IACSIT International Journal of Engineering and Technology Vol.1,No.5,December,2009
ISSN: 1793-8236

VEGA Based Routh-Padé Approximants For Discrete Time Systems : A Computer-Aided Approach

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***Abstract*—A VEGA (Vector Evaluated Genetic Algorithm) based method to derive a reduced order (th-order) model for a given stable discrete-time system is presented. In this method not only stability is preserved and the first time-moments/Markov-parameters are fully retained but also the errors between a set of subsequent time-moments/Markov-parameters of the system and those of the model are minimized. The method is useful as it guarantees improvement as well as alleviates the problems of deciding the values of number of error functions to be minimized. Furthermore, the operations of the proposed method are carried out entirely in domain. The search area for GA is very wide and it usually converges to a point near global optima.**

***Index Terms*—Discrete-time systems, Model reduction, Routh-Padé approximants.**

I. Introduction

The approximation of linear high-order systems by a low-order model has received considerable attention due to the advantages of reduced computational effort and increased understanding of the original systems. Consequently, a large number of time-domain and frequency-domain simplification techniques have been developed to suit different requirements [1,2]. Amongst them, a frequency-domain method is Padé approximation in which 2r terms of the power series expansion (time moments) of the high-order (nth-order) transfer function are fully retained in low-order (rth-order) model. In some cases, Padé approximant may turn out to be unstable even though the original system is stable. To overcome stability problem, a number of stable reduction methods [3-7] based on retention of only r terms have been developed for discrete-time systems. However, matching of only terms may not generally be sufficient to ensure a good overall time response approximation and it is also important to note that, for overall time response approximation, both time moments and Markov parameters should be considered [8,9]. In [10], a bilinear Routh approximation (BRA) method has been proposed as an extension of RA method [11] to discrete systems but it has been found that BRA method of [10] may fail to produce good approximants [12] as [11] again deals with r terms matching. Further improvement over [12] is suggested in [13]. However, the method of [13] does not possess any optimal properties.

In a recent publication [14], a suboptimal bilinear Routh approximation (SBRA) method is presented which is an improvement over BRA method and can be used to improve bilinear Schwarz approximation [15-17]. The SBRA method is based on combining Routh technique and minimization of ISE. The last and parameters of BRA method are replaced by new parameters so that the ISE of impulse response of the reduced model is locally minimized without destroying time moments fitting properties of BRA method. However, selecting the denominator coefficients arbitrarily and fixing time moments may bring a loss of considerable degree of freedom in optimization. It may also be noted that the methods [10-17] require bilinear transformation which is not an efficient operation as it involves extra computation and complexity especially for the systems with very high order. Thus, the essential problem is to obtain, avoiding bilinear transformation, a model which retains or near retains a few terms in excess of r terms while preserving stability.

In this note, a computer-oriented method based on the concept of Pareto-optimality is proposed for the solution of Routh-Padé approximation problem. The method is essentially a multi-objective optimization procedure in which VEGA [22,23] is used to generate Pareto-optimal solutions and the final solution is chosen based on the best fitness value of the objective function. The numerator polynomial of the model is obtained by fully retaining first time moments/Markov parameters of the system and the denominator polynomial is obtained by minimizing the errors between a set of next r time moments/Markov parameters (matching or near-matching of a total of 2r terms as in standard Padé approximation) of the system and those of the model while preserving stability. The operations of the method are carried out entirely in domain. Thus, use of bilinear transformation is avoided. Two numerical examples are included which bring out the systematic nature of the algorithm and the improvement achieved in the system approximation.

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II. PROCEDURE FOR PAPER SUBMISSION

The Routh-Padé problem is formulated by first calculating the time-moments and Markov-parameters of the system and the model. Consider that higher-order discrete-time system is expressed as:

$$G_n(z) = \frac{x(z)}{y(z)} = \frac{a_1 z^{n-1} + a_2 z^{n-2} + \cdots + a_n}{z^n + b_1 z^{n-1} + \cdots + b_n} \quad (1)$$

$$= t_1 + t_2(z-1) + t_3(z-1)^2 + \cdots \quad (2)$$

(expansion around $z = 1$)

$$= M_1 z^{-1} + M_2 z^{-2} + \cdots \quad (3)$$

(expansion around $z = 0$)

Assume that a reduced order (rth- order) model of the form:

$$\hat{G}_r(z) = \frac{\hat{x}(z)}{\hat{y}(z)} = \frac{\hat{a}_1 z^{r-1} + \hat{a}_2 z^{r-2} + \cdots + \hat{a}_r}{z^r + \hat{b}_1 z^{r-1} + \cdots + \hat{b}_r}, r < n (4)$$

$$= \hat{t}_1 + \hat{t}_2(z-1) + \hat{t}_3(z-1)^2 + \cdots \quad (5)$$

$$= \hat{M}_1 z^{-1} + \hat{M}_2 z^{-2} + \cdots \quad (6)$$

is to be constructed, where $t'_i s$ and $\hat{t}'_i s$ are time-moments around $(z = 1)$, $M_i' s$ and $\hat{M}_i's$ are Markov-parameters of the system and model respectively.

The Routh-Padé problem for discrete-time system is formulated by first calculating the time-moments and Markov-parameters of discrete-time system (1) and the model (4).

A. . Calculation of Time-Moments

Putting $z = p + 1$ in (1) and expanding about $p = 0$, (1), becomes:

$$G_n(p) = \frac{x(p)}{y(p)} = \frac{a_1(p+1)^{n-1} + \cdots + a_n}{(p+1)^n + b_1(p+1)^{n-1} + \cdots + b_n} \quad (7)$$

$$= \frac{U_1 p^{n-1} + U_2 p^{n-2} + \cdots + U_n}{p^n + V_1 p^{n-1} + V_2 p^{n-2} + \cdots + V_n'} \quad (8)$$

$$= t_1 + t_2 p + t_3 p^2 + \cdots \quad (9)$$

$$= t_1 + t_2(z-1) + t_3(z-1)^2 + \cdots \quad (10)$$

$$t_i = \begin{cases} t_i = \dfrac{U_n}{V_n} \\ \left(U_{(n+1-i)} - \sum_{j=1}^{i-1} V_{(n+j-i)} t_j\right) \Big/ V_n & i = 2,3,\cdots \end{cases} \quad (11)$$

where $U_i = 0$ for $i \le 0; V_0 = 1; V_i = 1$ for $i \le -1$. Hence, time-moments of the system (1) becomes:

$$T_i = \begin{cases} t_i & i = 1 \\ (-1)^{(i-1)} \sum_{j=1}^{i-1} \dfrac{1}{j!} (T_s)^j w_{(i-1)} j^t{}_{(j+1)} & i = 2,3,\cdots \end{cases} \quad (12)$$

where T_s is the sampling frequency and $w_{(i-1)j}$ is defined as:

$$w_{(i-1)j} = \begin{cases} w_{(i-2),(j-1)} + j w_{(i-2),j} & (i-1) > j \\ 0 & (i-1) > j \end{cases} \quad (13)$$

with $w_{(i-1)(i-1)} = w_{(i-1)1} = 1$ (14)

For the reduced-order model represented by (2.4), the respective time-moments $\hat{T}_i's$ take the form:

$$\hat{T}_i = \begin{cases} \hat{t}_i & i = 1 \\ (-1)^{(i-1)} \sum_{j=1}^{i-1} \dfrac{1}{j!} (T_s)^j w_{(i-1)j} \hat{t}_{(j+1)} & i = 2,3,\cdots \end{cases} \quad (15)$$

where $\hat{t}_i's$ are given by:

$$\hat{t}_i = \begin{cases} t_1 = \dfrac{\hat{U}_r}{\hat{V}_r} \\ \left(\hat{U}_{(n+1-i)} - \sum_{j=1}^{i-1} \hat{V}_{(r+j-i)} \hat{t}_j\right) \Big/ \hat{V}_r & i = 2,3,\cdot \end{cases} \quad (16)$$

and $\hat{U}_i = 0$ $i \le 0$ $\hat{V}_0 = 1$; $\hat{V}_i = 0$ for $. i \le -1$ Note that $\hat{U}_i's$ and $\hat{V}_i's$ are obtained for the model in the same manner as $\hat{U}_i's$ and $\hat{V}_i's$ are obtained for the system.

B. Calculation of Markov-Parameters

The Markov-parameters $(M_1, M_2, \cdots\cdots)$ of the system (1) are determined by expanding (1) around $z = 0$

The Markov-parameters $M_i's$ are given by:

$$M_i = \begin{cases} a_i \\ a_i - \sum_{j=1}^{i-1} b_{i-j} M_j \end{cases} \quad (17)$$

where $a_i = b_i = 0$ for $i = n+1, n+2 \cdots,$

Expanding the model (4) around $z = 0$, one has:

$$\hat{M}_i = \begin{cases} \hat{a}_i \\ \hat{a}_i - \sum_{j=1}^{i-1} \hat{b}_{i-j} \hat{M}_j \end{cases} \quad (18)$$

where $\hat{a}_i = \hat{b}_i = 0$ for $i = r+1, r+2, \cdots,$

C. Formulation of Objective Function

We seek a stable model so as to satisfy

$$\left.\begin{aligned} \hat{T}_i - \hat{T}_i = 0 \\ \hat{M}_i - M_i = 0 \end{aligned}\right\} \quad i = 1,\ldots,l;\ j = 1,\ldots,r-l \quad (19)$$

This arbitrariness in stability preservation can be exploited to minimize square of the errors of matching of r time moments/Markov parameters of the system with those of the model, namely, to minimize objective functions z^T_{l+k}, z^M_{r-l+l} given by

$$\left.\begin{aligned} z^C_{l+k} = \left(1 - \frac{\hat{T}_{l+k}}{T_{l+k}}\right)^2 \\ z^M_{r-l+l} = \left(1 - \frac{\hat{M}_{r-l+l}}{M_{r-l+l}}\right)^2 \end{aligned}\right\} k = 1,.,m;\ l = 1,.r-m \quad (20a)$$

Using (15), (18) and (19), (20a) takes the form belonging to

$$\left.\begin{aligned} z^T_{l+k}(\hat{\mathbf{b}}) = f(\hat{b}_1, \hat{b}_2,.,\hat{b}_r) \\ z^M_{r-l+l}(\hat{\mathbf{b}}) = f(\hat{b}_1, \hat{b}_2,.,\hat{b}_r) \end{aligned}\right\} k = 1,.,m;\ l = 1,.r-m$$

D. Formulation of Stability Constraints

The stability constraints are obtained using Jury's stability table, as applied to characteristic polynomial,

$$\hat{y}(z) = z^r + \hat{b}_1 z^{r-1} + \hat{b}_2 z^{r-2} + \cdots + \hat{b}_r. \qquad (21)$$

which is denominator of the model. Jury's stability table is shown below:

TABLE I: JURY'S STABILITY TABLE

$\hat{b}_r$	$\hat{b}_{r-1}$	$\hat{b}_{r-2}$	$\cdots$	$\hat{b}_2$	$\hat{b}_1$	1
1	$\hat{b}_1$	$\hat{b}_2$	$\cdots$	$\hat{b}_{r-2}$	$\hat{b}_{r-1}$	$\hat{b}_r$
d_{r-1}	d_{r-2}	d_{r-3}	$\cdots$	d_1	d_0	
d_0	d_1	d_2	$\cdots$	d_{r-2}	d_{r-1}	
e_{r-2}	e_{r-3}	e_{r-4}	$\cdots$	e_0		
e_0	e_1	e_2	$\cdots$	e_{r-2}		
		.	.			
p_3	p_2	p_1	p_0			
p_0	p_1	p_2	p_3			
q_2	q_1	q_0				

where

$$d_k = \begin{vmatrix} \hat{b}_r & \hat{b}_{r-1-k} \\ \hat{b}_0 & \hat{b}_{k+1} \end{vmatrix}, \quad \hat{b}_0 = 1$$

for $k = 0,1,\cdots\cdots,r-1$

$$e_k = \begin{vmatrix} \hat{d}_{r-1} & \hat{d}_{r-2-k} \\ d_0 & \hat{d}_{k+1} \end{vmatrix},$$

for $k = 0,1,\cdots\cdots,r-2$

$$q_k = \begin{vmatrix} p_3 & p_{2-k} \\ p_0 & p_{k+1} \end{vmatrix},$$

for $k = 0,1,2.$

A model with characteristic polynomial $\hat{y}(z)$ is stable if the following conditions are all satisfied:

$$\left.\begin{array}{l} g_1(\hat{b}) = 1 > \left|\hat{b}_r\right| \\ g_2(\hat{b}) = \hat{y}(z)\big|_{z=1} > 0 \\ g_3(\hat{b}) = \hat{y}(z)\big|_{z=-1} \begin{cases} \text{for } r \text{ even} \\ \text{for } r \text{ odd} \end{cases} \\ g_4(\hat{b}) = |d_{r-1}| > |d_0| \\ g_M(\hat{b}) = |q_2| > |q_0| \end{array}\right\} \qquad (22)$$

E. Problem Statement

The problem is to minimize (20) subject to (22).

III. APPLICATION OF V.E.G.A.

The vector evaluated genetic algorithm (VEGA) [22,23] is proposed herein for solving the above stated problem. VEGA is the simplest possible multi-objective GA [22,23] and is straightforward extension of a single-objective extension of multi-objective optimization. Since a number of objectives (say Q) have to be handled, GA population is divided at every generation into Q equal subpopulations randomly. Each subpopulation is assigned a fitness value based on different objective function.

After each solution is assigned a fitness, the selection operator, restricted among solutions of each subpopulation, is applied until the complete subpopulation is filled [22,23]. The following VEGA procedure is used [22,23].

Step 1 Set, for population size N, an objective function counter i = 1 and define $x = N/Q$

Step 2 For all solutions, $j = 1 + (i-1)*x$ to $j = i*x$, assign fitness as: $Z(\hat{b}^{(j)}) = z_i(\hat{b}^{(j)})$.

Step 3 Perform proportionate selection on all x solutions to create a mating pool P_i.

Step 4 If $i = Q$, go to *Step* 5. Otherwise, increment i by one and go to *Step* 2.

Step 5 Combine all mating pools together: $P = \mathbf{U}_{i=1}^{Q} P_i$. Perform crossover and mutation on P to create a new population.

A common and simple way to handle constraints is to ignore any solution that violates any of the assigned constraints. Penalty function approach is a popular constraint handling strategy. Minimization of objective function is assumed here. Before the constraint violation is calculated, all constraints are normalized. Thus the resulting constraint functions are the $g_j(x^{(i)}) \geq 0$ for j=1,2,3,…..,J for each solution $x^{(i)}$,the constraint violation for each constraint is calculated as follows

$$w_J(x^{(i)}) = \begin{cases} \left|g_j(x^{(i)})\right| & \text{if } g_j(x^{(i)}) < 0 \\ 0 & \text{otherwise} \end{cases}, \qquad (23)$$

Thereafter all constraints violation are added together to get overall constraints violation

$$\Omega(x^{(i)}) = \sum_{J=1}^{J} \mathrm{w}_J(x^{(i)}) \qquad (24)$$

This constraint violation is then multiplied with penalty parameter R_m and objective function values.

$$F_m(x^{(i)}) = f_m(x^{(i)}) + R_m \Omega(x^{(i)}) \qquad (25)$$

The function F_m take into account the constraints violation. Once penalized function (25) is formed ,any of the unconstrainted multi-objective optimization methods can be used with F_m . Since all penalized fuctions are to minimized,Gas should move into the feasible region and finally approach the pareto-optimal set.

Now, the problem is to minimize (20), satisfying (22). The vector evaluated genetic algorithm (VEGA) [22,23] is proposed herein for solving the above stated problem. VEGA is the simplest possible multi-objective GA [22,23] and is straightforward extension of a single-objective extension of multi-objective optimization. Since a number of objectives (say Q) have to be handled, GA population is divided at every generation into Q equal subpopulations randomly. Each subpopulation is assigned a fitness value based on different objective function.

After each solution is assigned a fitness value, the selection operator restricted among solutions of each subpopulation, is applied until the complete subpopulation is filled [22,23]. The following VEGA procedure is used [22,23].

In this VEGA, linear crossover operator is used. It creates three solutions, $0.5(\hat{b}_i^{(1,t)} + \hat{b}_i^{(2,t)})$, $(1.5\hat{b}_i^{(1,t)} - 0.5\hat{b}_i^{(2,t)})$, $(-0.5\hat{b}_i^{(1,t)} + 1.5\hat{b}_i^{(2,t)})$ from two parent solutions $\hat{b}_i^{(1,t)}$ and $\hat{b}_i^{(2,t)}$ at generation *t*, with the best two solutions being chosen as offspring. For performing mutation, random mutation is used. Instead of creating a solution from the entire search space, a solution in the vicinity of parent solution with a uniform probability distribution is chosen: $y_i^{(1,t+1)} = \hat{b}_i^{(1,t)} + (r_i - 0.5)\Delta_i$ where r_i is a random number in [0,1] and Δ_i is the user–defined maximum perturbation allowed in i-th decision variable.

IV. EXAMPLES

The performance of the algorithm is verified by application to the following numerical examples:

EXAMPLE 1

Suppose for a fourth-order system given by Younseok Choo [6]:

$$G_4(z) = \frac{0.8645z^3 - 1.9002z^2 + 1.3982z - 0.3106}{z^4 - 2.6z^3 + 2.66z^2 - 1.296z + 0.288} \qquad (26)$$

$T_1 = 1 \quad T_2 = 0.6288 \quad T_3 = -0.8773$

$M_1 = 0.8645, \quad M_2 = 0.3475, \quad M_3 = 0.00213$ a second order model of the form:

$$\hat{G}_2(z) = \frac{\hat{a}_1 z + \hat{a}_2}{z^2 + \hat{b}_1 z + \hat{b}_2} \qquad (27)$$

is desired. Equation (17) gives

$$\hat{t}_1 = \frac{\hat{a}_1 + \hat{a}_2}{1 + \hat{b}_1 + \hat{b}_2}, \quad \hat{t}_2 = \frac{\hat{a}_1 - (2 + \hat{b}_1)\hat{t}_1}{1 + \hat{b}_1 + \hat{b}_2} \qquad (28)$$

$$\hat{t}_3 = \frac{\hat{t}_1 + (2 + \hat{b}_1)\hat{t}_2}{1 + \hat{b}_1 + \hat{b}_2}$$

From (17) and (19), we obtain:

$$\hat{T}_1 = \hat{t}_1, \qquad \hat{T}_2 = -\hat{t}_2, \; \hat{T}_3 = \hat{t}_2 + \frac{1}{2}\hat{t}_3, \qquad (29)$$

$\hat{M}_1 = \hat{a}_1, \;\; \hat{M}_2 = \hat{a}_2 - \hat{M}_1\hat{b}_1, \; \hat{M}_3 = -\hat{M}_1\hat{b}_2 - \hat{M}_2\hat{b}_1$, The approximant $\hat{G}_2(z)$ is obtained by following steps given below:

Step 1: From the requirement first r terms matching, one finds:

$$\left.\begin{matrix} \hat{T}_1 = T_1 \\ \hat{M}_1 = M_1 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} \hat{a}_2 = (1 + \hat{b}_1 + \hat{b}_2) - \hat{a}_1 \\ \hat{a}_1 = 0.8645 \end{matrix}\right. \qquad (30)$$

Step 2: From (28), (29) together with (30), we obtain:

$$\hat{T}_2 = \frac{(-0.8645 - (2 + \hat{b}_1)1.0)}{1 + \hat{b}_1 + \hat{b}_2}$$

$$\hat{M}_2 = 1.0(1 + \hat{b}_1 + \hat{b}_2) - 0.8645(1 + \hat{b}_1) \qquad (31)$$

$$\hat{T}_3 = -\hat{T}_2 - \frac{(1.0 - (2 + \hat{b}_1)\hat{T}_2)}{2.0(1 + \hat{b}_1 + \hat{b}_2)}$$

$$\hat{M}_3 = -0.8645\hat{b}_2 - \hat{M}_2\hat{b}_1$$

***Step 3*:** Now the objective functions , (taking $m_1 = m_2 = 2$ takes the following form:

Minimize

$$\left.\begin{matrix} f_1 = \left(1 - \dfrac{\hat{T}_2}{0.6288}\right)^2 \\ f_2 = \left(1 - \dfrac{\hat{T}_3}{-0.8733}\right)^2 \end{matrix}\right\}$$

subject to constraints:

$$g_1(\hat{b}) = (1/|\hat{b}_2|) - 1 > 0,$$

$$g_2(\hat{b}) = 1 + \hat{b}_1 + \hat{b}_2 > 0 \qquad (32)$$

$$g_3(\hat{b}) = 1 - \hat{b}_1 + \hat{b}_2 > 0$$

To obtain the optimum values of $\hat{b}_1$ and $\hat{b}_2$ for which f_1 and f_2 take the minimum value:

Following GA parameter settings are used

Population size	6
Selection	Roulette-wheel selection operator
Croosover	Linear crossover (Elite preserving)
Mutation	Random mutation with $\Delta_i = 0.1$.

***Step 4*:** For the following population of initial conditions, the population after crossover and mutation operators are shown in following table: considering $R_1 = 2, R_2 = 10.R_1$, the resulting Pareto-optimal front for the penalized functions is closed to the true Pareto-optimal solution.

(Refer Table II and Table III)

Applying Pareto-Optimality and V.E.G.A. algorithm converges to the following optimal solution:

$$\hat{b}_1 = -0.811075, \quad \hat{b}_2 = 0.316536 \qquad (33)$$

Step 5: From (28) the numerator parameters of $\hat{G}_2(z)$ turns out as:

$$\hat{a}_1 = 0.8645 \qquad \hat{a}_2 = -0.359034 \qquad (34)$$

Step 6: Finally, $\hat{G}_2(z)$ takes the form:

$$\hat{G}_2(z) = \frac{0.8645z - 0.359034}{z^2 - 0.811075z + 0.316536}, \qquad (35)$$

Time- moments and Markov-parameters of the model are following

$\hat{T}_1 = 1.0, \hat{T}_2 = 0641840, \; \hat{T}_3 = -0.876181.$

$\hat{M}_1 = 0.8645, \hat{M}_2 = 0.342135, \hat{M}_3 = 0.003852$

For comparison purposes, a second order approximant by Younseok Choo [6] is found to be:

$$\hat{G}_2(z) = \frac{0.91321z - 0.57321}{z^2 - 0.87323z + 0.21388} \quad (36)$$

The step responses for original system (26) and its models (35) and (36) are shown in Fig. 1. It is observed that the proposed method gives better reduced model than the method by Younseok Choo [6].

EXAMPLE 2

Consider a fourth order system by Younseok Choo [7]:

$$G_4(z) = \frac{2z^4 + 1.8z^3 + 0.8z^2 + 0.1z - 0.1}{z^4 - 1.2z^3 + 0.3z^2 + 0.1z + 0.02} \quad (37)$$

$G_4(z)$ can be decomposed as:

$$G_4(z) = \tilde{G}(\infty) + G(z) \quad (38)$$

where $\tilde{G}(\infty) = 2$ and

$$G_4(z) = \frac{4.2z^3 + 0.2z^2 + 0.1z - 0.14}{z^4 - 1.2z^3 + 0.3z^2 + 0.1z + 0.02} \quad (39)$$

Time-moments and Markov-parameters of the system are following:

$T_1 = 18.90909$, $T_2 = 35.90909$, $T_3 = -33.078522$

$M_1 = 4.2$, $M_2 = 5.24$, $M_3 = 4.928$. Suppose a second order approximant of the form

$$\hat{G}_2(z) = \frac{\hat{a}_1 z + \hat{a}_2}{z^2 + \hat{b}_1 z + \hat{b}_2} \quad (40)$$

is desired.

The approximant $\hat{G}_2(z)$ is obtained by following steps given below:

Step1: From the requirement first r terms matching, one finds:

$$\left.\begin{matrix} \hat{T}_1 = T_1 \\ \hat{M}_1 = M_1 \end{matrix}\right\} \Rightarrow \begin{cases} \hat{a}_2 = 18.90909(1 + \hat{b}_1 + \hat{b}_2) - \hat{a}_1 \\ \hat{a}_1 = 4.2 \end{cases} \quad (41)$$

Step 2:

$$\hat{T}_2 = \frac{(-4.2 - (2 + \hat{b}_1)18.90909)}{1 + \hat{b}_1 + \hat{b}_2},$$

$$\hat{M}_2 = 18.90909(1 + \hat{b}_1 + \hat{b}_2) - 0.8645(1 + \hat{b}_1)$$

$$\hat{T}_3 = -\hat{T}_2 - \frac{(18.90909 - (2 + \hat{b}_1)\hat{T}_2)}{2.0(1 + \hat{b}_1 + \hat{b}_2)}$$

$$\hat{M}_3 = -0.8645\hat{b}_2 - \hat{M}_2\hat{b}_1. \quad (42)$$

Step 3: Now the objective functions (taking $m_1 = m_2 = 2$) take the following form: Minimize

$$\left.\begin{matrix} f_1 = \left(1 - \dfrac{\hat{T}_2}{35.90909}\right)^2 \\ f_2 = \left(1 - \dfrac{\hat{T}_3}{-33.078522}\right)^2 \end{matrix}\right\} \quad (43)$$

Step 4: For the population of initial conditions, the population after crossover and mutation operators,

Applying Pareto-Optimality and V.E.G.A., algorithm converges to the following optimal solution, considering $R_1 = 2, R_2 = 10.R_1$, the resulting Pareto-optimal front for the penalized functions is closed to the true Pareto-optimal solution given in equation (44)

$$\hat{b}_1 = -1.442346, \quad \hat{b}_2 = 0.616743. \quad (44)$$

Step 5: The numerator parameters of $\hat{G}_2(z)$ turns out as:

$$\hat{a}_1 = 4.2 \qquad \hat{a}_2 = -0.9023114. \quad (45)$$

Step 6: Finally, $\hat{G}_2(z)$ takes the form:

$$\hat{G}_2(z) = \frac{4.2z - 0.9023114}{z^2 - 1.442346z + 0.616743}, \quad (46)$$

Time-moments and Markov-parameters of the model are following:

($\hat{T}_1 = 18.90909$, $\hat{T}_2 = 36.381035$, $\hat{T}_3 = -32.427620$

$\hat{M}_1 = 4.2, \hat{M}_2 = 5.155532, \hat{M}_3 = 4.845744$)

For comparison purposes, a second order approximant by Younseok Choo [7] is found to be:

$$\hat{G}_2(z) = \frac{4.87768z - 2.55604}{z^2 - 1.50888z + 0.63166}, \quad (47)$$

The step responses of (39), (46) and (47) are plotted in Fig. 2. Clearly, (46) is a significant improvement over (47).

The ISE pertaining to unit step input corresponding to (46) and (47) are 0.014388 and 8.162988 respectively, which confirms the applicability of the present technique to realize improvement in system approximation.

V. CONCLUSIONS

In this paper, GA is used for finding the Routh-Padé approximants for discrete-time systems. It is shown that the numerator polynomial of the model is obtained by fully retaining the first terms (time-moment/Markov-parameters), of the system and the denominator polynomial is found by minimizing the errors between a set of subsequent time moments/Markov parameters of the system and those of the model while preserving stability. The effectiveness and superiority of proposed method has been illustrated with the help of two examples.

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