Inelastic Displacement Ratio Spectrum for Near-Fault Ground Motions

Gaylord Kabongo-Booto and George D. Hatzigeorgiou

Abstract—**This paper examines the spectrum of inelastic displacement ratio (*IDR*) for structures subjected to near-fault earthquakes. *IDR* can be defined as the ratio of the maximum inelastic to the maximum elastic displacement of a structure and allows the computation of its maximum inelastic displacement directly from the corresponding elastic counterpart. Extensive parametric analyses are carried out to obtain empirical expressions for this ratio, in terms of the period of vibration and ductility demands.**

Index Terms—**Inelastic displacement ratio spectrum, seismic analysis, ductility demands, near-fault earthquakes.**

I. Introduction

This study develops a simple and efficient method for the evaluation of inelastic displacement ratio (IDR) spectrum. Knowledge of IDR, i.e. the ratio of the maximum inelastic to the maximum elastic displacement of a structure, allows the computation of its maximum inelastic displacement directly from the corresponding elastic counterpart. Veletsos [1], Veletsos and Newmark [2] and Clough [3] found that in the low-frequency range of the spectrum, the maximum displacement of an inelastic system may be considered the same as the maximum displacement of the associated elastic system, according to the so-called 'equal-displacement rule'. Nonetheless, Miranda [4] and Chopra and Chintanapakdee [5] observed that IDR can differ considerably from unity in the moderately high to high frequency regions of spectrum. Furthermore, Hatzigeorgiou and Beskos [6] found that repeated earthquakes significantly affect the inelastic displacement ratio.

It is worth noticing that most of the seismic codes assumed far-fault earthquakes to describe the seismic loads. However, an earthquake that recorded closely to its fault at a station located toward the direction of the fault rupture is qualitatively quite different from the usual far-fault seismic records [7]. The categorization and analytical representation of near-fault earthquakes as well as the study of their effects on the seismic behavior of structures are very important research topics today. On e can mention here the works of Makris [8], Makris and Chung [9], Zhai *et al.*, [10], Ruiz-Garcia [11] and Iervolino et al. [12]. Although the development that has been accomplished so far, there is still a clear need to understand the behavior of structures subjected to near-source seismic ground motions. Thus, this study focuses on the evaluation of inelastic displacement ratio (IDR) spectrum of structures subjected to near-fault earthquakes. Without loss of generality, elastic - perfectly plastic models are adopted. These models are simple and can adequately describe steel or reinforced concrete structures with primarily flexural behavior. The influence of period of vibration and ductility demands are taken into account in constructing expressions for the IDR through extensive parametric studies and nonlinear regression analysis. A statistical investigation of 72,000 inelastic time-history analyses are carried out to study 200 SDOF models with 6 levels of ductility demands, excited by 60 earthquake accelerogram records from around the world, under various types of faults mechanisms such as strike-slip, reverse or oblique-reverse.

II. Description of Model

An elastic – perfectly plastic SDOF system with viscous damping is used to model the structure, as shown in Fig. 1. The dynamic equilibrium equation of this system is given by

$$m\ddot{u} + c\dot{u} + k^T u = -ma_g \tag{1}$$

where m is the mass, u the relative displacement, c the damping coefficient, k^T the tangent stiffness, a_g the acceleration of the ground motion while upper dots stand for time derivatives.
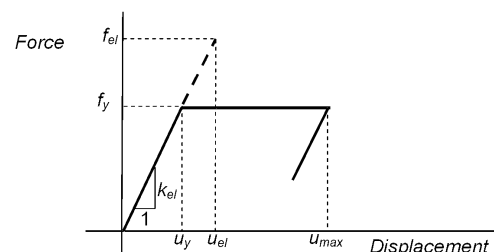

Fig. 1. Elastic - perfectly plastic model of a SDOF.

Using structural dynamics theory [6], these systems are defined here by their elastic vibration period T, ranging from 0.02 sec to 4.0 sec, and viscous damping ratio, ξ, assumed to be 5%. The yield force f_y can be expressed in terms of the yield displacement u_y and the elastic stiffness k_{el} as

$$f_y = k_{el} \cdot u_y \tag{2}$$

While the ductility μ is defined in terms of the maximum displacement u_{max} and the yield displacement u_y, as

$$\mu = \frac{u_{max}}{u_y} \qquad (3)$$

Finally, the inelastic displacement ratio is defined as the maximum lateral inelastic displacement u_{max} divided by the maximum lateral elastic displacement u_{el} for a system with the same mass and initial stiffness (i.e., same period of vibration) subjected to the same earthquake ground motion. This ratio is given by

$$IDR = \frac{u_{max}}{u_{el}} \qquad (4)$$

III. SEISMIC INPUT

The seismic excitations that have been used in this work consist of 60 pulse-like ground motions recorded closely to faults with strike-slip, reverse or oblique-reverse mechanisms. The examined near-fault ground motions, which have been recorded at a distance less than 10 km from the fault rupture, are characterized by intense velocity pulses of relatively long period that clearly distinguish them from typical far-field ground motions. The aforementioned 60 seismic records have been downloaded from the NGA-PEER strong ground motion database [13]. The complete lists of the aforementioned earthquakes are shown in Table I.

TABLE I: NEAR-FAULT EARTHQUAKES UNDER CONSIDERATION

Earthquake	Year	Station	Magn. M_w
Cape Mendocino	1992	Cape Mendocino	7.01
Cape Mendocino	1992	Petrolia	7.01
Chi-Chi, Taiwan	1999	CHY006	7.62
Chi-Chi, Taiwan	1999	CHY035	7.62
Chi-Chi, Taiwan	1999	TAP003	7.62
Chi-Chi, Taiwan	1999	TAP005	7.62
Chi-Chi, Taiwan	1999	TCU036	7.62
Coalinga-05	1983	Oil City	5.77
Coalinga-05	1983	Transmitter Hill	5.77
Coalinga-07	1983	Coalinga-14th - Elm	5.21
Erzican, Turkey	1992	Erzincan	6.69
Imperial Valley	1979	Aeropuerto Mexicali	6.53
Imperial Valley	1979	Agrarias	6.53
Imperial Valley	1979	Brawley Airport	6.53
Imperial Valley	1979	Meloland Overpass FF	6.53
Imperial Valley	1979	El Centro Array #10	6.53
Imperial Valley	1979	El Centro Array #6	6.53
Imperial Valley	1979	El Centro Array #8	6.53
Imperial Valley	1979	Holtville Post Office	6.53

IV. ANALYSIS

For each earthquake record, the period of the SDOF system is increased from 0.02 to 4.0 sec with an increment of

0.02 sec (i.e., 200 values of period), while the ductility demand is assumed to increase from 1.0 to 6.0 with an increment of 1.0 (i.e., 6 values of μ factors). Thus, 72,000 analyses are examined: (60 ground motions) $\times$ (200 periods, T) $\times$ (6 levels of ductility demands, μ). Every analysis serves to solve the nonlinear differential Eq. (1) by Newmark method and determine the response $u(t)$ in terms of various parameters of the problem.

TABLE I: CONTINUED

Earthquake	Year	Station	Magn. M_w
Kobe, Japan	1995	KJMA	6.90
Kobe, Japan	1995	Takarazuka	6.90
Kobe, Japan	1995	Takatori	6.90
Kocaeli, Turkey	1999	Arcelik	7.51
Kocaeli, Turkey	1999	Yarimca	7.51
Landers	1992	Lucerne	7.28
Landers	1992	Yermo Fire Station	7.28
Loma Prieta	1989	Alameda NA Stn Hanger	6.93
Loma Prieta	1989	Gilroy - Gavilan Coll.	6.93
Loma Prieta	1989	Gilroy - Historic Bldg.	6.93
Loma Prieta	1989	Gilroy Array #2	6.93
Loma Prieta	1989	LGPC	6.93
Loma Prieta	1989	Oakland - Outer Harbor	6.93
Loma Prieta	1989	Oakland - Title & Trust	6.93
Mammoth Lakes	1980	L. Valley Dam (Upr L)	5.94
Morgan Hill	1984	Coyote Lake Dam (SW)	6.19
Morgan Hill	1984	Gilroy Array #6	6.19
Nahanni, Canada	1985	Site 2	6.76
Northridge	1994	LA - Wadsworth Hosp.	6.69
Northridge	1994	LA Dam	6.69
Northridge	1994	Newhall - Fire Sta	6.69
Northridge	1994	Newhall - W Pico C. Rd.	6.69
Northridge	1994	Pacoima Dam (downstr)	6.69
Northridge	1994	Pacoima Dam (up. left)	6.69
Northridge	1994	Rinaldi Receiving Sta	6.69
Northridge	1994	Sylmar - Converter Sta E.	6.69
Northridge	1994	Sylmar - Olive View Med	6.69
N. Palm Springs	1986	North Palm Springs	6.06
San Salvador	1986	National Geografical Inst	5.80
Superstition Hills	1987	El Centro Imp. Co. Cent	6.54
Superstition Hills	1987	Kornbloom Road (temp)	6.54
Superstition Hills	1987	Parachute Test Site	6.54
Westmorland	1981	Parachute Test Site	5.90
Westmorland	1981	Westmorland Fire Sta	5.90
Whittier Narrows	1987	LB - Orange Ave	5.99
Yountville	2000	Napa Fire Station #3	5.00

A comprehensive nonlinear regression analysis is

performed on the basis of the data obtained by the aforementioned response analyses. Thus, the following empirical expression for *IDR–T–µ* is obtained

$$IDR(T,\mu)=1+(\mu-1)^{c_1}\left(\frac{c_2}{T^{c_3}}+c_4\right) \qquad (5)$$

This empirical expression was one of the simplest equations which better described the numerical data following downward and upward concave curves, obtained by a homemade program after testing hundreds of simple mathematical equations. More specifically, this program fits about one thousand of built-in equations and then ranks them with respect to the corresponding correlation factor between the 'exact' and 'predicted' results. Coefficients c_1-c_4 should be evaluated in order to minimize to divergence between the results of 'exact' dynamic inelastic analysis and those of empirical Eq. (5). This leads to the following expression

$$IDR(T,\mu)=1+(\mu-1)^{1.10}\left(\frac{0.16}{T^{0.45}}-0.11\right) \qquad (6)$$

where the correlation coefficient is R^2=0.964 and the standard deviation is σ=0.055.

V. COMPARISON WITH 'EXACT' RESULTS AND OTHER STUDIES

This section examines the effectiveness and accuracy of the proposed empirical Eq. (6) for the evaluation of inelastic displacement ratios. Furthermore, comparisons of the proposed method with other existent studies from the pertinent literature are also provided. Thus, Fig. 2 shows the whole set of results for the aforementioned 72,000 analyses using the proposed method versus the 'exact' results of dynamic inelastic analyses.

It is found that the proposed empirical expression can accurately evaluate the inelastic displacement ratios for structures subjected to near-fault earthquakes. This can also be observed examining inelastic displacement ratio spectra for specific ground motions. For example, Fig. 3 shows the IDR spectrum for a SDOF system with *µ*=4, subjected to Palm Springs Earthquake (07/08/1986). It is obvious that empirical Eq. (6) closely follows the 'exact' dynamic inelastic analyses results.
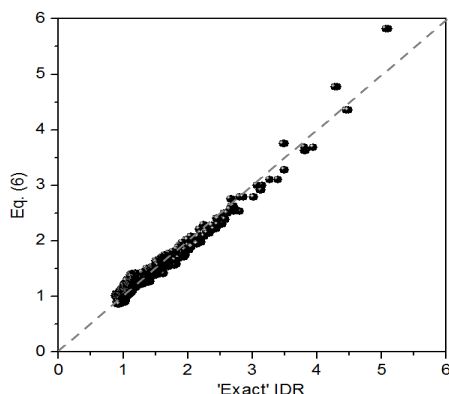

Fig. 2. Proposed method (Eq. 6) versus 'exact' results.

Miranda [14] proposed another empirical expression for the evaluation of inelastic displacement ratio, in the form

$$IDR(T,\mu)=\left[1+\left(\frac{1}{\mu}-1\right)exp\left(-12T\mu^{-0.8}\right)\right]^{-1} \qquad (7)$$

It is worth noticing that Eq. (7) has been proposed for traditional far-field seismic records and it is examined here to compare the structural response under far-field and near-fault ground motions. Thus, Fig. 4 presents the analyses results using the aforementioned empirical expression of Miranda [14] versus the 'exact' results of dynamic inelastic analyses.
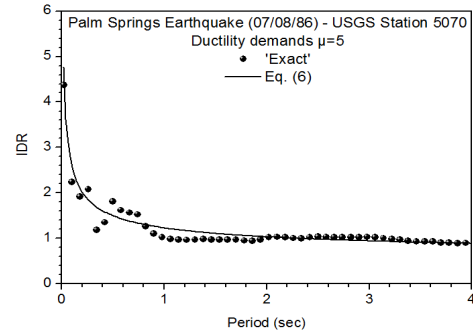

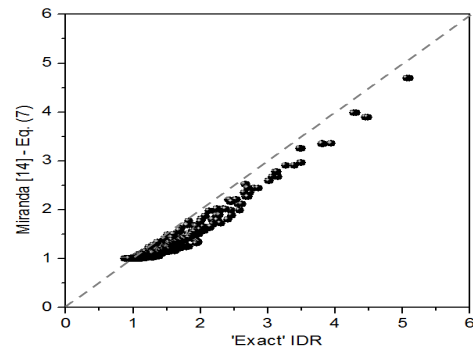Fig. 3. IDR spectrum for Palm Springs Earthquake (07/08/1986).

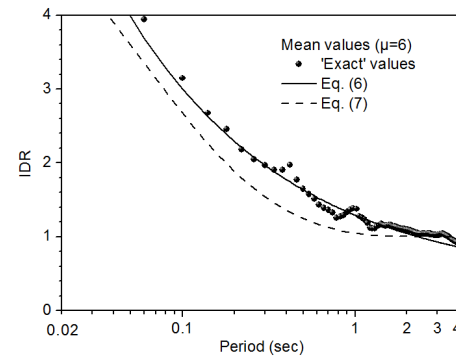Fig. 4. Miranda's approach [14] (Eq. 7) versus 'exact' results.

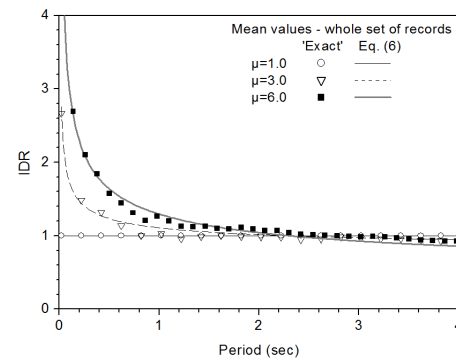Fig. 5. Empirical Eqs (6) and (7) versus 'exact' results for *µ*=6.

Fig. 6. Influence of ductility demands on *IDR*

Examining Fig. 4, it can be observed that almost the whole set of results appears to be under the diagonal of the diagram. This behavior means that near-fault ground motions generally lead to higher values of inelastic displacement ratios in comparison with far-field seismic records. This is also obvious in Fig. 5, which depicts the *IDR* mean 'exact' values (for the whole set of records) for μ=6, as well as the values from the proposed method (Eq. 6) and the approach of Miranda [14] (Eq. 7).

Furthermore, it is obvious that the inelastic displacement ratio is strongly affected by the period of vibration, where the higher the period the lower the *IDR*. Additionally, Fig. 6 shows that inelastic displacement ratios are also influenced by ductility demands.

It is found that, generally, the higher the level of ductility demands the higher the *IDR*.

VI. CONCLUSIONS

This study proposes a new method for evaluating inelastic displacement ratios of SDOF systems subjected to near-fault earthquakes on the basis of empirical expressions obtained after extensive parametric studies. The influence of period of vibration and ductility demands are carefully examined and discussed. A detailed study of the influence of the various parameters of the problem on the inelastic displacement ratio leads to the following conclusions:

1) The increase of ductility demands always leads to increased inelastic displacement ratios and vice versa. Furthermore, these ratios extremely depend on the structural period of the SDOF system, especially in the short-period range, say up to 1.0 sec. In this case, the lower the period, the higher the inelastic displacement ratio.

2) Near-fault seismic records lead to different inelastic displacement ratios in comparison with traditional far-field ground motions. More specifically, the former type of records leads to higher values of IDR in comparison with the latter one, especially in the short-period range, say up to 1.0 sec. Therefore, the pulse character of near-fault records should be taken into account.

3) Theoretical background examining all the parameters and characteristics of mechanisms of seismic faults, i.e., reverse, strike-slip, oblique-reverse etc., as well as appropriate empirical expressions for the corresponding inelastic displacement ratios are currently being derived and will be presented in a future paper by the authors.

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