Model for the Location of Transfer Hub in Clustered Cities

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***Abstract*—The location method about transfer hub in clustered city is different from the method in ring type city because the different layout of the land in the two type cities. In this paper, the defect about the one element analytic model method and the multivariate optimisation analysis model produce unsuitable result when used for location of transfer hub in clustered cities is pointed out, and the location of transfer hub in clustered cities is supposed as location of semidefinite form, the bilevel programming model for the location of transfer hub in clustered cities is built based on the land condition and the transportation condition, and the Interactive relationship between the distribution of transfer hub and traffic network and traffic flow, the solve step is given, and the example about the two clusters of one clustered cities is gave to test and verify the model，the result make clear that the model can supply theoretical support for the location of transfer hub in clustered cities.**

***Index Terms*—Traffic engineering, clustered cities, Location of transfer hub, location of semidefinite form, bilevel programming model.**

I. Introduction

Transfer hub is the important center for passenger transfer in internal and external city, and passenger transfer for different modes of transportation in internal city. The localization of transfer hub is affected by many factors, as well as involvement of wide range. [1], [2] It determines the efficiency of passenger transfer directly and is one of the hot issues of the transportation sector. At present domestic and foreign the methods to site the transfer hub include the one element analytic model method and the multivariate optimisation analysis model method, both the methods are mainly used for ring type city, and will produce unsuitable result when used in clustered cities such as the result is just suitable for individual but not the whole clustered cities. [2] Besides, The existing models of the localization do not consider the influences of the whole traffic network because they all assume that the traffic network already be in a consistent state in the whole planning area when the transfer hub had built. It Will produce and attract a large quantity of traffic flow in the range of services to cause excessive traffic pressure of the surrounding road network which it is called gathered effects and of course it is needed to widen or built the roads and result in the change of the road network. [3], [4] We should base on the conditions of construction land and traffic, combine with the layout of transfer hub, traffic network construction and traffic flow to analyse the interaction of them, [5], [6] so that, we can find the proper method to site the transfer hub.

II. Basic Assumptions

We assume that the problem of site selection of transfer hub between the clustered city belongs to half stereotyped location problem. Then each of clustered cites that were connected with several service channels will build a transfer hub. In this way, road network of each group can be simplified to a weighted graph $W=(N, A, P)$ In this equation, N is stand for a set of points in graph W which represent the set of road network; A is stand for a set of arcs in graph W which represent the set of road sections; P is stand for weighting matrix in graph W which represent the impedance between each of the road sections. In this kind of network, “r” and “s” stand for sets of point; the road node which arc length is “a” can also be replaced by the form such as (r,s); X_a represents the passenger flow of the section; P_a that means the cost of transport in the section represents the weight value and monetary unit of Yuan is used as dimension in the balance of charge.[7] Besides, the cost in one section can only be associate with its own traffic flow, that’s to say $P_a = P_a(X_a)$.

In the road network, a certain amount of traffic volume that come from each nodes will be carried by the hole network. [8] Variable named b_r is defined. b_r =1 shows that node “r” is selected as a hub. On the contrary, it means the node is ordinary and $r \in N$. All such variables are gathered by set B. When the hub is completed, the mass mode of transport is adopted among the passages of the hubs. The effect of the mass mode can decrease the average trip cost of passengers. It reduces impedance of the travel route. In other words, a transport corridor will come into being among the different travel routes that exist between the two nodes “r” and “s”. The cost function of the section will also turn into $p = P_a(X_a)$. A is a set of the sections which is made up of a and it forms the transport corridor between “r” and node “s”. In addition, A^* is selected as the set of sections which is named $a*$ and $a*$ is out of the corridor. f_r is defined as the fixed value of the cost of each hub. Distributive volume that is get by the size effect of each sections should consider the possible extension caused by

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the volume and we define g_a as the unit cost.

III. Modeling Process

The external passenger demand and external transport corridors between clusters were considered, and the interactive feedback relationship of transfer hub layout、construction of transport network and traffic stream in road network was analyzed. The Bi-level Programming location model of transfer hub in clustered cities was established.

A. The Establishment of the Upper Model

It is assumed that the collection of alternative transfer hubs in clustered city is obtained by calculating the size of the transfer hub, the object region is divided into n partitions, only one transfer hub can be established in each partition, and the total quantity of established transfer Hubs is m. So the cost of construction of m transfer hubs is required:

$$\min U=\sum_{i=1}^{M}\sum_{r=1}^{n}b_{ir}z_{ir} \tag{1}$$

where Z is an M×N order matrix about Z_{ir}, Z_{ir} is decision variable and Z_{ir} is 0 or 1;B is an M×N order matrix about b_{ir} , and b_{ir} is the cost of construction land use when i hub is located in r partition.

The total cost of the expansion of road section and travel of passenger in the planning object located in its cluster:

$$\min(Y,Z)=\lambda\left\{\sum_{a'\in A'}x_{a'}\cdot t_{a'}\left(x_{a'}\right)+\left[\sum_{a''\in A''}x_{a''}\cdot t_{a''}\left(x_{a''},y_{a''}\right)+\sum_{a'''\in A'''}x_{a'''}\cdot t_{a'''}\left(x_{a'''},y_{a'''}\right)\right]\right\}+\left[\sum_{a''\in A''}g_{a''}\left(y_{a''}\right)+\sum_{a'''\in A'''}g_{a'''}\left(y_{a'''}\right)\right] \tag{2}$$

Constraint condition:

$$s.t.\quad q'_{rs}=q_{rs}+\sum_{i=1}^{M}z_{ir}P_{is}+\sum_{i=1}^{M}\sum_{j=1}^{M}z_{ir}z_{js}u_{ij} \tag{3}$$

$$Z_{ij}=0 \text{ or } 1\ \ (1\le i\le M\ ,1\le r\le K) \tag{4}$$

$$y_{a''}=2(c_{a''}-x_{a''})\ge 0 \qquad (1\le a''\le m) \tag{5}$$

$$y_{a'''}=c_{a'''}-x_{a'''}\ge 0 \qquad (1\le a'''\le m) \tag{6}$$

$$\sum_{a'\in A'}x_{a'}t_{a}\left(x_{a'}\right)\ge\left[\sum_{a''\in A''}x_{a''}t_{a''}\left(x_{a''},y_{a''}\right)+\sum_{a'''\in A'''}x_{a'''}t_{a'''}\left(x_{a'''},y_{a'''}\right)\right] \tag{7}$$

where $x_a{}'$ is the traffic volume of road section a' between general nodes; $x_a{}''$ is the traffic volume of road section a'' between transfer hubs in different clusters; $x_a{}'''$ is the traffic volume of road section a''' between transfer hubs in a cluster; $t_{a'}(x_{a'})$ is the travel time of road section a' between general nodes; $t_{a''}(x_{a''})$ is the travel time of road section a'' between transfer hubs in different clusters; $t_{a'''}(x_{a'''})$ is the travel time of road section a''' between transfer hubs in a cluster; λ is a dimensionless conversion coefficient; $y_a{}''$ is the increase of road section capacity in different clusters; $y_a{}'''$ is the increase of road section capacity in a cluster; $g_a{}''$ is the cost of the expansion of road section in different clusters; $g_a{}'''$ is the cost of the expansion of road section in a cluster;

The main trunk road connecting each cluster is the Key sections in whole traffic network, if transfer hubs are located at both ends of the main trunk road connecting different clusters, the transfer hubs will make a strengthened links between clusters, the intensity descripping contact between different clusters of optimization model:

$$\max E=\sum_{i,j=1}^{m}\sum_{r,s=1}^{n}e_{ij}z_{ir}z_{js} \tag{8}$$

$$e_{ij}=\sum_{i,j\in a''}\frac{x_{ij}}{t_{ij}^{\beta}} \tag{9}$$

where e_{ij} is the intensity descripping contact between different clusters; x_{ij} is the traffic volume of road section between transfer hubs in different clusters; t_{ij} is the impedance of road section between transfer hubs in different clusters.

Multiple objective optimization can be obtained by merging the objective functions above. So the location Model of transfer hub in clustered cities is:

$$\min F=\lambda\left\{\sum_{a'\in A'}x_{a'}\cdot t_{a'}\left(x_{a'}\right)+\left[\sum_{a''\in A''}x_{a''}\cdot t_{a''}\left(x_{a''},y_{a''}\right)+\sum_{a'''\in A'''}x_{a'''}\cdot t_{a'''}\left(x_{a'''},y_{a'''}\right)\right]\right\}+\left[\sum_{a''\in A''}g_{a''}\left(y_{a''}\right)+\sum_{a''\in A''}g_{a'''}\left(y_{a'''}\right)\right]+\sum_{i=1}^{M}\sum_{r=1}^{n}b_{ir}z_{ir}-\mu\sum_{i,j=1}^{m}\sum_{r,s=1}^{n}e_{ij}z_{ir}z_{js} \tag{10}$$

where F is the comprehensive objective function about the travel time of passenger in clustered cities, the cost of the expansion of road, he cost of constructing transfer hubs and the optimal intensity of contact between different clusters.

B. Mathematical Programming Problem of the User Equilibrium Assignment

The model is:

$$\min : F\left(x\right)=\sum_{a\in A}\int_{0}^{x_a}t_a\left(w,y\right)d_w \tag{11}$$

$$s.t.\ :\sum_{k}f_k^{rs}=q'_{rs},\qquad \forall r,s \tag{12}$$

$$x_a = \sum_{r,s}\sum_{k} f_k^{rs} \cdot \delta_{a,k}^{rs} \qquad \forall_a \tag{13}$$

$$f_k^{rs} \ge 0, \qquad \forall r,s \qquad \forall k \tag{14}$$

The BPR Model can be used to indicate the impedance of road section in formulas above.

$$t_a = t_a^0 \left[1 + \theta \left(\frac{f_a}{c_a'} \right)^{\varphi} \right] \tag{15}$$

where t_a^0 is the necessary time of Vehicle free driving when the traffic volume is 0 in road section; c_a' is the actual capacity of road section a; θ is a parameter, θ normally values 0.15; 、φ is also a parameter, φ normally values 4.0.

IV. Solution of the Model

Step1：initialize the expansion of the amount of the planned traffic network sections $y''_{a''}$ 、$y'''_{a'''}$ ，initial value：$y''_{a''} = (0, 0 \cdots, 0)$ ，$y'''_{a'''} = (0, 0 \cdots, 0)$ that is, the initialization of the transport network is the status quo network；Make $k = 0$ ；

Step2：Take $y''^{k}_{a''}$ 、$y'''^{k}_{a'''}$ into the upper objective function, and the problem is converted to a general hub location problem.

Step3：List all eligible matrix Z .

Step4：solve the lower users' equilibrium assignment problem for x 、t under the current traffic network conditions.

Step5：take x 、t into the objective function to get solutions of all F and get the Z^{K+1} which makes Z minimum.

Step6：take Z^{K+1} into the objective function and the problem is transformed into a network design problem.

Step7: Combining simple network design algorithm with experts' advice, we proposed expansion of some sections of the collection $y''^{k+1}_{a''}$ 、$y'''^{k+1}_{a'''}$.

Step8: Examine if there were significant differences among $y''^{k}_{a''}$ 、$y'''^{k}_{a'''}$,$y''^{k+1}_{a''}$ and $y'''^{k+1}_{a'''}$. If there were , set $k = k + 1$ and return Step 2 ; If not , $y''^{k+1}_{a''}$ 、$y'''^{k+1}_{a'''}$ and Z^{K+1} were the answer ,and all is over.

V. Analysis of Examples

We assume that there is a clustered city consisted by at least two groups. Between the groups, one is made up of eight nodes and twelve sections, and the other one is made by twelve nodes and sixteen sections. Those two groups are contacted by two roads. Now, we choose two nodes as the spare hubs in the transportation network of the groups. Besides, each sections is made to two-way traffic and the two sides of the section have the same volume. Detailed datum show on the next Table I. The costs of each hub are made to fixed value, and the cost matrix is F1=[f1r]1×8=[80 110 100 90 80 60 100 90]，F2=[f2r]2×12=[90 100 110 80 90 70 80 90 110 90 100 70], used the million as unit. Then the cost is set to pa(xa), pa(xa)=p0+Axa. When the hub is finished, the cost function of the sections which consist of the contact corridor between two groups is $\hat{p}_a(x_a) = p_0 + \frac{A}{10}$.
In this function, P_0 represents the constant of various sections of function, it is called empty flow matrix $P_1 = [p_{10}]_{1\times12}$ =[3 4 5 2 6 5 3 2 5 4 3 2] ，$P_2 = [p_{20}]_{2\times16}$ =[4 3 3 2 4 3 5 6 3 4 2 4 3 4 5 3], used the yuan as unit. The extended cost of each section have a matrix named G: G1=[g1a]1×12=[3 2 1 4 5 8 5 4 6 3 4 4]，G2=[g2a]2×16=[4 1 3 3 5 2 6 5 4 4 3 4 5 2 4 3 5], used the million as unit. Constant terms: A=25 Yuan/(per time), capacity of each section c_a =1800 million times per year.

The distributions of the traffic flow show on the Table II.

TABLE I: Result about Traffic Flow Distribution in Road Section Million Annual

No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14
x_a	3500	2550	1280	600	1200	1900	1850	2200	900	1150	900	2180	900	1200
No.	15	16	17	18	19	20	21	22	23	24	25	26	27	28
x_a	1600	1990	1850	2120	800	550	1200	1090	1100	1890	2100	1980	1560	1790

The results reveal that it will get the minimum value when to choose node 4 and node 20 as the hubs between group one and group two. The objective function can get the lowest cost which is 876280000 yuan.

VI. Conclusions

Papers to build a bi-level programming of Group city hubs, According to the group city special land use and traffic conditions, and With half finalize the design location problem as the prerequisite, considering of each group`s inward and outward of the access road and external passenger demand, and hubs layout and traffic network construction, network traffic flow`s interactive feedback relations . And then shows the method of calculation of the model and make use of specific examples to explain. The results show that the model can tour the city transfer hub site to provide theoretical support.

TABLE II: OD MATRIX MILLION ANNUAL

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
1		200	180					160			255	250						170			100		80				120	
2	200			150	380				100				300				120			210					100			90
3	180					230						200				180			200					200				
4		150					350		250	90	200		330	120	200							80				120		
5		380						350					210					170									180	
6			230									300					100				180							
7				350						110			210										100					120
8	160				350						280		200												120			
9		100		250								120					70							70				
10				90			110								250					100			90					
11	255			200				280				70						70									160	
12	250		200			300			120		70		90			120		150						80				
13		300		330	210		210	200				90		120		220			160			90						120
14				120									120		280			210			90					310		
15				200						250				280		140				130			90				110	
16			180									120	220		140		120					70			210			280
17		120				100			70							120			150			170		210		190		
18	170				170						70	150		210					200	220					160			
19			200										160				150	200				310		270			60	
20		210								100					130			220			90		180					170
21	100					180								90						90		120			150			
22				80									90			70	170		310		120		130			180		
23	80						100			90					90					180		130			290			100
24			200						70			80					210		270						70		90	
25		100						120								210		160			150		290	70		100		
26				120										310			190					180			100		90	
27	120				180						160				110				60					90		90		120
28		90					120						120			280				170			100				120	

tags：In the table,“0” is for empty ，nodes from1to12 is in the nodes of group 1 whilenodes from 13 to 28 is in group 2.

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