

Dimensional Synthesis of Kinematic Dexterity and Workspace for 3-DOF Parallel Manipulator

Ruixia Li and Fenxia Li

Abstract—In this paper, dimensional synthesis of a novel 3-PUU parallel manipulator, which is applied for a parallel kinematic machine (PKM) tool, is proposed. Firstly, kinematics analysis of the parallel manipulator is established, and the inverse solutions of the kinematics are systematically analyzed and geometric description is given. Secondly, the Jacobian matrix is deduced and the kinematic dexterity index and the workspace are introduced. In order to require better dexterity performance and larger workspace, multi-objective optimizations based on non-dominated sorting genetic algorithm II (NSGA-II) are performed in terms of dexterity performance and workspace of the proposed parallel manipulator. By optimizing the design variables including geometric dimensions of the parallel manipulator, the dexterity and workspace of the proposed parallel manipulator have been greatly improved. The analytic results are verified by simulation to show that the parallel manipulator has a good application prospect.

Index Terms—Dimensional synthesis, parallel manipulator, jacobian matrix, dexterity, workspace.

I. INTRODUCTION

Compared with traditional serial robotics, the parallel manipulators have some significant advantages including higher accuracy, higher stiffness, higher loading capacity, less error accumulation, and more convenient for inverse kinematic control. Up to now, parallel manipulators as an emerging area have shown great application potential for the development of machine tools in virtue of their excellent mechanical properties mentioned above [1]-[4]. However, the main disadvantage of the parallel manipulators is the limited workspace, because these legs will collide to each other and each leg has several different joints which limit the motion of parallel manipulators as well. Therefore, it is quite necessary to perform the dimensional syntheses optimum analysis in the early design stage for the next-generation robotic systems for industrial applications with higher accuracy and higher speed, more robustness and adaptability, and so on.

Dexterity is one of the most important performances to evaluate the parallel kinematic machines (PKM) good or bad [5]-[7], which reflects on the dexterous capability over the task workspace. Meanwhile, dimensional synthesis aims to improve the performance of a mechanical robotic system and

is an important procedure of parallel manipulators from their schematic diagram to the structural design, in which two challenging issues associated with kinematic performance index and optimal design method are involved [8]. Therefore, it is quite necessary to perform the dexterity analysis and to conduct optimal design of the parallel kinematic machines in the early design stage. The determinant and condition number of Jacobian matrix have been widely used in performance evaluation and optimal design of parallel manipulators [9], [10]. The workspace as the other most important index of the parallel manipulator is computed considering the link length and angle constraints on the basis of the inverse solution of the kinematics. It is particularly important to notice that, we should consider workspace during the design of the mechanism which determines the size of the parallel manipulator operating range actuator. And it is the foundation for the subsequent optimization. The workspace of parallel manipulator is mainly restricted by driven chains and constraint chain, as well angle constraints. Li researched the workspace of the five degrees of freedom parallel manipulator based on the Monte Carlo simulation method [11]. Yet, Li *et al.* [12] pointed out that the unit inconsistent problem arises when these indices are applied to parallel manipulators with mixed DOFs of translations and rotations. Apart from the kinematic performance index associated with Jacobian matrix, some efforts were made to evaluate the motion or force transmissibility using screw theory [13]. Tsai [14] defined the virtual coefficient between the transmission wrench screw and the input velocity screw, while Wang *et al.* [15] proposed the input or output transmission index and dealt with kinematic evaluations of a 3-DOF parallel manipulator successfully.

Upon the architecture of presented novel parallel manipulator that can generate three translations degrees of freedom, its kinematic model and mathematical model are established and the Jacobian matrix is derived for the first time. In which two essential procedures are involved, one is the kinematic dexterity performance index by means of condition number of Jacobian matrix, the other is the design method adopted to perform the multi-objective dimensional synthesis using an artificial intelligence approach that is non-dominated sorting genetic algorithm II (NSGA-II). Conclusions are provided in the last section of this paper.

II. KINEMATICS ANALYSIS

A. Structure of the Parallel Manipulator

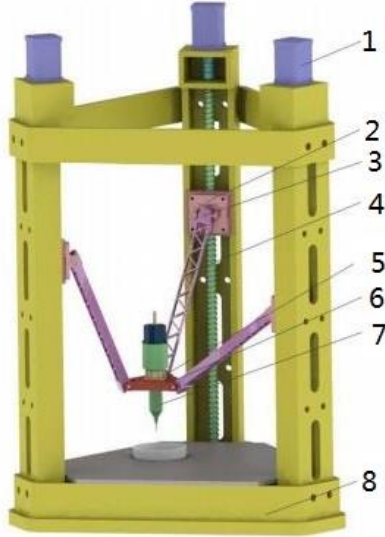
The CAD and schematic model of the proposed parallel manipulator in Fig. 1 and 2 show that it consists of a fixed base, a moving platform and the three identical legs which are

Manuscript received December 8, 2018; revised April 12, 2019. This work was supported by the financial support of the Teaching Reform and Innovation Project for Shanxi Universities under Grant No.J2018136 and General Project of Changzhi Medical College under Grant QDZ201655.

Ruixia Li is with the Faculty of Biomedicine Engineering at Changzhi Medical College, Changzhi, Shanxi, China (e-mail: liruixiaczh@126.com).

Fenxia Li is with the Faculty of NC Engineering at Shanxi Institute of Mechanical and Electrical Engineering, Changzhi, Shanxi, China (e-mail: lfx5168@163.com).

featured as PUU in sequence (where P is prismatic joint, and U is universal joint, respectively). In addition, the active P joint on the fixed base along the rail is driven by a lead screw linear actuator. The radius of moving platform at universal joints A_1, A_2, A_3 is r_a , the fixed base $\Delta B_1 B_2 B_3$ is an equilateral triangle whose circumradius is r_b . The sliders of P joints C_1, C_2, C_3 are restricted to move along the sliding rails. The motion distance of active joint can be expressed as q_i ($i=1,2,3$), and the constant length rod of length is l_i . The parallel manipulator has three degrees of freedom, that is, translation along x -axial, y -axial and z -axial.



1.Motor 2.Primstaic joints 3.Universal Joint 4.body frame
 5. Universal Joint 6. Moving platform 7.Machine Tool 8.
 Fixed Platform
 Fig. 1. The CAD model of 3-PUU parallel kinematic
 machine tool.

For the purpose of analysis, as represented in Fig. 2. We assign a fixed Cartesian coordinate system O - xyz at the centered point O of the fixed base $\Delta B_1 B_2 B_3$, and the x -axial is coincident with point B_1 , and the z -axis is perpendicular to the platform whose direction is downward, and the y -axial direction is determined by the right-hand rule. Similarly, we establish a moving coordinate system O' - $x'y'z'$ whose origin point O' is the geometric center of the moving platform, and let x' axis and x axis are parallel to each other initially and x axis is coincident with point A_1 . Meanwhile, z' axis direction is downward, and y' axial direction is determined by the right-hand rule.

B. Inverse kinematics and Jacobian Analysis

The position vectors of A_i points are expressed in the moving coordinate frame as the following:

$$A_1 = [0 \quad r_a \quad 0]^T \quad (1a)$$

$$A_2 = [r_a \sin 60^\circ \quad r_a \cos 60^\circ \quad 0]^T \quad (1b)$$

$$A_3 = [-r_a \sin 60^\circ \quad r_a \cos 60^\circ \quad 0]^T \quad (1c)$$

Similarly, the position vectors of points B_i are expressed in the reference frame as the following:

$$B_1 = [0 \quad -r_b \quad q_1]^T \quad (2a)$$

$$B_2 = [r_b \sin 60^\circ \quad r_b \cos 60^\circ \quad q_2]^T \quad (2b)$$

$$B_3 = [-r_b \sin 60^\circ \quad r_b \cos 60^\circ \quad q_3]^T \quad (2c)$$

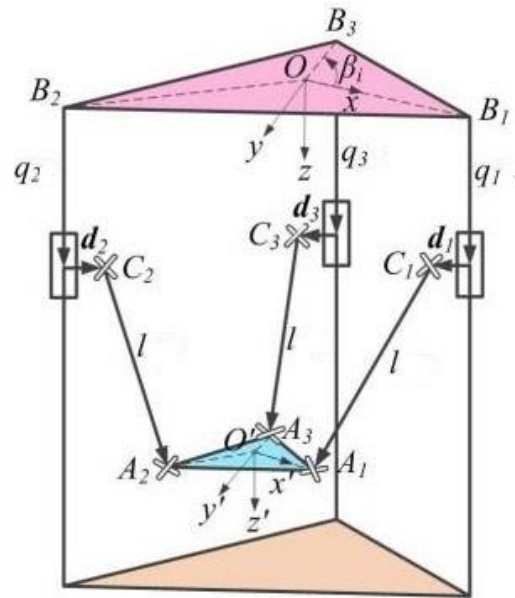


Fig. 2. The schematic of 3-PUU parallel manipulator.

where q_1, q_2, q_3 represents the position of the three slides.

Since the parallel manipulator has three translational degrees of freedom, the moving coordinate system has no rotation relative to the fixed coordinate system, so the position vectors of points A_i are expressed in the fixed coordinate frame as the following:

$$A_{iO} = A_{iO'} + OO' \quad (i=1,2,3) \quad (3)$$

where the position vector of the reference point O' in the reference coordinate frame can be written as $[x \quad y \quad z]^T$.

The inverse position solution of the parallel manipulator can be represented by the constraint equation of the fixed length leg

$$L = \|C_{iN} - A_{iN}\| \quad (i=1,2,3) \quad (4)$$

where L denote the vector of the three identical legs. We also can write as

$$\|A_i C_i\| = \|{}^O A_i - {}^O C_i\| = l_i^2 \quad (5)$$

From the definition of the vector model, we can get the equation:

$$q_1 = z - \sqrt{L^2 - x^2 - (y - r_a + r_b)^2} \quad (6a)$$

$$q_2 = z - \sqrt{L^2 - [x + (r_a - r_b) \sin 60^\circ]^2 - [(y + (r_a - r_b) \cos 60^\circ)]^2} \quad (6b)$$

$$q_3 = z - \sqrt{L^2 - [x - (r_a - r_b) \sin 60^\circ]^2 - [(y + (r_a - r_b) \cos 60^\circ)]^2} \quad (6c)$$

Once the structure parameters r_a, r_b are sure, and the position parameters (x, y, z) with respect to the moving

platform are given, the driven chain lengths are obtained by the equation (6), that is, the inverse solution of kinematics position.

The operational velocity vector of the end-effector can be expressed as

$$v = [v_x \quad v_y \quad v_z]^T \quad (7)$$

Equation (6) of the time derivative can be rewritten in matrix form

$$\dot{q}_i = \mathbf{J}v \quad (8)$$

In which \mathbf{J} is the 3 by 3 velocity Jacobian matrix of the parallel manipulator.

III. PERFORMANCE INDEX

In order to make the parallel manipulator has good kinematics performance in the workspace, the kinematics dexterity and workspace optimization was studied and the global condition index and the workspace were introduced as the evaluation index [16].

A. Dexterity

Condition number of Jacobian matrix at the point is used to measure the dexterity [17], which can be calculated as follows:

$$\kappa = \frac{\sqrt{\lambda_{\max}(\mathbf{J}^T \mathbf{J})}}{\sqrt{\lambda_{\min}(\mathbf{J}^T \mathbf{J})}} \quad (9)$$

where $\lambda_{\max}(\mathbf{J}^T \mathbf{J})$ and $\lambda_{\min}(\mathbf{J}^T \mathbf{J})$ are the maximum and minimum eigenvalue, respectively.

When $k=1$, the mechanism owns the best transmission performance, in this case, the configuration is called isotropic. When the condition number trends to infinite, the mechanism is in a singular configuration.

In order to obtain the kinematics performance in the whole workspace, Gosselin and Angeles [18] proposed the global condition index, which is a measure of its kinematics precision and control accuracy performance of the parallel manipulator tool. The dexterity of the manipulator over the entire workspace or reasonable central portion of the workspace should be considered. The global condition index can be defined as the ratio of the integral of the inverse condition numbers calculated in the whole workspace, dived by the volume of the workspace, namely

$$\kappa_J = \frac{\int_W kdW}{\int_W dW} \quad (10)$$

In which W is the workspace of parallel mechanism, and κ is the local condition number defined as the reciprocal of the condition of the Jacobian matrix at a particular position. It is noteworthy that the global condition number index is bounded as $(0, 1)$. If the κ_J approach zero, the mechanism has a bad global performance and as the κ_J approaches one the mechanism has a good global performance. Therefore, we should make the optimization objective κ_J maximization.

B. Workspace

The workspace is the set of all points where the reference point of the moving platform can achieve in the space, which is the area of the robot manipulator, and is a pretty important index to measure the performance of the parallel manipulator. The workspace of a parallel manipulator can be defined as a reachable region of the origin of a coordinate system attached to the center of the moving platform. The algorithm to define the workspace is based on the boundary of the workspace in a Cartesian coordinate system with constrains of the limitation of actuators and angle of joints.

In this paper, we established the global condition index based on the workspace according the previous section, so we need to solve the workspace firstly. The workspace can be expressed as

$$W = \{(x, y, z) \in R \mid f(x, y, z) \leq 0\} \quad (11)$$

where $f(x, y, z) \leq 0$ denoted the constraint condition, that is:

(1) The active chains length constraints can be expressed by

$$q_{\min} \leq q_i \leq q_{\max} \quad (12)$$

where q_{\max} denoted the maximum movement position, q_i denoted the slider position of the i -th leg, and q_{\min} denoted the minimum movement position.

(2) The mechanism was non-singular configuration, namely, the determinant of the Jacobian matrix was not equal to zero.

IV. DIMENSIONAL SYNTHESIS

In general, a constrained multi-objective optimization problem can be explained as solving the problem of searching for the vector of decision variables to optimize the objective functions. A genetic algorithm is a nontraditional optimization method based on the Darwinian survival-of-the-fittest evolutionary theory. The method is accomplished using genetic operators such as selection, crossover, and mutation until it converges towards an optimal solution. In this paper, a non-dominated sorting-based multi-objective dimensional synthesis method called non-dominated sorting genetic algorithm II (NSGA-II), is adopted to find the optimum solution [19-20].

A. The Optimization Design Space

The scale parameters of the parallel manipulator were radii of the moving platform r_a and the fixed platform r_b . Select the link length of the active chains l_i , the searching workspace range is defined in $x^2 + y^2 \leq 0.15^2, 0.2 \leq z \leq 1.8$, and the optimization objective functions for κ_J and the workspace W , then the multi-objective optimization model can be expressed as

$$\begin{cases} \max f_1 = \kappa_J(r_a, r_b, l) \\ \max f_2 = W(r_a, r_b, l) \end{cases} \quad (13)$$

$$\text{s.t.} \begin{cases} 0.03 \leq r_a \leq 0.06 \\ 0.08 \leq r_b \leq 0.16 \\ 0.02 \leq l_i \leq 0.4 \\ 0.1 \leq q_i \leq 1.5 \end{cases} \quad (14)$$

B. The Optimization Design Algorithm

The algorithm parameters are set to: Maximum number of generation: 40; Population size: 30; Mutation rate: 0.05; Crossover rate: 0.9; Selection strategy: Tournament; Tournament size: 2; Crossover type: Intermediate; Crossover ratio: 1; Mutation function: Adaptive feasible.

The multi-objective optimization of dexterity and workspace have been done separately, however, sometimes the dexterity and workspace have to be considered simultaneously, thus the multi-objective optimization must be taken into account, which is required to find all the possible tradeoffs among the conflicting objective functions. To achieve that, a GA based multi-objective optimization toolbox in MATLAB was used to optimize the dexterity and workspace. The optimization results corresponding design variables are shown in Table I (units: m).

TABLE I: ARCHITECTURE PARAMETERS

r_a	0.45m
r_b	1.25m
l	0.35m

For illustration of the dexterity and workspace of the parallel manipulator presented in this paper, the numerical simulation solutions should be calculated to analyze the performance index characteristic of the mechanism. So next, we will have some cases analysis and discussion.

V. CASE ANALYSIS AND DISCUSSION

For the optimum design of the robotic manipulators, the idea of global index is proposed based on averaging the local index in a workspace and providing the changing trend under different variables.

A. Simulation Results and Discussion

On the basis of the inverse kinematics and derivation of the dexterity calculation formula, as well as the constraints, the dexterity and workspace distribution of the crossbar parallel machine tool are solved by using the MATLAB program.

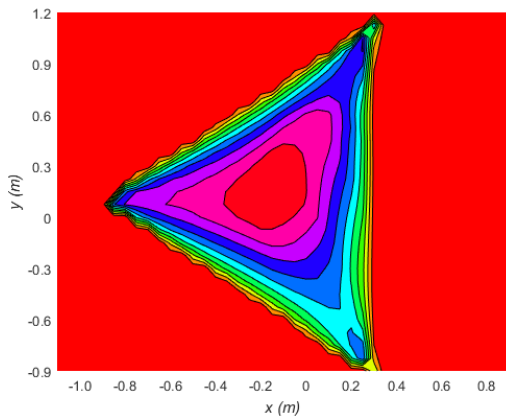


Fig. 3. The dexterity before optimization when $z=1.0m$

parallel manipulator, which demonstrate that the dexterity performance of the parallel manipulator improved and the smoothness also increased, so the optimization design is feasible. It can be seen from the comparison of the Fig. 3 and Fig. 4 that the optimized parallel manipulator has more stable dexterity capability than before optimization.

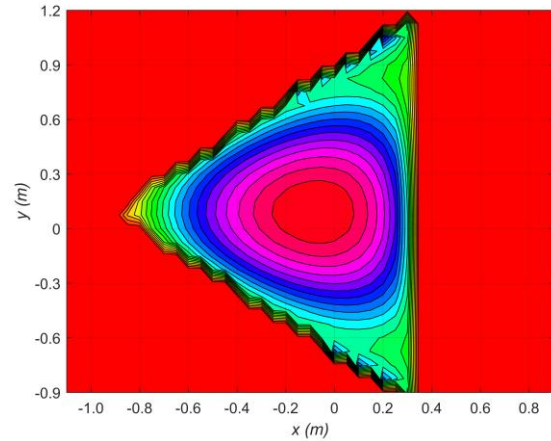


Fig. 4. The dexterity after optimization when $z=1.0m$.

Through the inverse position and the constraint condition, the discretizing position of the moving platform can be numerically determined and depicted in Fig. 5-Fig. 8.

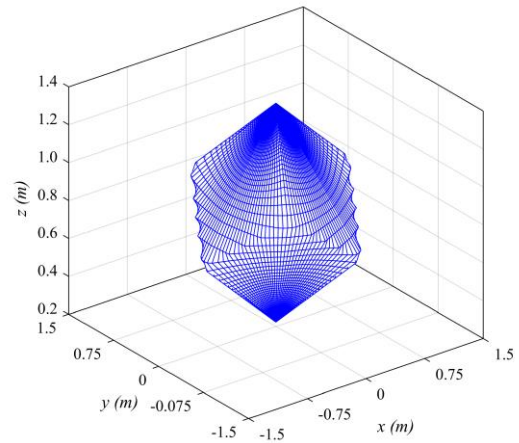


Fig. 5. The 3D view of the workspace before optimization.

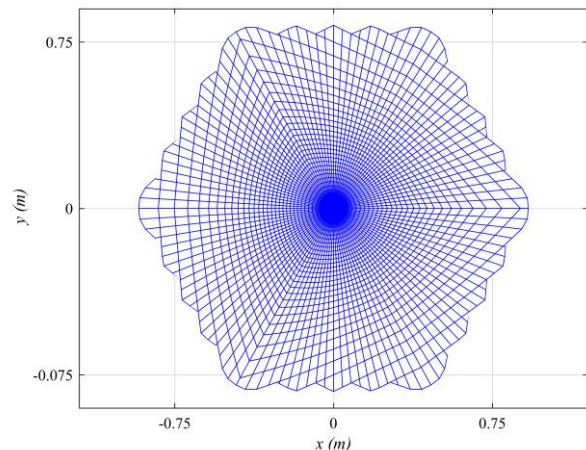


Fig. 6. The top view of the workspace before optimization.

The Fig.3 and Fig.4 show the dexterity distribution in the area of $-1.2 m < x < 1.0 m$, $-1.2 m < y < 1.0 m$ and $z=1.0 m$ of the

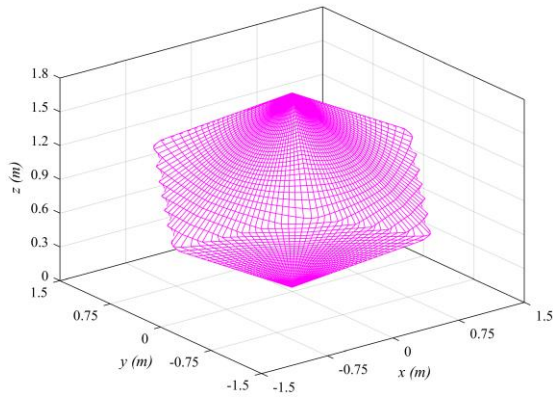


Fig. 7. The 3D view of the workspace after optimization.

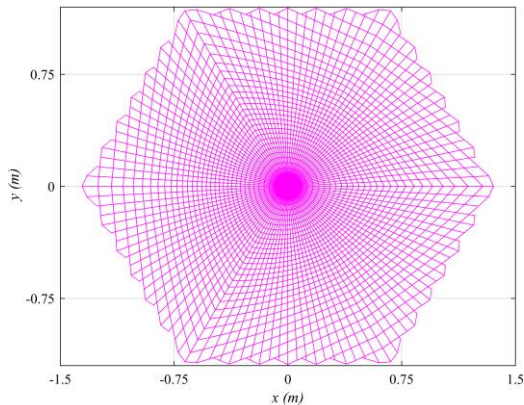


Fig. 8. The top view of the workspace after optimization.

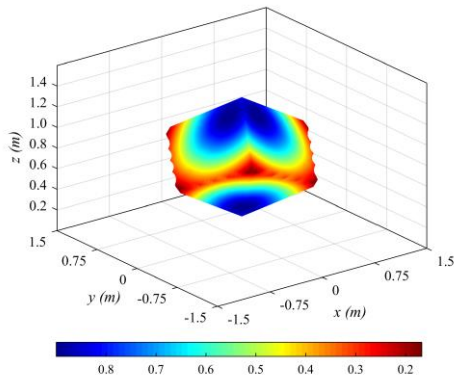


Fig. 9. The dexterity distribution in workspace before optimization (3D view).

The Fig. 5 and 7 show the 3D view workspace of parallel manipulator before and after optimization. In particular, the optimized workspace can contain cube with 0.15 m length and 1.8 m height, while before, the contain cube with 0.75 m length and 1.4 m height. Furthermore, according to the proposed optimal design method, it is obvious that the parallel manipulator after optimization has much larger boundary workspace region than before optimization through the comparison between the Fig. 6 and 8. Thus, the optimal design results for prescribed workspace provide some structural improvements of this type of parallel manipulator.

In the following, we evaluate the dexterity performance distribution in the workspace before and after optimization

for different structural parameters of the parallel manipulator. The dexterity distributions for different elevation ranges before and after optimization are depicted in Fig. 9-Fig. 12, respectively.

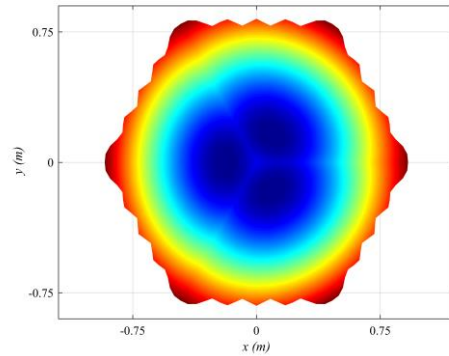


Fig. 10. The dexterity distribution in workspace before optimization (top view).

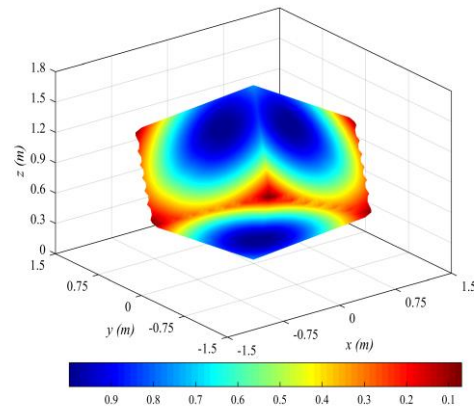


Fig. 11. The dexterity distribution in workspace after optimization (3D view).

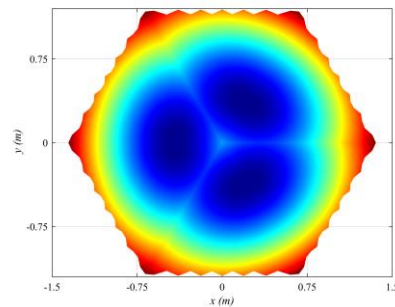


Fig. 12. The dexterity distribution in workspace after optimization (top view).

Based on the analysis of dexterity and workspace, the multi-objective optimization issues are addressed. Fig.10 and Fig.12 illustrated the dexterity distribution in the workspace. Obviously, by revising and improving the structural parameters, the dexterity and workspace of the parallel manipulator are significantly enhanced and have much better dexterity performance regions. In the range of dexterity, with the position increasing, the condition number gradually raised, indicating the dexterity of the parallel manipulator is getting worse. However, in the search space, the range of the condition number varies between 0.1 and 0.9, and is far away from the ill

condition region, indicating that the parallel manipulator has no singular configuration in the ideal workspace, and has good kinematic dexterity in the whole reachable workspace.

VI. CONCLUSIONS

This paper develops a 3-DOF parallel manipulator multi-objective dimension optimization design that aims to increase the dexterity and workspace of the proposed parallel manipulator. The main contributions of this paper are summarized as follows:

- 1) This paper is focused on the multi-objective optimization of a parallel manipulator with three degrees of freedom that aims to simultaneously increase the dexterity and the workspace.
- 2) The kinematics analysis and Jacobian matrix is derived and multi-objective optimizations are implemented by searching for the relationship between the dimensional parameters and the objective functions.
- 3) The computation results showed that the performance of the proposed manipulator can be largely enhanced and the proposed methodology of multi-objective optimization can be also applied to the dimensional synthesis of other parallel manipulators.

In future works, dynamics problem and motion control will be further considered, and a prototype will be manufactured and several relevant experiments will be conducted.

ACKNOWLEDGMENT

The authors would like to acknowledge the financial support of the Teaching Reform and Innovation Project for Shanxi Universities under Grant No.J2018136 and General Project of Changzhi Medical College under Grant QDZ201655. Furthermore, the work described in this paper is considerably revised and added some more information by our team. The author would like to express her gratitude to Ms. Fenxia Li who provides useful information in mechanical structure design and Ms. Chunhua Shi, Ms. Ping Li and Mr. Dianhua Bian who help us to collect the data in study area.

REFERENCES

- [1] G. Yu, L. Wang, J. Wu *et al.*, "Stiffness modeling approach for a 3-DOF parallel manipulator with consideration of nonlinear joint stiffness," *Mechanism and Machine Theory*, 2018, vol. 123, pp. 137-152.
- [2] B. Bounab, "Multi-objective optimal design based kineto-elastostatic performance for the delta parallel mechanism," *Robotica*, 2016, vol. 34, no. 2, pp. 258-273.
- [3] R. Aidan, J. Padayachee, and G. Bright, "Research and development of a 5-axis hybrid kinematic CNC machine," in *Proc. 24th International Conference on Mechatronics and Machine Vision in Practice*, Auckland, 2017, pp. 1-6.
- [4] J. Wu, Y. Gao, B. Zhang *et al.*, "Workspace and dynamic performance evaluation of the parallel manipulators in a spray-painting equipment," *Robotics and Computer Integrated Manufacturing*, 2017, vol. 44, pp. 199-207.
- [5] X. F. Fang, S. C. Zhang, Q. H. Xu *et al.*, "Optimization of a crossbar parallel machine tool based on workspace and dexterity," *Journal of Mechanical Science and Technology*, 2015, vol. 29, no. 8, pp. 3297-3307.
- [6] G. Carretero and A. Juan, "Formulating jacobian matrices for the dexterity analysis of parallel manipulators," *Mechanism and Machine Theory*, 2006, vol. 41, no. 12, pp. 1505-1519.
- [7] J. Kim, F. C. Park, S. J. Ryu *et al.*, "Design and analysis of a redundantly actuated parallel mechanism for rapid machining," *IEEE Transactions on Robotics and Automation*, 2001, vol. 17, no. 4, pp. 423-434.
- [8] X. J. Liu, Q. M. Wang, and J. Wang, "Kinematics, dynamics and dimensional synthesis of a novel 2-DoF translational manipulator," *Journal of Intelligent and Robotic Systems*, 2005, vol. 41, no. 4, pp. 205-224.
- [9] W. Liping, X. Huayang, and G. Liwen, "Kinematics and inverse dynamics analysis for a novel 3-PUU parallel mechanism," *Robotica*, 2017, vol. 35, no. 10, pp. 2018-2035.
- [10] J. Merlet, "Jacobian, manipulability, condition number, and accuracy of parallel robots," *Journal of Mechanical Design*, 2006, vol. 128, no. 128, pp. 199-206.
- [11] R. X. Li and F. X. Li, "Workspace performance investigation of a multi-DOF parallel robot manipulator," *International Journal of Engineering and Technology*, vol. 2019, vol. 11, no. 1, pp. 1-5.
- [12] Q. Li, Q. Chen, C. Wu *et al.*, "Geometrical distribution of rotational axes of 3-[P][S] parallel mechanisms," *Mechanism and Machine Theory*, 2013, vol. 65, no. 7, pp. 46-57.
- [13] H. T. Liu, T. Huang, A. Kecskeméthy, D. G. Chetwynd, and Q. Li, "Force/motion transmissibility analyses of redundantly actuated and overconstrained parallel manipulators," *Mechanism and Machine Theory*, 2017, vol. 109, pp.126-138.
- [14] M. J. Tsai and H. W. Lee, "Generalized evaluation for the transmission performance of mechanisms," *Mechanism and Machine Theory*, 1994, vol. 29, no. 4, pp. 607-618.
- [15] J. Wang, C. Wu, and X. J. Liu, "Performance evaluation of parallel manipulators: Motion/force transmissibility and its index," *Mechanism and Machine Theory*, 2010, vol. 45, no. 10, pp. 1462-1476.
- [16] Z. Gao and D. Zhang, "Performance analysis, mapping, and multiobjective optimization of a hybrid robotic machine tool," *IEEE Transactions on Industrial Electronics*, 2015, vol. 62, no. 1, pp. 423-433.
- [17] X. F. Fang, S. C. Zhang, Q. H. Xu *et al.*, "Optimization of a crossbar parallel machine tool based on workspace and dexterity," *Journal of Mechanical Science and Technology*, 2015, vol. 29, no. 8, pp. 3297-3307.
- [18] C. Gosselin and J. Angeles, "A global performance index for the kinematic optimization of robotic manipulators," *ASME Journal of Mechanical Design*. 1991, vol. 113, pp. 220-226.
- [19] H. Li and Q. Zhang, "Multi-objective optimization problems with complicated pareto sets, MOEA/D and NSGA-II," *IEEE Transactions on Evolutionary Computation*, 2009, vol. 13, no. 2, pp. 284-302
- [20] B. Lian, T. Sun, Y. Song *et al.*, "Stiffness analysis and experiment of a novel 5-DoF parallel kinematic machine considering gravitational effects," *International Journal of Machine Tools and Manufacture*, 2015, vol. 95, pp. 82-96.



Ruixia Li was born in Qinshui, Shanxi, on June 26, 1979. She received the B.E. degree in mechanical engineering from the Taiyuan University of Technology, Taiyuan, China, in 2001; the M.E. degree in mechanical and electronic engineering from the Taiyuan University of Technology, Taiyuan, China, in 2007.

She is currently an associate professor with the Department of Biomedicine Engineering, Changzhi Medical College, Shanxi. As the Academic leader of Rehabilitation equipment engineering, her current research interests include parallel mechanism, multi-objective optimization, control strategy research, rehabilitation robot and medical instruments.



Fenxia Li received the B.E. degree from the Taiyuan University of Technology, Taiyuan, China, in 2000, the M.E. degree from the Taiyuan University of Technology, Taiyuan, China, in 2008.

She is currently a professor with the Department of NC Engineering, Shanxi Institute of Mechanical and Electrical Engineering. As a leader of the Huang danian's style teacher team, she takes the lead in innovation and solves technical problems. Her

current research interests include NC processing, NC technology, mechanical processing.