Time Dependent Spectrum Analysis of Transient Signals: Application to Seismic Engineering

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***Abstract*—A data windowing approach is developed to estimate the evolutionary power spectra of strong motion earthquakes. The main properties and the engineering significance of the evolutionary power spectrum of seismic accelerations are first presented. Specific guidelines for optimal windowing to be used in time dependent spectrum analyses of transient signals are then formulated. Based on these guidelines, three moving data window functions and their characteristics are examined with the perspective of studying their effects on both the spectral resolution and the stability of spectral estimates of earthquake strong ground motions. Specific parameter values for which particular data window functions are more suitable to be used in order to obtain reliable estimates of the evolutionary spectral amplitudes are discussed.**

***Index Terms*—Data window functions, nonlinear response spectrum, random process, strong ground motions.**

I. Introduction

A substantial amount of work has been done in recent years to develop various techniques for the estimation of the evolutionary power spectra of earthquake strong motions[1, 2]. This problem is of importance in relation to investigations on characterization of site effects, strong ground motion simulation[3], and moving resonance effects [4].

The assumption of stationary ground spectral characteristics seems satisfactory in analyzing seismic response of linear structures to ground motions in which only the spectral components near the natural periods are important. However, for nonlinear stiffness degrading systems subjected to strong motions, the natural periods are increased and later arrival of the low frequency wave components may induce large structural response. Clearly, this resonance – type response would not have occurred in the case of stationary ground motions. Equally sensitive to the nonstationary ground motion spectral content is the low cycle fatigue response of some structural components [5]. In this case, the duration of the spectral component at the period of structural response becomes of crucial importance. All the same, if consideration is to be given to the response of some light attachments and their secondary systems mounted on some nuclear power plant walls. The functional failure of these systems may drastically affect the overall performance of the system.

In this paper, a data windowing approach is developed to effectively compute the evolutionary power spectra of strong motion earthquakes. based on specific guidelines, three moving data windows and their characteristics are examined. in addition, specific parameter values for which particular data window functions are more suitable to be used in order to obtain reliable estimates of the evolutionary spectral amplitudes are discussed. Finally, conclusions of engineering significance are presented on the basis of results relative to the effects of windowing characteristics on the spectral resolution and the stability of spectral estimates of earthquake strong ground motions.

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II. Formulation of Evolutionary Power Spectrum

The relationship between a zero mean random process $X(t)$, its evolutionary power spectrum $P_{xx}(f, t)$ (called also physical spectrum [6]) and its mean square value $E[X^2(t)]$ can be formulated as follows :

$$P_{xx}(f\,,t) = E\left[\left|\int_{-\infty}^{+\infty} X(s)w(t\text{-}s)exp(j2\pi fs)\;ds\right|^2\right] \tag{1}$$

$$\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} P_{xx}(f\,,t)\;df\;\;dt = \int_{-\infty}^{+\infty} E\left[X^2(t)\right]dt \tag{2}$$

where $E[X^2(t)]$ represents the expectation operator, $w(t\text{-}s)$ a selected window function normalized such that its total energy equals unity; f and t the frequency and time variables, s a dummy variable and j such that $j^2 = -1$.

The right hand side of Eq. (1) clearly indicates that the evaluation of the evolutionary power spectrum is dependent on the shape and the characteristics of the selected window function. It is seen from Eq. (1) that, as t varies, the window time function is shifted such that a seismic wave component with frequency f can be conveniently traced. Eq. (1) also states that the total power is associated with the volume covered by the physical spectrum when plotted in the frequency – time plane. Using energy considerations, it can be shown that the physical spectrum of the ground acceleration process $X(t)$ offers the additional flexibility to be simply related to engineering parameters such as, the Arias intensity, the Housner spectral intensity and the RMS acceleration at the site.

III. Guidelines for Optimal Windowing

A. Basic Considerations

In the previous section, the evolutionary power spectrum was introduced for tracing the time evolution of any wave component of a given accelerogram. However, it should be

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understood that inherent in such a decomposition is the truncation of the target accelerogram into a number of time sequences, the Discrete Fourier Transform (DFT) of which are to be evaluated. These DFT's are then combined in a judicious way so as to yield the DFT of the entire record as described by the FFT algorithm. It follows that the overall truncation operation is performed at two levels:

1) The first one is inherent in the definition of the total duration and happens before the DFT is taken.
2) The second one takes place during the process of windowing $x(t)$ via the selected data window.

For more details on this subject, the reader is referred to any book of digital filters ; the point to be made here, being that the true ground motion spectrum is practically impossible to obtain and only a smoothed one can be evaluated instead. To further clarify from a quantitative perspective the truncation effect on the spectrum, two cases are now considered:

1) Assume the hypothetical case of an accelerogram $x(t)$ of infinite DURATION ; this is the same as looking at the record $x(t)$ through a window of infinite extent, i.e., the time window amplitude is equal to unity at any time t. It follows from the convolution theorem that:

$$\int_{-\infty}^{+\infty} x(t)\, w(t)\, exp(-j2\pi ft)\, dt = \int_{-\infty}^{+\infty} F(f')\, \delta(f-f')\, df' = F(f) \quad (3)$$

i.e., the true spectrum is recovered. In the above equation, $\delta(f - f')$ represents the Dirac delta function evaluated at $f = f'$.

2) Consider now the real accelerogram of finite duration T. This is equivalent to applying the rectangular window function rect(t/L), with $L = T$, to $x(t)$. Then by a similar argument to case a) above, one obtains:

$$\int_{-\infty}^{+\infty} x(t)\, w(t)\, exp(-j2\pi ft)\, dt = \int_{-\infty}^{+\infty} F(f') \left[\mathrm{sinc} 2\pi\left(f-f'\right)\frac{L}{2}\right] df' \neq F(f) \quad (4)$$

which clearly shows that unlike the previous example, the true spectrum is not recovered. Instead $F(f)$, one has obtained the convolution of $F(f)$ with a sinc function: an operation which represents a certain smoothing of the real spectrum and spurious effects due to the side lobes of the window spectrum. The amount of smoothing is obviously dependent on the frequency f and the window length $L = N.\ \Delta t$ where Δt is the sampling time interval and N the total number of sampling points (see Fig. 1).

A clear, pictorial interpretation of this fact is provided by Fig. 1 above. It is also seen that the shorter the window length, the larger the smoothing effect and the lower the spectral resolution on the spectrum.

B. Main Properties of Data Window Function

As demonstrated in the previous section, data windowing is always accompanied by a distortion of the true spectrum and the corrected spectrum is practically unrecoverable. To maintain the distortion level as minimum as possible, the following guidelines are recommended:

1) Ideally, we would like to have a window with a spectral width of the same order of magnitude as the narrowest detail to be investigated in the spectrum. The smaller the bandwidth, the lower the stability of spectral estimates and the higher is the spectral resolution. This is again clearly illustrated in Fig. 1.
2) The window spectrum should have very small (if not insignificant) side lobes, the effect of which introduces spurious SPECTRAL lines (i. e. the stability of spectral estimates is altered). As can be inferred from Eq. (4), this condition is ideally accomplished by using the Dirac delta function.

Condition 1) suggests, accordingly to the uncertainty principle, the use of a broad data window function. Condition 2) implies no discontinuity in the derivative of the window function, i.e., a gradual tapering off the tails of the window. Simply stated, the window should have no sharp corner.

In summary, it may be concluded from Sections III-1 and III-2 that once a window functional form is selected, one has to make a compromise between two main factors : resolution and stability of spectral estimates. From Fig. 2, it is clearly seen that the value of L = 128 Δt leads to an excellent agreement (in amplitude and maximum peak location) between the nonlinear acceleration response spectra, used herein as decision criterion, of the simulated and target accelerogram.

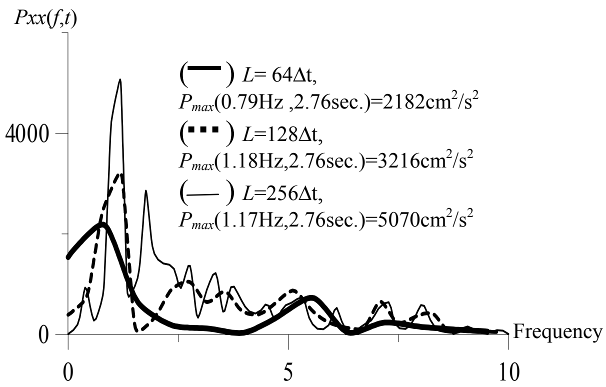


Fig. 1. Effect of window length L (El – Centro Earthquake, Gaussian window)

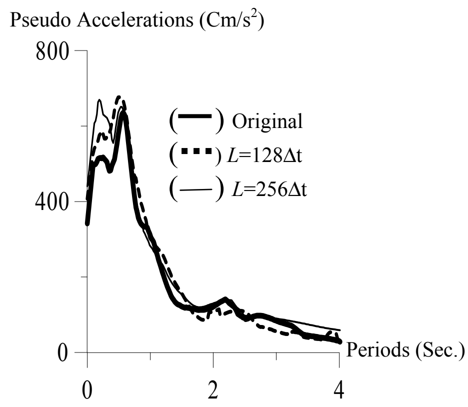


Fig. 2. Effect of window length L on nonlinear response spectra (El-Centro earthquake, Gaussian window)

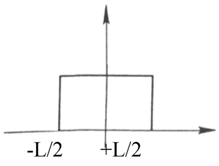
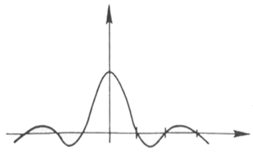
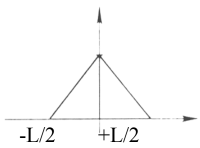
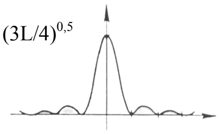
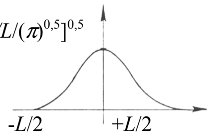
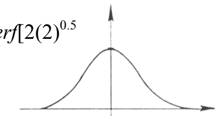
To further illustrate the desirable properties of moving data window functions (symmetry, small side lobes, high concentration in the main lobe and so on), the characteristics of three moving data window functions are determined and reported in Table I below.

It should be underlined that the functional forms of these window functions commonly share the important property to converge to the Dirac delta function as the spectral width L_f

tends to zero or equivalently, as the window length in the time domain tends to infinity.

From Eqs. (1) and (4) on the one hand and based on the numerical and analytical results of Fig. 1 and Table I respectively, on the other hand, it can be concluded that resolution and stability of spectral estimates depend not only on the functional form of the selected data window function but also and more importantly on its spectral.

TABLE I: CONSIDERED WINDOWS AND THEIR CHARACTERISTIC PARAMETERS

Normalized window in time domain $\int_{-\infty}^{+\infty} w^2(t)\,dt = 1$	**Nominal length**	**Nominal width**	**Window in frequency domain** $\int_{-\infty}^{+\infty} w^2(f)\,df = 1$
$w(t)=(1/L)^{0.5}\,\mathrm{rect}(-L/2,+L/2)$ 	L	$L_f=1/L$	$W(f) = L^{0,5}\,\mathrm{sinc}(\pi f L)$ 
$w(t) = (3/L)^{0,5}\,\mathrm{rect}\,(-L/2, +L/2)$ 	L	$L_f=1/L$	$W(f) = (3L/4)^{0,5}\,\mathrm{sinc}^2(\pi f l/4)$ $(3L/4)^{0,5}$ 
$w(t) = [4/L/(\pi)^{0,5}]\,\exp(-8t^2/L^2)$ $[4/L/(\pi)^{0,5}]^{0,5}$ 	L	$L_f=1/L$	$W(f)=(2)^{0,5} exp[-(\pi f L)^2/8]\; erf[2(2)^{0.5}]$ $[(2)^{0,5}\, erf[2(2)^{0.5}$ 
$L = \frac{1}{w(0)}\int_{-\infty}^{+\infty} \lvert w(t)\rvert\, dt$	$LL_f = 1$		$L_f = \frac{1}{W(0)}\int_{-\infty}^{+\infty} \lvert W(f)\rvert\, df$

IV. CONCLUSIONS

The major results obtained in this investigation can be summarized as follows:

- The evolutionary power spectrum can be simply related to ground motion intensity parameters such as the RMS acceleration and Arias intensity at a given site.
- The evolutionary power spectra of strong ground motions can be accurately and effectively computed by means of the method used in the present study.
- Resolution and stability of the evolutionary power spectrum estimates are two conflicting requirements that can be optimally determined from a critical study of various factors such as the functional form of the selected data window and more importantly its spectral width (or equivalently the window length in the time domain). The latter should be of the same order of the narrowest detail to be investigated in the ground motion spectrum.

The applicability of the above – mentioned analytical developments to near field strong ground motion analysis and their extension to ground motion simulation will be numerically assessed in a forthcoming work It is hoped that the analytical developments presented in this study provide an opportunity for stimulating further interest in the important subject of nonstationary characterization and simulation of earthquake strong ground motions.

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