Optimum Cost Design of Reinforced Concrete Retaining Walls Using Hybrid Firefly Algorithm

R. Sheikholeslami, B. Gholipour Khalili, and S. M. Zahrai

***Abstract*—This paper develops a novel optimization method namely hybrid firefly algorithm with harmony search technique (IFA–HS), to obtain the optimal cost of the reinforced concrete retaining walls satisfying the stability criteria. The hybrid IFA–HS is utilized to find the economical design adhering to provisions of *ACI 318-05*. Also Coulomb lateral earth pressure theory is used to derive the lateral total thrust on the wall. Some design examples are tested using the new method. The results carried out on these examples confirm the validity of the proposed algorithm. The IFA–HS method can be considered as an improvement of the recently developed firefly algorithm. The improvements include the utilizing of a memory that contains some information extracted online during the search, adding of pitch adjustment operation in harmony search serving as mutation operator during the process of the firefly updating, and modifying the movement phase of firefly algorithm. The detailed implementation procedure for this improved meta–heuristic method is also described.**

***Index Terms*—Concrete retaining wall, firefly algorithm, harmony search.**

I. Introduction

Retaining walls are designed to withstand lateral earth and water pressures and for a service life based on consideration of the potential long–term effects of material deterioration on each of the material components comprising the wall. Permanent retaining walls should be designed for a minimum service life of 50 years. Temporary retaining walls should be designed for a minimum service life of 5 years.

The cantilever wall is the most common type of retaining walls. This type of wall is constructed of reinforced concrete. They can be used in both cut and fill applications. They have relatively narrow base widths. They can be supported by both shallow and deep foundations. The position of the wall stem relative to the footing can be varied to accommodate right–of–way constraints. They are most economical at low to medium wall heights. The cantilever wall generally consists of a vertical stem, and a base slab, made up of two distinct regions,means a heel slab and a toe slab. All three components behave like one–way cantilever slabs: the stem acts as a vertical cantilever under the lateral earth pressure; the 'heel slab' and the 'toe slab' act as a horizontal cantilever under the action of the resulting soil pressure.

Conventional design of concrete retaining walls is highly dependent on the experience of engineers. The structure is defined on a trial–and–error basis. Tentative design must satisfy the limit states prescribed by concrete codes. This process leads to safe designs, but the cost of the reinforced concrete retaining walls is, consequently, highly dependent upon the experience of the designer. Therefore, in order to economize the cost of the concrete retaining walls under design constraints, it is advantageous for designer to cast the problem as an optimization problem.

Optimum design of retaining walls has been the subject of a number of studies. Saribas and Erbatur [1] presented a detailed study on reinforced concrete cantilever retaining walls optimization using cost and weight of walls as objective functions. In their study, they controlled overturning failure, sliding failure, shear and moment capacities of toe slab, heel slab, and stem of wall as constraints. Ceranic and Fryer [2] proposed an optimization algorithm based on simulated annealing (SA). Sivakumar and Munwar [3] introduced a target reliability approach (TRA) for design optimization of retaining walls. Ahmadi and Varaee [4] proposed an optimization algorithm based on the particle swarm optimization(PSO) foroptimum design of retaining walls. Ghazavi and Bazzazian Bonab [5] applied a methodology to arrive at the optimal design of concrete retaining wall using the ant colony optimization (ACO). Camp and Akin [6] developed a procedure for designing low–cost or low–weight cantilever reinforced concrete retaining walls using the big bang–big crunch algorithm (BB–BC). Kaveh *et al.*, [7] used the heuristic big bang–big crunch algorithm (HBB–BC) for the optimum design of gravity retaining walls subjected to seismic loading. Also Talatahari *et al.*, [8] proposed a method based on the charged system searchalgorithm (CSS) for optimum seismic design of retaining walls.

In this paper, a novel hybrid swarm intelligence algorithm, namely hybrid firefly algorithm with harmony search (IFA–HS), based on the combined concepts of firefly algorithm (FA) and harmony search (HS) technique, is proposed to solve design problems of reinforced concrete retaining walls. The main idea of the hybrid IFA–HS algorithm is to integrate the HS operators into the FA algorithm, and thus increasing the diversity of the population and the ability to have the FA to escape the local minima. The remainder of this paper is organized as follows: in Section II the design procedure of retaining walls and problem formulation is described then the IFA–HS algorithm and its implementation details are presented in Section III. Section IV presents our computational studies. Finally conclusions in Section V close the paper.

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R. Sheikholeslami is with Departmant of Civil and Environmental Engineering, Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran (e-mail: razi.sheikholeslami@aut.ac.ir).

B. G. Khalili is with Departmant of Civil Engineering, University of Tehran, Iran (e-mail: b.g.khalili@ut.ac.ir).

C. S. M. Zahrai is with Engineering and Management of civil Infrastructures, School of Civil Engineering, Faculty of Engineering University of Tehran, Iran (e-mail: mzahrai@ut.ac.ir).

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II. Optimal Design of Reinforced Concrete Cantilever Retaining Walls

Consider a retaining wall shown in Fig. 1 Typically, three failure modes are considered in the analysis of the retaining structure: overturning, sliding, and bearing capacity. The overturning moment about the toe of the wall is a balance of the force caused by the active soil pressure of the retained soil weight and the self–weight of the concrete structure, the soil above the base, and the surcharge load. For the sliding mode of failure, only the horizontal component of the active force is considered. Horizontal resisting forces result from the weight of wall and soil on the base, surcharge load, friction between soil and base of wall, and passive force owing to soil on the toe and base shear key sections.

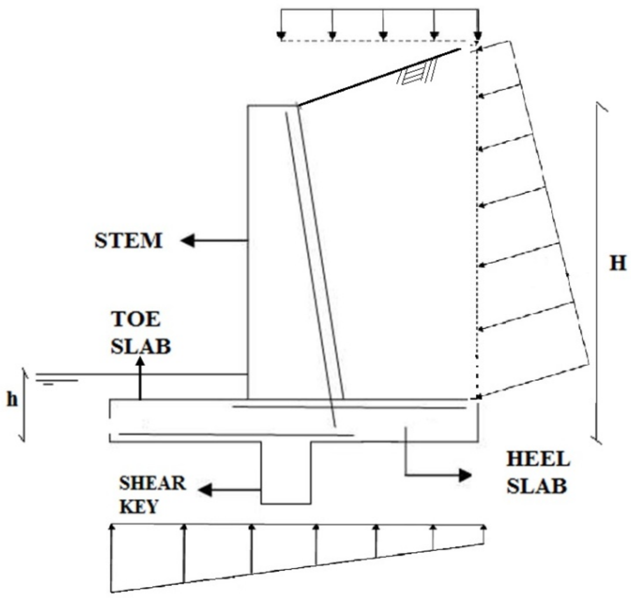

Fig. 1. A schematic view of a cantilever retaining wall.

On the other hand the stem of the wall will bend as cantilever, so that tensile face will be towards the backfill. The heel slab of the wall will have net pressure acting downwards, and will bend as a cantilever, having tensile face upwards. Hence, considering a concrete retaining wall,the four primary concerns relating to the design of these walls are [9]:

1) That it has an acceptable factor of safety with respect to overturning.
2) That the allowable soil bearing pressures are not exceeded.
3) That it has an acceptable factor of safety with respect to sliding.
4) That the stresses within the components (stem and footing) are within code allowable limits to adequately resist imposed vertical and lateral loads.

These safety factors can be expressed as
Check for overturning:

$$FS_O = \frac{\sum M_R}{\sum M_O} \tag{1}$$

Check for sliding along the base:

$$FS_S = \frac{F_R}{F_d} \tag{2}$$

Check for bearing capacity failure:

$$FS_b = \frac{q_u}{q_{\max}} \tag{3}$$

where:
ΣM_O = sum of the moments of forces that tend to overturn about toe
ΣM_R = sum of the moments of forces that tend to resist overturning about toe
F_R = sum of the horizontal resisting forces
F_d = sum of the horizontal driving forces
q_u= ultimate bearing capacity
q_{max} = maximum bearing pressure

The optimal cost design of a concrete cantilever retaining wall is proposed to be determined by the minimum of the costs of concrete and steel reinforcement. The objective function can be expressed as follows

$$Cost = C_1 \times V_{conc} + C_2 \times W_{steel} \tag{4}$$

where V_{conc} and W_{steel} are the volume of concrete (m^3/m) and the weight of reinforcement steel in the unit of length (kg/m), respectively; C_1 is the cost of the concrete (unit/m^3), and C_2 is the cost of steel (unit/kg).

As we mentioned before theoptimal design of cantilever retaining walls is a constraint problem. These constraints may be classified into four groups of: stability, capacity, reinforcement configuration, and geometric limitations which are defined as

$$\begin{aligned} &FS_o \geq 1.5 \ , \ FS_S \geq 1.5 \ \text{and} \ FS_b \geq 3 \\ &\frac{M_u}{M_n} \leq 1 \ \text{and} \ \frac{V_u}{V_n} \leq 1 \end{aligned} \tag{5}$$

In which FS_o, FS_s and FS_b are the factors of safety against overturning, sliding and bearing capacity, respectively; M_u and V_u are the design moment and design shear strength in the stem, toe, or heelof the retaining wall, respectively; and M_n and V_n are the flexural and shear strength, respectively.

Here, shears and moments (V, M) are calculated based on *ACI 318–05* codes [10]. The moment capacity of any reinforced concrete wall section (stem, toe, or heel) should be greater than the design moment of the structure. In the same way, shear capacities of wall sections should be greater than the design shear forces. The flexural and shear strength are calculated as

$$M_n = \phi_b A_s f_y \left(d - \frac{a}{2} \right) \tag{6}$$

$$V_n = \phi_v 0.17 bd \sqrt{f_c'} \tag{7}$$

In which $\phi$is the nominal strength coefficient (ϕ_b= 0.9 and ϕ_v= 0.85); $A_s$is the cross–sectional area of steel reinforcement; $f_y$is the yield strength of steel; $d$is the distance from compression surface to the centroid of tension steel;$a$is the depth of stress block; f'_c is the compression strength of concrete; and $b$is the width of the section.

The design variables for the reinforced concrete retaining wall are shown in Fig. 2 These variables are categorized into two groups: the geometric variables that prescribe the dimensions of the wall cross section (Xj, j = 1,…, 7) and those related to the steel reinforcement ($A_s i$, i = 1,…, 4). In total there are eleven design variables.

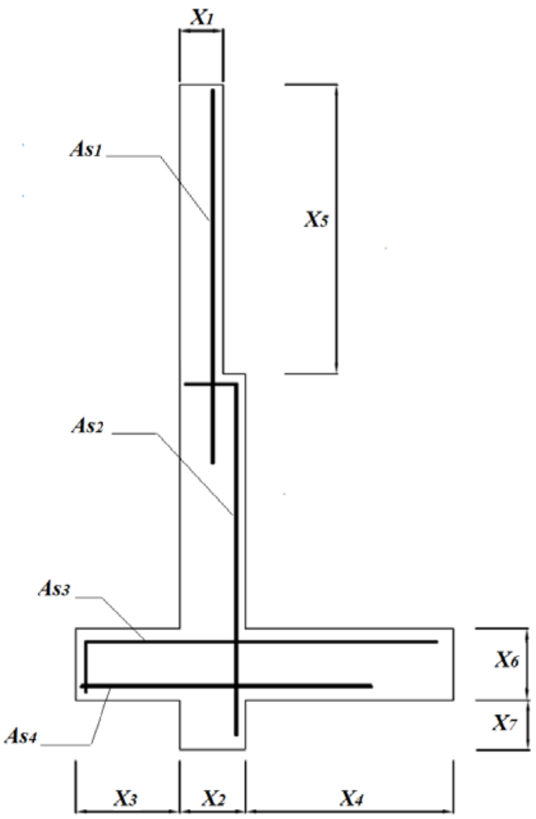

Fig. 2. Design variables.

The optimization algorithm initiates the design process by selecting random values for the design variables. Then the algorithm checks the wall for stability and if the dimensions satisfy stability criteria, the algorithm calculate the required reinforcement and checks the strength. In this procedure choosing design parameters that fulfill all design requirements and have the lowest possible cost is concerned. In order to handle the constraints, a penalty approach is utilized. In this method, the aim of the optimization is redefined by introducing the penalized cost function as

$$F_{\cos t} = Cost\left(1 + \varphi\sum_{k=1}^{n} g_k\right) \tag{8}$$

where φ is a penalty constant, g_k is the amount of violation of k–th constraint, and n is the total number of constrains. Herethe penalty constant isselected in a way that it decreases the penalties and reduces the variables. Thus, in the first steps of the search process, φ is set to 1.5 and then ultimately increased to 5.

III. The Hybrid Firefly Algorithm with Harmony Search

A. Preliminary

To begin with, a brief background on thefirefly algorithm (FA) and harmony search (HS) approach isprovided in this section.

Among the phenomenon–mimicking methods, algorithm inspired from the collective behavior of species such as ants, bees, wasps, termite, fish, and birds are referred as swarm intelligence algorithms. Recently, Yang [11] proposed the firefly algorithm (FA) as a novel swarm intelligence algorithm which mimics the natural behavior of fireflies.Recently, various applications of the FA in different research areas are reported. Gandomi *et al.*, [12] used a FA based approach for solving mixed continuous/discrete structural optimization problems. The study revealed the efficiency of the FA algorithm in the field of structural optimization. Gomez [13] employed FA for sizing and shape optimization of truss structures with dynamic constraints. Also Kazemzadeh Azad and Kazemzadeh Azad [14] employed an improved FA (IFA) algorithm for optimum design of planar and spatial truss structures with both sizing and shape design variables and reported promising results.

As mentioned before FA is a nature–inspired heuristic search technique based on natural behavior of fireflies. According to [15], to develop the FA, natural flashing characteristics of fireflies have been idealized using the following three rules,

1) All of the fireflies are unisex; therefore, one firefly will be attracted to other fireflies regardless of their gender.
2) Attractiveness of each firefly is proportional to its brightness, thus for any two flashing fireflies, the less bright firefly will move towards the brighter one. The attractiveness is proportional to the brightness and they both decrease as their distance increases. If there is no brighter one than a particular firefly, it will move randomly.
3) The brightness of a firefly is determined according to the nature of the objective function.

The attractiveness of a firefly is determined by its brightness or light intensity which is obtained from the objective function of the optimization problem. However, the attractiveness β, which is related to the judgment of the beholder, varies with the distance between two fireflies. The attractiveness β can be defined by

$$\beta = \beta_0 \exp(-\gamma\, r^2) \tag{9}$$

where r is the distance of two fireflies, β_0 is the attractiveness at $r = 0$, and γ is the light absorption coefficient. The distance between two firefliesi and j at x_i and x_j, respectively, is determined using the following equation

$$r_{ij} = \left\|x_i - x_j\right\| = \sqrt{\sum_{k=1}^{k=d}\left(x_{i,k} - x_{j,k}\right)^2} \tag{10}$$

where $x_{i,k}$ is the k–th parameter of the spatial coordinate x_i of the i–th firefly. In the firefly algorithm, the movement of a firefly i towards a more attractive (brighter) firefly j is determined by the following equation

$$x_i = x_i + \beta_0 \exp(-\gamma\, r_{ij}^{\,2})\left(x_j - x_i\right) + \alpha\varepsilon_i \tag{11}$$

where the second term is related to the attraction, while the third term is randomization with the vector of random variables ε_i using a normal distribution[16].

An improved version of the FA (IFA) was proposed and applied to design optimization of truss structures by Kazemzadeh Azad and Kazemzadeh Azad [14]. The performance of the IFA was investigated in typical design optimization examples of truss structures and satisfactory results were reported. For movement stage of the FA, the following equation is used

$$x_i = x_j + \beta_0 \exp(-\gamma r_{ij}^2)(x_j - x_i) + \alpha\varepsilon_i \qquad (12)$$

In the original FA, the movement of a firefly i towards a brighter firefly j is determined by (11). Since x_j is brighter than x_i , in (12) instead of moving firefly i towards j, searching the vicinity of firefly j which is a more reliable area is proposed to update the position of firefly $i$based on the current position of firefly j. To do this, x_i is replaced by x_j and the above equation is implemented for movement stage of the FA. In (12), ε_i is a randomly generated number using a normal distribution and α is a scaling parameter. Normal distribution has two parameters: a mean value and a standard deviation. In this study the mean value of the normal distribution is set to zero and the standard deviation is taken as the standard deviation of k–th parameter of all fireflies in each generation.

In the IFA to avoid missing the brighter fireflies of the population, the position of a firefly is updated only if the new position found is better than the old one. Therefore, in the process of optimization each candidate design will be replaced only with a better design. It is apparent that (12) may generate fireflies outside the bounds of design variables. In order to remove this problem, the parameters of fireflies which are not created within the bounds of design variables are rounded into the boundary values.

The harmony search (HS) method is another optimization algorithm that inspired by the working principles of the harmony improvisation from music. Similar to the other nature–inspired approaches, HS is a random search technique. It does not require any prior domain knowledge, such as gradient information of the objective function. However, different from those population–based approaches, it only utilizes a single search memory to evolve. Therefore, the HS method has the distinguished feature of algorithm simplicity [17]. HS is a meta–heuristic search technique without the need of derivative information, and with reduced memory requirement. In comparison with other meta–heuristic methods, HS is computationally effective and easy to implement for solving various kinds of engineering optimization problems. There are four principal steps in this algorithm [18]:

Step 1. Initialize a harmony memory (HM). The initial HM consists of a certain number of randomly generated solutions for the optimization problem under consideration. For an n dimension problem, a HM with the size of *HMS* can be represented as follows

$$\text{HM} = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_n^1 \\ x_1^2 & x_2^2 & \cdots & x_n^2 \\ \cdots & \cdots & \cdots & \cdots \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_n^{HMS} \end{bmatrix} \qquad (13)$$

where (x^i_1, x^i_2,..,x^i_n), (i=1, 2,.., HMS) is a candidate solution. *HMS* is typically set to be between 10 and 100.

Step 2. Improvise a new solution (x'_1, x'_2,..,x'_n) from the HM. Each component of this solution, x'_j, is obtained based on the harmony memory considering rate (*HMCR*). The *HMCR* is defined as the probability of selecting a component from the HM members, and 1-*HMCR* is, therefore, the probability of generating it randomly. If x'_j comes from the HM, it can be further mutated according to the pitching adjust rate (*PAR*). The *PAR* determines the probability of a candidate from the HM to be mutated.

Step 3. Update the HM. First, the new solution from Step 2 is evaluated. If it yields a better fitness than that of the worst member in the HM, it will replace that one. Otherwise, it is eliminated.

Step 4. Repeat Step 2 to Step 3 until a termination criterion (e.g., maximal number of iterations) is met.

The usage of harmony memory (HM**)** is important because it ensures that good harmonies are considered as elements of new solution vectors. In order to use this memory effectively, the HS algorithm adopts a parameter $HMCR\in(0,1)$, called harmony memory considering (or accepting) rate. If this rate is too low, only few elite harmonies are selected and it may converge too slowly. If this rate is extremely high, near 1, the pitches in the harmony memory are mostly used, and other ones are not explored well, leading not into good solutions. Therefore, typically, we use *HMCR* = 0.7 ~ 0.95 [18]. Note that a low *PAR* with a narrow bandwidth (*bw*) can slow down the convergence of HS because of the limitation in the exploration of only a small subspace of the whole search space. On the other hand, a very high *PAR* with a wide *bw* may cause the solution to scatter around some potential optima as in a random search. Furthermore large *PAR* values with small *bw* values usually cause the improvement of best solutions in final generations which algorithm converged to optimal solution vector.

B. The Hybrid IFA–HS Method

The hybrid IFA–HS algorithm combines the optimization capabilities of HS and IFA. In HS algorithm the diversification is controlled by random selection. Random selection explores global search space more widely and efficiently while the pitch adjustment makes the new solution good enough and near the existing good solutions. The intensification in HS algorithm is controlled by memory consideration, leading the searching process toward the searching space of good solutions [19]. Also the use of the HM in HS allows the selection of the best vectors that may represent different regions in the search space. On the other hand the disadvantages of the basic FA algorithm are premature convergence and sometimes not obtaining efficacious experiences between solutions in a population. In order to obtain a high quality solution we combine the above mentioned strategies. Since FA algorithms is memory less, there is no information extracted dynamically during the search, while the hybrid IFA–HS uses a memory that contains some information extracted online during the search. In other word some history of the search stored in a memory can be used in the generation of the candidate list of solutions and the selection of the new solution. Using the original configuration of the IFA, we generate the new harmonies based on the newly generated firefly each iteration after firefly's position has been updated. The updated harmony vector substitutes the newly generated firefly only if it has better fitness. This selection scheme is rather greedy which often overtakes original HS and FA. The proposed IFA–HS algorithm involves two phases of optimization: The IFA algorithm using heuristic search technique, The HS algorithm using memory consideration, random selection and pitch adjustment.

The hybrid IFA–HS algorithm has another beneficial feature; it iteratively explores the search space by combining multi–search space regions to visit a single search space region. The IFA–HS iteratively recombines the characteristics of many solutions in order to make one solution. It is able to fine tune this solution to which the algorithm converges using neighborhood structures. Throughout the process recombination is represented by memory consideration, randomness by random consideration, and neighborhood structures by pitch adjustment and variation of firefly's attractiveness. Therefore IFA–HS algorithm has the advantage of combining key components of population–based and local search–based methods in a simple optimization model.

IV. Design Examples

In this Section, two numerical examples are optimized with the proposed method. The final result of the IFA–HS is compared to the solution of the other standard algorithms to demonstrate the performance of the present approach. For the proposed algorithm, a population size of 100 and harmony memory size of 70 have been used. The HS parameters have been set either equal to $HMCR = 0.95$, and $PAR = 0.35$ for all the examples. The maximum number of evaluations has been 10,000.Note that for these parameters the IFA–HS algorithms exhibited good performance in solution quality and required a reasonably small computational overhead.

A. Example I

To check the performance, robustness, and accuracy of the above algorithm, a retaining wall studied by Saribasand Erbatur [1] is considered. The details of this wall and other necessary input parameters are given in Table I.

TABLE I: Input Parameters for Example I

Parameter	Value	Unit
Height of stem	4.5	m
Yield strength of reinforcing steel	400	MPa
Compressive strength of concrete	21	MPa
Surcharge load	30	kpa
The angle of wall friction	15	degree
Internal friction angle of retained soil	36	degree
Internal friction angle of base soil	34	degree
Unit weight of retained soil	17.5	kN/m^3
Unit weight of base soil	18.5	kN/m^3
Unit weight of concrete	23.5	kN/m^3
Cohesion of base soil	100	kpa
Depth of soil in front of wall	0.75	m
Cost of steel	0.40	\$/kg
Cost of concrete	40	$\$/m^3$

It is noted that all the values given in this table are for a unit length of the wall.

The results of the cost optimization design for theIFA–HS, ACO [5], andRETOPT [1] are summarized in Table II.

As shown in Table II, the minimum cost for the IFA–HS algorithm is 180.064 (\$/m), while the best cost of ACO [5] and RETOPT [1] is 201.185 and 189.546 (\$/m), respectively.

TABALE II: Optimal Design Comparison for Example I

Design Variables	IFA–HS	ASO [5]	RETOPT [1]
$X1$ (m)	0.250	0.250	N/A
$X2$ (m)	0.378	0.251	N/A
$X3$ (m)	1.181	1.143	N/A
$X4$ (m)	1.700	1.385	N/A
$X5$ (m)	2.820	4.500	N/A
$X6$ (m)	0.450	0.400	N/A
$X7$ (m)	0.300	-	N/A
A_s1 (cm^2)	28.85	29.50	N/A
A_s2 (cm^2)	30.34	29.50	N/A
A_s3 (cm^2)	7.24	14.00	N/A
A_s4 (cm^2)	13.98	14.00	N/A
Minimum cost	180.064	201.185	189.546
Mean cost	214.667	N/A	N/A

The best cost of IFA–HSobtained after 4,200 function evaluations. In addition, the average cost of 30 different runs for the IFA–HS algorithm is 214.667 (\$/m).

B. Example II

For further validation of the developed optimization method, another example is considered and the results are compared with IFA and HS methods. A wall with heights of 5.5 m is considered. Other specifications for the design of this retaining wall are presented in Table III.

TABLE III: Input Parameters for Example II

Parameter	Value	Unit
Height of stem	5.5	m
Yield strength of reinforcing steel	400	MPa
Compressive strength of concrete	21	MPa
Surcharge load	25	kpa
The angle of wall friction	10	degree
Internal friction angle of retained soil	36	degree
Internal friction angle of base soil	0	degree
Unit weight of retained soil	17.5	kN/m^3
Unit weight of base soil	18.5	kN/m^3
Unit weight of concrete	23.5	kN/m^3
Cohesion of base soil	120	kpa
Depth of soil in front of wall	0.75	m
Cost of steel	0.40	\$/kg
Cost of concrete	40	$\$/m^3$

TABLE IV: Optimal Design Comparison for Example II

Design Variables	IFA–HS	IFA	
$X1$(m)	0.250	0.250	0.250
$X2$(m)	0.450	0.450	0.475
$X3$(m)	2.00	2.100	2.125
$X4$(m)	1.500	1.445	1.850
$X5$(m)	3.200	3.200	3.200
$X6$(m)	0.500	0.500	0.500
$X7$(m)	0.250	0.350	0.255
A_s1(cm^2)	38.00	36.00	30.00
A_s2(cm^2)	30.00	40.00	40.00
A_s3(cm^2)	8.47	7.89	11.78
A_s4(cm^2)	9.68	10.25	10.61
Minimum cost	211.841	216.860	228.310
Mean cost	245.825	282.784	304.102
No. of analyses	5,600	5,900	6,300

Table IV reports the best results and the required number of evaluation for convergence in the present algorithm compared with IFA and HS. The IFA–HS found the best feasible solutionof 211.841 ($/m) after 5,600 function evaluations while the IFA and HS found the best solutions of 216.860 and 228.310 ($/m) spending 5,900and 6,300 evaluations, respectively. The optimum cost of IFA and HS is 2.31% and 7.21% expensive design in comparison with the optimum cost obtained by IFA–HS. In addition, the average cost of the IFA–HS is 245.825 ($/m), while it is 282.784 and 304.102 ($/m) for the IFA and HS, respectively. These results demonstrate the effectiveness and efficiency of the proposed method.

V. Conclusion

A novel hybrid IFA–HS algorithm, on the basis of concepts of natural behavior of fireflies and harmony improvisationis proposed and applied to the minimum cost design of reinforced concrete retaining structures. In this algorithmIFA is used to fine tune the vectors stored in HM. Actually, HM vectors become as IFA population, then the evolving process is performed as the usual IFA procedure. Another improvement in this algorithm is adding pitch adjustment operation in IFA as mutation operator with the aim of speeding up convergence, thus making the approach more feasible for a wider range of practical applications while preserving the attractive characteristics of the basic FA.

Through a series of design examples, the hybrid IFA–HS algorithm demonstrated that it was both computationally efficient and capable of generating least–cost retaining wall designs that satisfy safety, stability, and material constraints.The proposed optimum design model enables engineering to find optimal/near–optimal designs.

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Razi Sheikholeslami was born in Tabriz, Iran, in 1989. He received his B.Sc. in structural engineering in 2011 from the Department of Civil Engineering at the University of Tabriz. Afterward he studied Hydraulic Structures at the Amirkabir University of Technology (Tehran Polytechnic), Tehran, Iran and got his M.Sc. degree in 2013.

His main research interests lie in the application of advanced optimization methods, particularly meta-heuristics, to difficult real-world problems in a variety of application areas, and in the effective exploitation and integration of optimization methods within the engineering design process. He also has interests in chaos theory and nonlinear time series analysis.

Behnam Gholipour Khalili was born in 1989 in Tabriz, Iran. He obtained his B.S. degree in civil engineering from Engineer ing Facility of Azerbaijan Shahid Madani University, Tabriz, Iran, in 2011. At present, he is a M.Sc student of Structural Engineering in the University of Tehran.

His research interests include computational mechanics, design and behavior of reinforced concrete structures, and construction management.

S. M. Zahrai was born in Tehran, Iran in 1964. He received his B.Sc. in civil engineering from Tehran Polytechnic in 1987, his M.Sc. in structural engineering from the Univ. of Tehran in 1990 and his Ph.D. in structural engineering from the Univ. of Ottawa in 1997.

He has been a professor at the school of Civil Eng., the Univ. of Tehran since 1999. He has authored many publications including 10 books, more than 200 journal and conference papers, so far supervised about 100 theses and research projects, managed and directed lots of large steel and concrete building tower or bridge projects in design and construction phases, served a few conferences as the chairman, and many international journals as member of editorial board or reviewer.

Zahrai has been also a member of many scientific societies and innovation advisory committees in Iran, awarded a few times for teaching or research and uniquely recognized for about 300 workshops and educational sessions held on different aspects of seismic design, retrofit and construction of steel and concrete building and bridge structures.