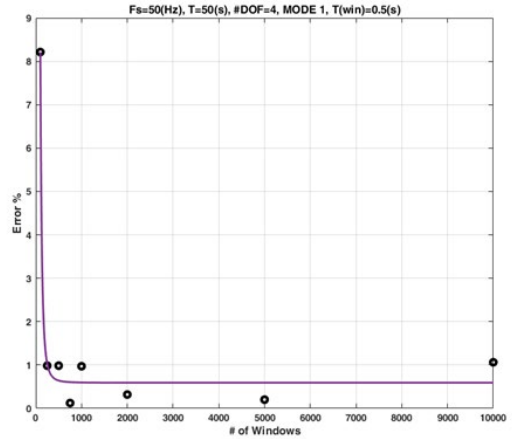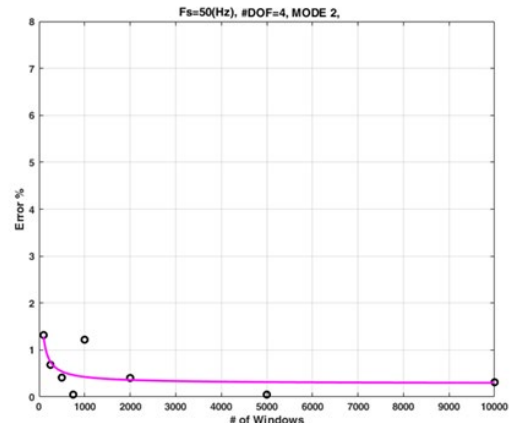

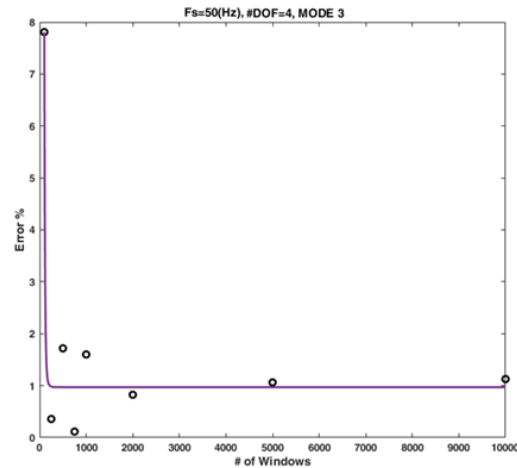Fig. 1. SVD spectrum of the structure using 1000 windows.

It can be concluded from the results that the minimum number of windows is 1000 and the convenient number is 2000. With this consideration, the error remains about 1%.
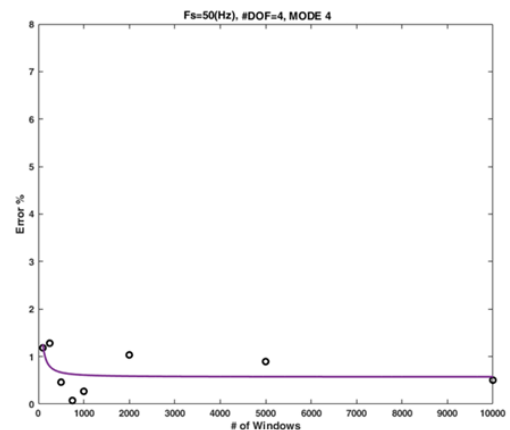

Fig. 2. The error of the modal frequency estimation for modes (a) 1, (b) 2, (c) 3 and (d) 4.

B. Sampling Frequency Effect

The sampling frequency is an important parameter in signal processing. If the sampling frequency is taken too large and the number of samples increases, the processing would be time-consuming. Keeping the number of windows and the window length equal to 1000 and 0.5 sec for the 6-DOF structure of Table I, the effect of sampling frequency is investigated. The damping ratio is taken as 0.03 for all of the modes. The signal processing is performed for 10, 25, 50, 100 and 200 Hz sampling rates. Fig. 3 represents the singular value spectra for 10 and 200 Hz rates. The estimated modal frequencies and their error are represented in Table III and Table IV respectively. Fig. 4 illustrates the error-sampling rate diagram for the 5th mode of vibration.
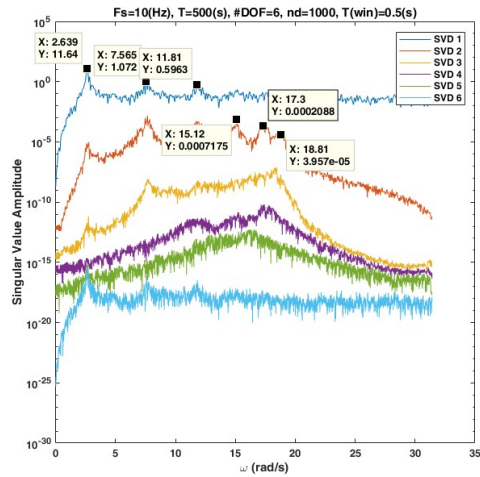
(a)
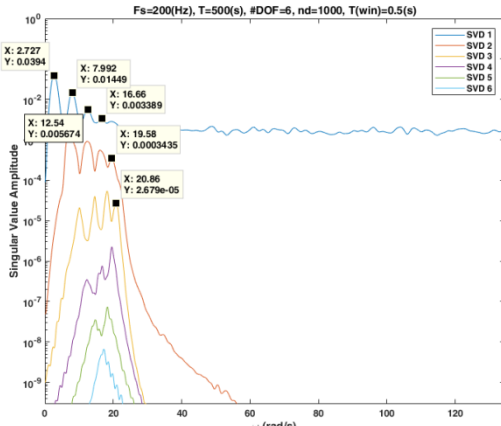
(b)

Fig. 3. SVD spectra for (a) 10 Hz and (b) 200 Hz sampling frequencies.

TABLE III: THE VALUES OF ESTIMATED MODAL FREQUENCIES WITH RESPECT TO THE SAMPLING RATES FOR ALL MODES

Fs (Hz)	Estimated Natural Frequency (rad/s)					
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
10	2.639	7.565	11.81	15.12	17.3	18.81
25	2.677	7.741	12.18	16.11	19.23	20.76
50	2.677	7.854	12.57	16.96	19.48	21.09
100	2.689	7.917	12.67	16.36	19.42	21.31
200	2.727	7.992	12.54	16.66	19.58	20.86
Real F (rad/s)	2.660	7.825	12.53	16.51	19.54	21.42

It can be noticed from the results that the errors are acceptable for sampling frequencies more than a specified

value. For the structure under investigation, this value is observed to be 50 Hz. Indeed, for many cases the lowest error happens at 50 Hz which is slightly greater than twice the greatest modal frequency. This observation confirms the considerations corresponding to the Nyquist frequency.

TABLE IV: THE ESTIMATION ERRORS IN MODAL FREQUENCY WITH RESPECT TO DIFFERENT SAMPLING RATES

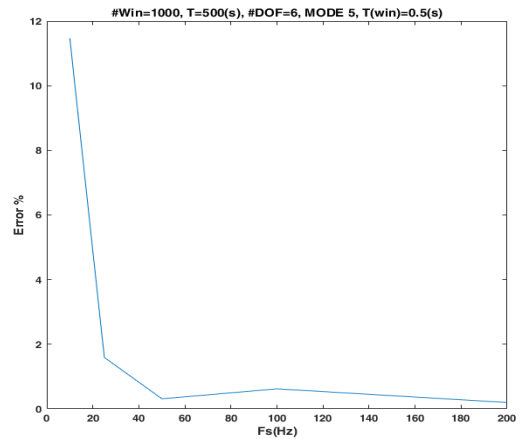| Fs (Hz)	Error %					
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5	Mode 6
10	0.7932	3.3325	5.7965	8.4684	11.469	12.216
25	0.6353	1.0836	2.8452	2.4753	1.5925	3.1165
50	0.6353	0.3603	0.2656	2.6702	0.3131	1.5764
100	1.0864	1.1653	1.0632	0.9619	0.6202	0.5497
200	2.5149	2.1237	0.0263	0.8541	0.1985	2.6498

Fig. 4. The 5th mode natural frequency estimation errors for different sampling rates.

C. The Effect of the Signal Nature

Several instruments exist for ambient vibration recording on structures. These instruments might record displacement, velocity or acceleration on buildings. For this purpose, the response of a structure due to a random signal is calculated in displacement, velocity and acceleration terms using Newmark method. These responses are used as the input to the FDD method for comparison. The 4-DOF structure of Table I is used for the analysis. Constant damping ratio, sampling frequency, the number of windows and the window length are assumed to be 0.03, 50 Hz, 1000 and 0.5 sec. The resulting singular value spectra for displacement, velocity and acceleration are available in Fig. 5.

The results show that all of the displacement, velocity and acceleration signals are proper for FDD processing. It can be noted from the spectra that the peaks are clearest in the acceleration spectra and the peaks in the velocity spectra are clearer than the displacement ones.

D. The Effect of The Number of DOF's

In this section, the influence of the number of DOF's on the accuracy of FDD is investigated. In other words, it is of interest to check if FDD is as precise in higher modes as the fundamental modes. To achieve the goal, a few structures are analyzed (structures mentioned in Table I). The modal damping ratio is taken as 0.03 for all structures and modes. The SVD spectra after analysis are given as Fig. 6. The obtained estimations for different structures are shown as Table V.
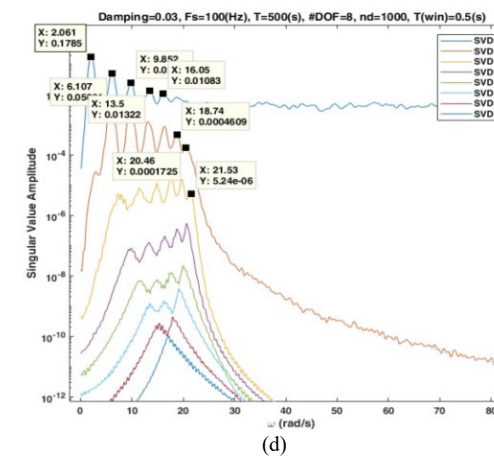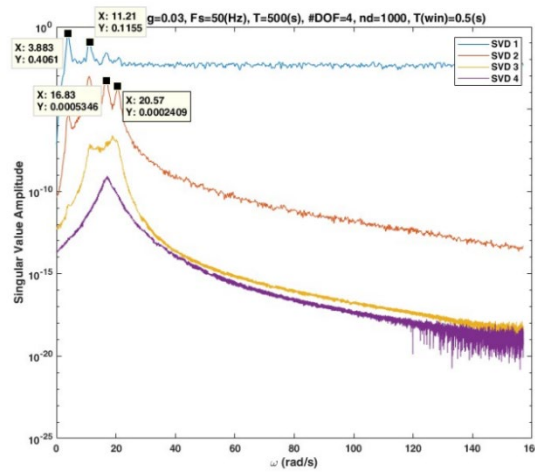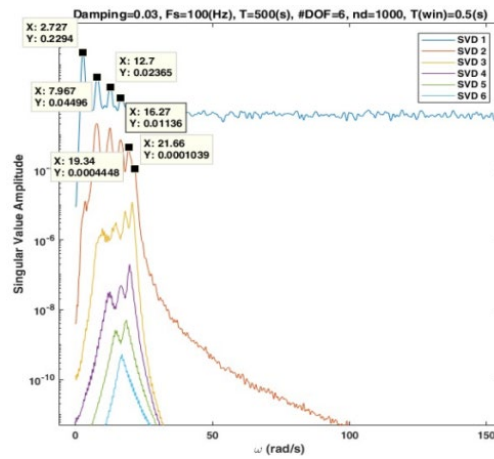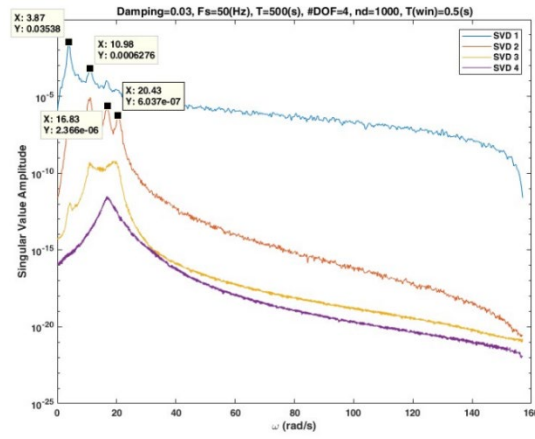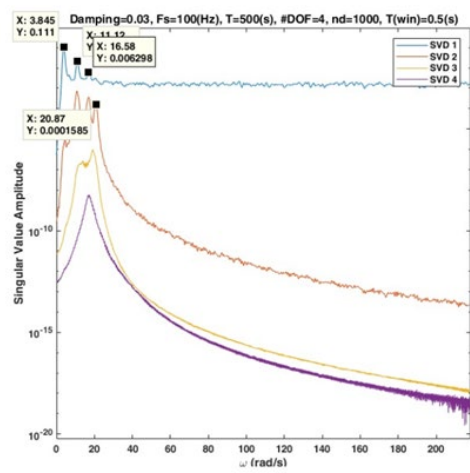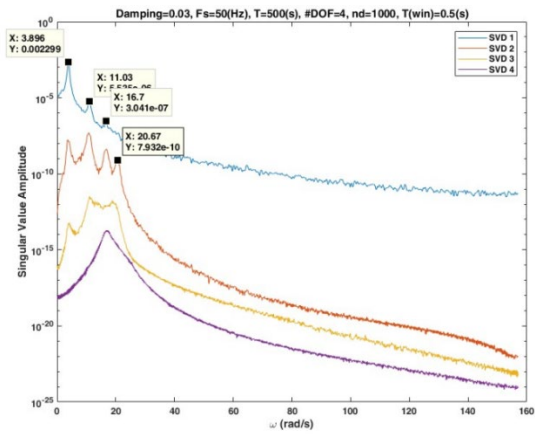
(a)

(b)

(c)

Fig. 5. SVD spectra obtained from (a) displacement, (b) velocity and (c) acceleration signals.
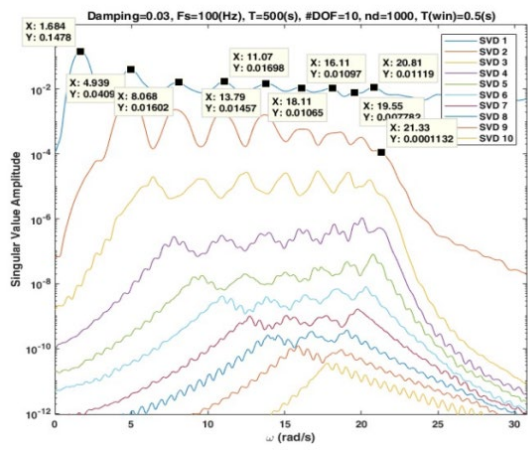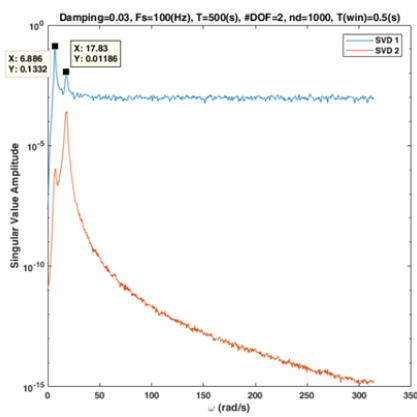

(a)

(b)

(c)

(d)

(e)

Fig. 6. SVD spectra for (a) 2-, (b) 4-, (c) 6-, (d) 8- and (e) 10-DOF structures.

TABLE V: ESTIMATED FREQUENCIES AND THE ESTIMATION ERROR USING FDD METHOD FOR MULTI-DEGREE-OF-FREEDOM STRUCTURES

# of DOFs	Mode	Natural Frequency (rad/s)	Estimated Frequency (rad/s)	Frequency Error (%)
2	1	6.82	6.886	0.97
	2	17.86	17.83	0.17
4	1	3.833	3.845	0.31
	2	11.03	11.12	0.82
	3	16.91	16.58	1.95
	4	20.73	20.87	0.68
6	1	2.66	2.727	2.52
	2	7.83	7.967	1.75
	3	12.54	12.7	1.28
	4	16.52	16.27	1.51
	5	19.53	19.34	0.97
	6	21.42	21.66	1.12
8	1	2.04	2.061	1.03
	2	6.04	6.107	1.11
	3	9.84	9.852	0.12
	4	13.3	13.5	1.50
	5	16.3	16.05	1.53
	6	18.75	18.74	0.05
	7	20.56	20.46	0.49
	8	21.67	21.17	2.31
10	1	1.65	1.684	2.06
	2	4.91	4.939	0.59
	3	8.06	8.068	0.10
	4	11.03	11.07	0.36
	5	13.75	13.79	0.29
	6	16.16	16.11	0.31
	7	18.21	18.11	0.55
	8	19.85	19.55	1.51
	9	21.05	20.81	1.14
	10	21.78	21.33	2.07

It can be noted from the results that the modes with lower frequencies can be read from the spectrum of the first singular values. However, other singular value spectra than the first one should be used for modes with higher natural frequencies. It was also observed that if a peak is clear in two different singular value spectra, the accuracy of the upper one is more.

It can be concluded that the accuracy of the results is still acceptable for higher modes for MDOF structures. Nevertheless, it must be noticed that in this research noises do not exist. In addition, as it was mentioned in the previous section, the more the damping ratios, the less the accuracy of the results are.

V. CONCLUSION

In this research the accuracy of FDD method was investigated with respect to signal recording and processing parameters. Using synthetic signals and simulation of structures, a basis was provided to compare the estimated results with the exact values. By this, the efficiency of the method was examined.

FDD is an acceptable method for modal identification of structures with rather small resulting errors. The number of time windows was observed to mostly influence the results accuracy in the frequency domain decomposition method. A number of 2000 time windows is observed to be sufficient to estimate the natural frequencies of MDOF systems in the OMA method. A sampling frequency of 50~100 Hz is proper for acceptable error in estimation of natural frequencies. The accuracy of FDD reduces as the damping ratio of the structure increases. For damping ratios of civil structures, the error is acceptable. Acceleration, velocity and displacement responses can be used in structures with any number of DOF's to estimate the natural frequencies using FDD method.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

AUTHOR CONTRIBUTIONS

M. Hadianfard: Project administration, Supervision, Conceptualization, Methodology, Validation and Editing. S. Kamali: Writing original draft, Methodology, Software, Investigation, Validation and Formal analysis; all authors had approved the final version.

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