Dimensional Synthesis of Kinematic Dexterity and Workspace for 3-DOF Parallel Manipulator

Ruixia Li and Fenxia Li

Abstract—In this paper, dimensional synthesis of a novel 3-PUU parallel manipulator, which is applied for a parallel kinematic machine (PKM) tool, is proposed. Firstly, kinematics analysis of the parallel manipulator is established, and the inverse solutions of the kinematics are systematically analyzed and geometric description is given. Secondly, the Jacobian matrix is deduced and the kinematic dexterity index and the workspace are introduced. In order to require better dexterity performance and larger workspace, multi-objective optimizations based on non-dominated sorting genetic algorithm II (NSGA-II) are performed in terms of dexterity performance and workspace of the proposed parallel manipulator. By optimizing the design variables including geometric dimensions of the parallel manipulator, the dexterity and workspace of the proposed parallel manipulator have been greatly improved. The analytic results are verified by simulation to show that the parallel manipulator has a good application prospect.

Index Terms—Dimensional synthesis, parallel manipulator, jacobian matrix, dexterity, workspace.

I. INTRODUCTION

Compared with traditional serial robotics, the parallel manipulators have some significant advantages including higher accuracy, higher stiffness, higher loading capacity, less error accumulation, and more convenient for inverse kinematic control. Up to now, parallel manipulators as an emerging area have shown great application potential for the development of machine tools in virtue of their excellent mechanical properties mentioned above [1]-[4]. However, the main disadvantage of the parallel manipulators is the limited workspace, because these legs will collide to each other and each leg has several different joints which limit the motion of parallel manipulators as well. Therefore, it is quite necessary to perform the dimensional syntheses optimum analysis in the early design stage for the next-generation robotic systems for industrial applications with higher accuracy and higher speed, more robustness and adaptability, and so on.

Dexterity is one of the most important performances to evaluate the parallel kinematic machines (PKM) good or bad [5]-[7], which reflects on the dexterous capability over the task workspace. Meanwhile, dimensional synthesis aims to improve the performance of a mechanical robotic system and is an important procedure of parallel manipulators from their schematic diagram to the structural design, in which two challenging issues associated with kinematic performance index and optimal design method are involved [8]. Therefore, it is quite necessary to perform the dexterity analysis and to conduct optimal design of the parallel kinematic machines in the early design stage. The determinant and condition number of Jacobian matrix have been widely used in performance evaluation and optimal design of parallel manipulators [9], [10]. The workspace as the other most important index of the parallel manipulator is computed considering the link length and angle constraints on the basis of the inverse solution of the kinematics. It is particularly important to notice that, we should consider workspace during the design of the mechanism which determines the size of the parallel manipulator operating range actuator. And it is the foundation for the subsequent optimization. The workspace of parallel manipulator is mainly restricted by driven chains and constraint chain, as well angle constraints. Li researched the workspace of the five degrees of freedom parallel manipulator based on the Monte Carlo simulation method [11]. Yet, Li et al. [12] pointed out that the unit inconsistent problem arises when these indices are applied to parallel manipulators with mixed DOFs of translations and rotations. Apart from the kinematic performance index associated with Jacobian matrix, some efforts were made to evaluate the motion or force transmissibility using screw theory [13]. Tsai [14] defined the virtual coefficient between the transmission wrench screw and the input velocity screw, while Wang et al. [15] proposed the input or output transmission index and dealt with kinematic evaluations of a 3-DOF parallel manipulator successfully.

Upon the architecture of presented novel parallel manipulator that can generate three translations degrees of freedom, its kinematic model and mathematical model are established and the Jacobian matrix is derived for the first time. In which two essential procedures are involved, one is the kinematic dexterity performance index by means of condition number of Jacobian matrix, the other is the design method adopted to perform the multi-objective dimensional synthesis using an artificial intelligence approach that is non-dominated sorting genetic algorithm II (NSGA-II). Conclusions are provided in the last section of this paper.

II. KINEMATICS ANALYSIS

A. Structure of the Parallel Manipulator

The CAD and schematic model of the proposed parallel manipulator in Fig. 1 and 2 show that it consists of a fixed base, a moving platform and the three identical legs which are
The position vectors of the three sliding rails can be expressed as \( q_i \) \((i = 1, 2, 3)\), and the constant length rod of length \( l \) is \( r \). The parallel manipulator has three degrees of freedom, that is, translation along \( x \)-axial, \( y \)-axial and \( z \)-axial.

For the purpose of analysis, as represented in Fig. 2. We assign a fixed Cartesian coordinate system \( O-x_1y_1z_1 \) at the centered point \( O \) of the fixed base \( \Delta B_1B_2B_3 \), and the \( x \)-axial is coincident with point \( B_1 \), and the \( z \)-axial is perpendicular to the platform whose direction is downward, and the \( y \)-axial direction is determined by the right-hand rule. Similarly, we establish a moving coordinate system \( O' - x'y'z' \) whose origin point \( O' \) is the geometric center of the moving platform, and let \( x' \) axis and \( x \) axis are parallel to each other initially and \( x \) axis is coincident with point \( A_1 \). Meanwhile, \( z' \) axis direction is downward, and \( y' \) axial direction is determined by the right-hand rule.

### B. Inverse kinematics and Jacobian Analysis

The position vectors of \( A_i \) points are expressed in the moving coordinate frame as the following:

\[
A_1 = \begin{bmatrix} 0 & r_a & 0 \end{bmatrix}^T \\
A_2 = \begin{bmatrix} r_a \sin 60 & r_a \cos 60 & 0 \end{bmatrix}^T \\
A_3 = \begin{bmatrix} -r_a \sin 60 & r_a \cos 60 & 0 \end{bmatrix}^T
\]  

Similarly, the position vectors of points \( B_i \) are expressed in the reference frame as the following:

\[
B_i = \begin{bmatrix} 0 & -r_b & q_i \end{bmatrix}^T \\
B_2 = \begin{bmatrix} r_b \sin 60 & r_b \cos 60 & q_2 \end{bmatrix}^T \\
B_3 = \begin{bmatrix} -r_b \sin 60 & r_b \cos 60 & q_3 \end{bmatrix}^T
\]

where \( q_1, q_2, q_3 \) represents the position of the three slides.

Since the parallel manipulator has three translational degrees of freedom, the moving coordinate system has no rotation relative to the fixed coordinate system, so the position vectors of points \( A_i \) are expressed in the fixed coordinate frame as the following:

\[
A_{ii} = A_{ii'} + OO' \quad (i = 1, 2, 3)
\]

where the position vector of the reference point \( O' \) in the reference coordinate frame can be written as \( \begin{bmatrix} x & y & z \end{bmatrix}^T \).

The inverse position solution of the parallel manipulator can be represented by the constraint equation of the fixed length leg

\[
L = \|C_{ii'} - A_{ii'}\| \quad (i = 1, 2, 3)
\]

where \( L \) denote the vector of the three identical legs. We also can write as

\[
\|A_{ii'}\| - \|A_{ii'} - C_{ii'}\| = l_i^2
\]

From the definition of the vector model, we can get the equation:

\[
q_i = z - \sqrt{L^2 - x^2 - (y - r_a + r_b)^2} \\
q_2 = z - \sqrt{L^2 - [x + (r_a - r_b) \sin 60]^2 - [(y + (r_a - r_b) \cos 60]^2} \\
q_3 = z - \sqrt{L^2 - [x - (r_a - r_b) \sin 60]^2 - [(y + (r_a - r_b) \cos 60]^2}
\]

Once the structure parameters \( r_a, r_b \) are sure, and the position parameters \((x, y, z)\) with respect to the moving
platform are given, the driven chain lengths are obtained by the equation (6), that is, the inverse solution of kinematics position.

The operational velocity vector of the end-effector can be expressed as

\[ \mathbf{v} = \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} \] (7)

Equation (6) of the time derivative can be rewritten in matrix form

\[ \dot{\mathbf{q}}_i = \mathbf{Jv} \] (8)

In which J is the 3 by 3 velocity Jacobian matrix of the parallel manipulator.

III. PERFORMANCE INDEX

In order to make the parallel manipulator has good kinematics performance in the workspace, the kinematics dexterity and workspace optimization was studied and the global condition index and the workspace were introduced as the evaluation index [16].

A. Dexterity

Condition number of Jacobian matrix at the point is used to measure the dexterity [17], which can be calculated as follows:

\[ \kappa = \frac{\lambda_{\text{max}}(\mathbf{J'}\mathbf{J})}{\lambda_{\text{min}}(\mathbf{J'}\mathbf{J})} \] (9)

where \( \lambda_{\text{max}}(\mathbf{J'}\mathbf{J}) \) and \( \lambda_{\text{min}}(\mathbf{J'}\mathbf{J}) \) are the maximum and minimum eigenvalue, respectively.

When \( k=1 \), the mechanism owns the best transmission performance, in this case, the configuration is called isotropic. When the condition number trends to infinite, the mechanism is in a singular configuration.

In order to obtain the kinematics performance in the whole workspace, Gosselin and Angeles [18] proposed the global condition index, which is a measure of its kinematics precision and control accuracy performance of the parallel manipulator tool. The dexterity of the manipulator over the entire workspace or reasonable central portion of the workspace should be considered. The global condition index can be defined as the ratio of the integral of the inverse condition numbers calculated in the whole workspace, dived by the volume of the workspace, namely

\[ \kappa_j = \frac{\int_{w} \frac{\lambda_{\text{min}}}{\lambda_{\text{max}}(\mathbf{J'}\mathbf{J})} \text{d}W}{\int_{w} \text{d}W} \] (10)

In which W is the workspace of parallel mechanism, and \( \kappa_j \) is the local condition number defined as the reciprocal of the condition of the Jacobian matrix at a particular position. It is noteworthy that the global condition number index is bounded as \( (0, 1) \). If the \( \kappa_j \) approach zero, the mechanism has a bad global performance and as the \( \kappa_j \) approaches one the mechanism has a good global performance. Therefore, we should make the optimization objective \( \kappa_j \) maximization.

B. Workspace

The workspace is the set of all points where the reference point of the moving platform can achieve in the space, which is the area of the robot manipulator, and is a pretty important index to measure the performance of the parallel manipulator. The workspace of a parallel manipulator can be defined as a reachable region of the origin of a coordinate system attached to the center of the moving platform. The algorithm to define the workspace is based on the boundary of the workspace in a Cartesian coordinate system with constrains of the limitation of actuators and angle of joints.

In this paper, we established the global condition index based on the workspace according the previous section, so we need to solve the workspace firstly. The workspace can be expressed as

\[ W = \{(x, y, z) \in \mathbb{R} | f(x, y, z) \leq 0\} \] (11)

where \( f(x, y, z) \leq 0 \) denoted the constraint condition, that is:

1. The active chains length constraints can be expressed by

\[ q_{\text{min}} \leq q_i \leq q_{\text{max}} \] (12)

where \( q_{\text{max}} \) denoted the maximum movement position, \( q_i \) denoted the slider position of the i-th leg, and \( q_{\text{min}} \) denoted the minimum movement position.

2. The mechanism was non-singular configuration, namely, the determinant of the Jacobian matrix was not equal to zero.

IV. DIMENSIONAL SYNTHESIS

In general, a constrained multi-objective optimization problem can be explained as solving the problem of searching for the vector of decision variables to optimize the objective functions. A genetic algorithm is a nontraditional optimization method based on the Darwinian survival-of-the-fittest evolutionary theory. The method is accomplished using genetic operators such as selection, crossover, and mutation until it converges towards an optimal solution. In this paper, a non-dominated sorting-based multi-objective dimensional synthesis method called non-dominated sorting genetic algorithm II (NSGA-II), is adopted to find the optimum solution [19-20].

A. The Optimization Design Space

The scale parameters of the parallel manipulator were radii of the moving platform \( r_\ell \) and the fixed platform \( r_i \). Select the link length of the active chains \( l_i \), the searching workspace range is defined in \( x^2 + y^2 \leq 0.15^2, 0.2 \leq z \leq 1.8 \), and the optimization objective functions for \( k_j \) and the workspace \( W \), then the multi-objective optimization model can be expressed as

\[ \begin{align*}
\text{max} & f_1 = k_j(r_\ell, r_i, l_i) \\
\text{max} & f_2 = W(r_\ell, r_i, l_i) \\
\text{s.t.} & 0.03 \leq r_i \leq 0.06 \\
& 0.08 \leq r_\ell \leq 0.16 \\
& 0.02 \leq l_i \leq 0.4 \\
& 0.1 \leq q_i \leq 1.5
\end{align*} \] (13)
B. The Optimization Design Algorithm

The algorithm parameters are set to: Maximum number of generation: 40; Population size: 30; Mutation rate: 0.05; Crossover rate: 0.9; Selection strategy: Tournament; Tournament size: 2; Crossover type: Intermediate; Crossover ratio: 1; Mutation function: Adaptive feasible.

The multi-objective optimization of dexterity and workspace have been done separately, however, sometimes the dexterity and workspace have to be considered simultaneously, thus the multi-objective optimization must be taken into account, which is required to find all the possible tradeoffs among the conflicting objective functions. To achieve that, a GA based multi-objective optimization toolbox in MATLAB was used to optimize the dexterity and workspace. The optimization results corresponding design variables are shown in Table I (units: m).

TABLE I: ARCHITECTURE PARAMETERS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r_s)</td>
<td>0.45 m</td>
</tr>
<tr>
<td>(r_b)</td>
<td>1.25 m</td>
</tr>
<tr>
<td>(l)</td>
<td>0.35 m</td>
</tr>
</tbody>
</table>

For illustration of the dexterity and workspace of the parallel manipulator presented in this paper, the numerical simulation solutions should be calculated to analyze the performance index characteristic of the mechanism. So next, we will have some cases analysis and discussion.

V. CASE ANALYSIS AND DISCUSSION

For the optimum design of the robotic manipulators, the idea of global index is proposed based on averaging the local index in a workspace and providing the changing trend under different variables.

A. Simulation Results and Discussion

On the basis of the inverse kinematics and derivation of the dexterity calculation formula, as well as the constraints, the dexterity and workspace distribution of the crossbar parallel machine tool are solved by using the MATLAB program.

The Fig.3 and Fig.4 show the dexterity distribution in the area of \(-1.2 \text{ m} \leq x \leq 1.0 \text{ m}, -1.2 \text{ m} \leq y \leq 1.0 \text{ m}\) and \(z=1.0 \text{ m}\) of the parallel manipulator, which demonstrate that the dexterity performance of the parallel manipulator improved and the smoothness also increased, so the optimization design is feasible. It can be seen from the comparison of the Fig.3 and Fig.4 that the optimized parallel manipulator has more stable dexterity capability than before optimization.

Through the inverse position and the constraint condition, the discretizing position of the moving platform can be numerically determined and depicted in Fig.5-Fig.8.
for different structural parameters of the parallel manipulator. The dexterity distributions for different elevation ranges before and after optimization are depicted in Fig. 9–Fig. 12, respectively.

The Fig. 5 and 7 show the 3D view workspace of parallel manipulator before and after optimization. In particular, the optimized workspace can contain cube with 0.15 m length and 1.8 m height, while before, the contain cube with 0.75 m length and 1.4 m height. Furthermore, according to the proposed optimal design method, it is obvious that the parallel manipulator after optimization has much larger boundary workspace region than before optimization through the comparison between the Fig. 6 and 8. Thus, the optimal design results for prescribed workspace provide some structural improvements of this type of parallel manipulator.

In the following, we evaluate the dexterity performance distribution in the workspace before and after optimization.
condition region, indicating that the parallel manipulator has no singular configuration in the ideal workspace, and has good kinematic dexterity in the whole reachable workspace.

VI. CONCLUSIONS

This paper develops a 3-DOF parallel manipulator multi-objective dimension optimization design that aims to increase the dexterity and workspace of the proposed parallel manipulator. The main contributions of this paper are summarized as follows:
1) This paper is focused on the multi-objective optimization of a parallel manipulator with three degrees of freedom that aims to simultaneously increase the dexterity and the workspace.

2) The kinematics analysis and Jacobian matrix is derived and multi-objective optimizations are implemented by searching for the relationship between the dimensional parameters and the objective functions.

3) The computation results showed that the performance of the proposed manipulator can be largely enhanced and the proposed methodology of multi-objective optimization could be also applied to the dimensional synthesis of other parallel manipulators.

In future works, dynamics problem and motion control will be further considered, and a prototype will be manufactured and several relevant experiments will be conducted.

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REFERENCES


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