To A Probabilistic Approach of Reinforced Slope Stability Analysis

L. Menaa and Z. Talaaiche

***Abstract*—A developed probabilistic model to analyze reinforced slopes is presented in this paper where, a log-normal distribution law and a first order of second moment method "FOSM" are used. Moreover, an interactive code, using C ++ language, based on a developed model, is carried out.**

With idea to validate the developed model, a parametric study was conducted, considering three types of current soils, taking into account the following parameters: cohesion "C", internal friction angle "φ ", the unit weight "γ" and apparent soil-reinforcement friction angle "α". Expected variables were: reliability index "β", the probability of failure (ruin) "P_r" and the safety factor " F_S".

Obtained results show that the friction angle φ and cohesion C seem most significant while, the reinforced soil unit weight "γ" and the soil-reinforcement interface friction angle "α" show a slightly sensitive influence. Besides, it is clearly showed that safety factor increases with the increase of reinforcement length.

***Index Terms*—Slope stability, performance function, reliability index, probability of ruin.**

I. Introduction

Soils are generally inhomogeneous mediums with frequently complex texture. Consequently, soil properties, mainly the cohesion and angle of internal friction are recurrently affected by uncertainties. For a more reliable calculations, these uncertainties dues to soil heterogeneity and sometimes to all errors affecting them (action, model, etc ...), should be preferably introduced in structural analysis.

Beside conventional deterministic methods, which consider soil parameters as deterministic, panoply of probabilistic methods are developed, considering not only mean values of depending parameters but also their uncertainties. These methods take into account the variability of site characteristics; therefore, the decision-making related to the potential risk that could affect the structures, themselves, would then be better evaluated.

In the geotechnical engineering, slope stability is evaluated by calculating a safety factor, where uncertainties of soil parameters have a direct impact on the safety of the structure. The probabilistic analysis dealing with slope stability was earlier conducted in the 70s [1], [2]. Since that, in the context of geostatistics, these methods have more evolved in both stability analysis and structural design fields [3], [4], [5], [6], [7].

In this context, the presented contribution will focus on the reliability analysis of reinforced slopes, using flexible material (geotextiles), as long reinforced earths consider not only soil properties but also the soil-reinforcement interaction uncertainties. Recently, much emphasis has been achieved by using of probabilistic method. At the beginning, several authors, such Kornell, Matsuo and Kuroda, have already contributed in the development and foundation of reliability analysis methods of the stability of unreinforced slopes. Kornell considered the shear strength along the slip line as linear combination of the coordinates of the considered point; he calculated the mean and variance of the safety factor F_S knowing the mean and variance of resisting and driving moments. Matsuo and Kuroda, adopted some assumptions for soil properties, by fitting slope surface and sliding circle using two functions f_1 *(x)* and f_2 *(x)*; after integrating over the two surfaces, they can calculate the expression of the safety factor F_S [6]. Among others, dealing with reinforced slopes, we can refer to Byung Sik Chun, Kyung Min Kim and Deok Ki Min; by using the method of limit equilibrium, they studied the reliability of reinforced earth retaining wall and the sensitivity of some parameters on the failure probability [8].

The aim of presented work is to develop a model which enables us to introduce the spatial variability of soil properties constituting the slope, taking into account the effect of flexible reinforcement in the probabilistic analysis of slopes stability. For this purpose, the first order of second moment method (FOSM) is handled; this method seems more appropriate for the considered topic [9]. All statistical characteristics of introduced parameters, such as mean and standard deviation were calculated. A function of the probability density is then properly chosen.

The probabilistic analysis of slope often go through the search for the critical state, giving the minimum safety factor by handling any explicit method (i.e. : Bishop, limit equilibrium, …); in this work, Caquot-Taylor explicit method is used, giving the expression of the safety factor and the performance function [5].

An interactive code, using C ++ language, which is not exposed in the present paper, is carried out. Flow chart of the developed code is illustrated by Fig. 1. Furthermore, some examples of resolved problems, exposed in literature, are redone in this work, in order to validate developed model [5]. For the same objective, panoply of software, like: Slide, visual slope, etc., can be used for this validation [10].

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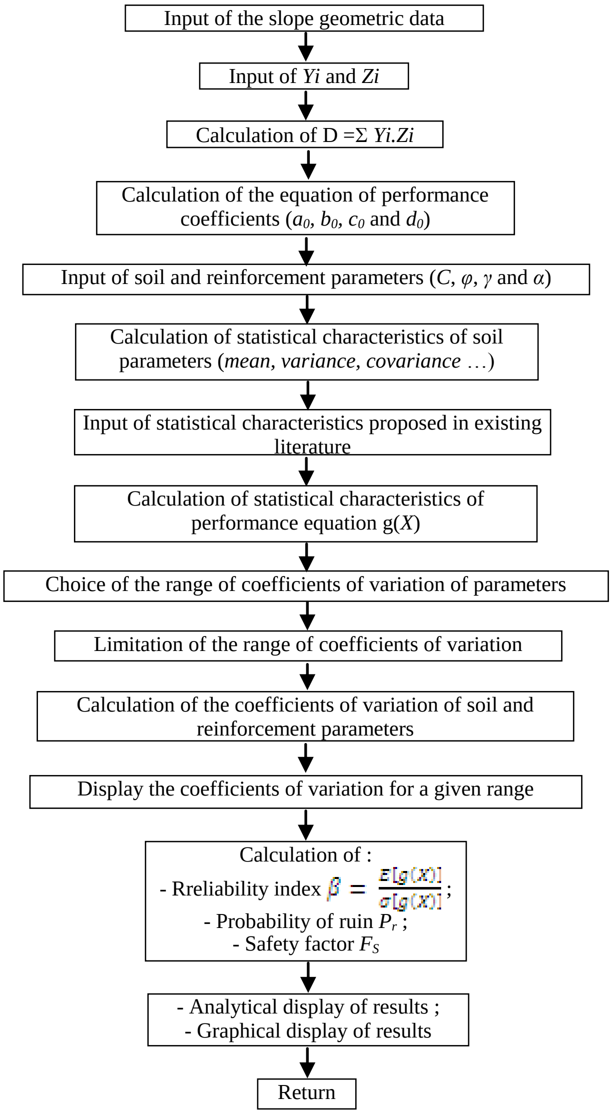

Fig. 1. Flow chart of developed calculus model.

II. Problematic and State Equation

The slope stability analysis can be carried out by establishing of equilibrium condition of the whole forces system (Fig. 2). This leads to calculate a safety factor of slope stability, noted "F_S". The limit equilibrium method can be used, adopting the assumptions of homogeneous soil.

When calculated safety factor is insufficient, it can be improved by increasing of resisting forces by introducing a reinforcing material. Hence, the expression of the safety factor, considering the rotational slump, may be given by the following general expression [11], [12]:

$$F_S = \frac{M_{\text{Res.}}(Soil) + M_{\text{Res.}}(Reinforcement)}{M_{Sld.}} \quad (1a)$$

Otherwise:

$$F_S = \frac{M_{\text{Res.}}(Soil)}{M_{Sld.}} + \frac{M_{\text{Res.}}(\text{Reinforcement})}{M_{Sld.}} \quad (1b)$$

where

F_S: Safety Factor, $M_{Res.}(Soil)$: Resistant moments and $M_{Sld.}$: Sliding moment.

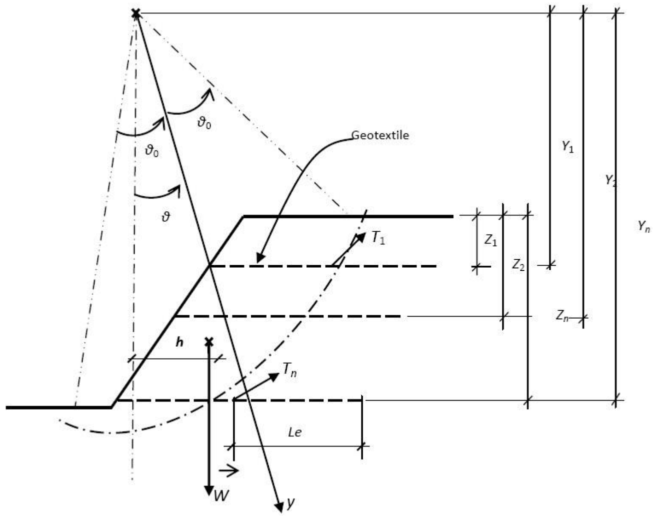

Fig. 2. Shape of analyzed reinforced slope.

Consequently, the global safety factor F_S can be considered as a superposition of both materials (slope soil and reinforcement), thus:

$$F_S = F_S\,(unreinforced\ soil) + F_S\,(Reinforcement) \quad (1c)$$

Let's use the explicit expression of slope safety factor given by Caquot-Taylor [4]:

$$W.h \leq \left[2\theta_0.r^2.C + \frac{\theta_0}{\sin\theta}.r.W_y.\tan\varphi\right] / F \quad (2a)$$

where W_y , the y projection of W (Fig. 2) .

Hence, safety factor for unreinforced slope becomes:

$$F_S\,(Soil) = \frac{2\theta_0.r^2.C + \frac{\theta_0}{\sin\theta}.r.W.\cos\theta.\tan\varphi}{W.h} \quad (2b)$$

where

φ, internal friction angle of slope material ; C, cohesion of slope material ; r, radius of failure circle ; h, horizontal distance between W force and the point "o"; θ_o, bisects of the angle formed by the slip plane; $W = \gamma\, S.1$, Weight of the slip prism (wedge); S, sliding prism area ; γ, unit weight of slope soil.

Safety factor for reinforcement becomes:

$$F_S(Reinforcement) = \frac{\sum T_i.Y_i}{W.h} = F_g \quad (2c)$$

where T_i : developed tension in the (i) strip layer ; Y_i : lever arm of the moment of T_i with respect to the point "o".

The expression of the tension in the reinforcing strip, depending on the shear stress "τ" at the interface and the anchor length "Le" [13], [14] can be written as:

$$T_i = 2.\tau_a.L_e \quad , \quad (3)$$

with

$$\tau_a = \sigma_v.\tan\delta_a + C_a \quad . \quad (4)$$

Handling relationship [13]:

$$C_i = \frac{\sigma_v.\tan\delta_a + C_a}{\sigma_v.\tan\varphi + C} \quad (5)$$

where C_i: coefficient or factor of soil-reinforcement (geotextile) interaction which is obtained from the extraction tests of a band of geotextile fabric at the laboratory, equal between $\frac{2}{3}$ and 1;

σ_v : Vertical soil stress [KN/m^2] = $\gamma . z$;

δ_a : The soil-reinforcement interface apparent friction angle.

Expression (3) becomes:

$$T_i = 2.C_i(\sigma_v.\tan\varphi + C).L_e \tag{6a}$$

In our case (flexible reinforcement) [15], [16] :

$$T_i = \tfrac{4}{3}.\sigma_v.\tan\varphi.\alpha.L_e \tag{6b}$$

where $\alpha = tan\delta_a$.

For multiple layers of reinforcement, expression of F_g becomes:

$$F_g = \frac{\sum_{i=1}^{n} \frac{4}{3}.\gamma.\tan\varphi.\alpha.L_e.Z_i.Y_i}{W.h} = \frac{\frac{4}{3}.\gamma.\tan\varphi.\ \alpha.L_e.D}{W.h} \tag{7}$$

where $D = \sum_{i=1}^{n} Z_i.Y_i$.

Finally, the expression (1) becomes:

$$F_S = \frac{2\theta_0.r^2.C + \frac{\theta_0}{\sin\theta}.r.W.\cos\theta.\tan\varphi + \frac{4}{3}.\gamma.\tan\varphi.\ \alpha.L_e.D}{W.h} \tag{8}$$

Which represents, for us, the explicit form of the equation of state describing the safety factor "F_S" for slope stability analysis.

III. Probabilistic Model Formulation

A. Performance Function

First, let's write the basic function of limit state or performance function:

$$g(X) = G = R - S \tag{9}$$

this allows to write the probability of failure P_r :

$$P_r = P(g(X) \le 0) = \int_{g(X)\le 0} f(X)\ dX \tag{10}$$

where $f(X)$, the probabilistic density function of random variable X.

For studied reinforced slope, the performance equation (9) takes the form:

$$g(X) = 2\theta_0.r^2.C + \frac{\theta_0}{\sin\theta}.r.\ (S.\gamma).\ \cos\ \theta.\tan\varphi\ + \frac{4}{3}.\gamma.\tan\varphi.\alpha.L_e.D - (S.\gamma).\ h \tag{11a}$$

This can be written as:

$$g(X) = a_0.X_1 + a_1.X_2.X_3 + a_2.X_2.X_3.X_4 + a_3.X_3 \tag{11b}$$

where

$$X_1 = C\ ;\quad X_2 = \tan\varphi\ ;\quad X_3 = \gamma\ ;\quad X_4 = \alpha$$

$$a_0 = 2\theta_0.r^2\ ;\quad a_1 = \frac{\theta_0}{\sin\theta_0}.r.S.\cos\theta\ ;\quad a_2 = \tfrac{4}{3}.L_e.D\ ;$$

$$a_3 = -\ S.h \tag{12}$$

B. Reliability Index Probability of Ruin Safety Factor.

By using of FOSM method, with centered and reduced lognormal distribution, the basic function is approximated by a Taylor's first order polynomial development; the mathematical Esperance ($E[g]$) and standard deviation (σ_g) of the basic function can be easily calculated from the mean and variance of remaining variables. For a given base function $g(X)$, the reliability index can then be defined as the ratio of the first two statistical moments of this function [16], so:

$$\beta = \frac{E[g(X)]}{\sigma[g(X)]} = \frac{g(\overline{X})}{\sqrt{Var[g(X)]}} \tag{13}$$

The probability of ruin, estimated using the method of second moments is given by:

$$P_r = \Phi(-\beta) = 1 - \Phi(\beta) \tag{14}$$

where $\Phi(\beta)$, is the distribution function of the standard normal distribution.

The safety factor F_S is given as:

$$F_S = \frac{a_0.\overline{X_1} + a_1.\overline{X_2}.\overline{X_3} + a_2.\overline{X_2}.\overline{X_3}.\overline{X_4}}{a_3.\overline{X_3}} \tag{15}$$

It is noted that the probability of ruin depends on the safety factor which can more or less important. This correspondence is simplified when assuming the dispersion of variable is mall relatively to the importance of self-weight. This corresponds to the major studied structures in geotechnics.

Koerner R. M. had already proposed a correspondence between the probability of ruin "P_r" and "F_S" using abacus forms, for a large number of construction damages like earth dams and many other works; a normal distribution and a lognormal distributions were applied [17].

IV. Parametric Study Using Developed Model

A. Validation of the Developed Model

In order to validate a developed model, an example of probabilistic slope stability, achieved by Jean-Louis Favre, handling method Caquot-Taylor [5] has been redone twice through a presented work, by using the developed model and software "Slide". It is about a compacted silt clay fill, as a platform of 8 m height and 2/3 slope. The problem data are shown in Table I. Comparative results of safety factor "F_S", reliability index "β" and the failure probability "P_r" are shown in Tables II and III.

TABLE I: ANALYZED SLOPE SOIL CHARACTERISTICS

	γ_h (kN/m^3)	C (kPa)	φ (°)
Average values	21	9	27
Coefficient of variation, Cv (.)	7 %	25 %	15 %
Coefficient of correlation, $\rho_{c,\varphi}$		- 0,4	

TABLE II: SLOPE ALONE COMPARATIVES RESULTS

	Developed code	Software «Slide»	
	Log-Normale Law	*Normale Law*	*Log-Normale law*
F_s	1,54	1.46*	
β	3,42	3,59	4,282*
P_r (%)	0,000	0,000	0,000*

* : compared results

TABLE III: REINFORCED SLOPE COMPARATIVES RESULTS.

	Developed code	Software «Slide»	
	Log-Normale Law	*Normale Law*	*Log-Normale law*
F_s	1,706	1,747*	
β	4,444	6,475	8,424*
P_r (%)	0,000	0,000	0,000*

* : compared results

The validation results for an unreinforced slope have shown a good agreement with those obtained by usual methods; for example, the difference between the values of the safety factor calculated using the developed model and software "Slide" is about of 5%. The safety factor F_S calculated by the developed model is greater than that calculated by "Slide", which allows conclude, in this case, that the developed model is more optimistic.

For the reinforced slope, the results validation also shows good agreement with those obtained by usual methods; the difference between the values of the safety factor calculated by the two methods is about of 2%. The safety factor F_S calculated by the developed model is lower than that calculated by "Slide". So, the developed model seems more pessimistic for reinforced structures.

In all cases, obtained results by developed model seem comparables to those given by currently used methods.

B. Parametric Study

In this parametric study, we will discuss the effect of each variable on the safety factor "F_S", reliability index "β" and the ruin probability "P_r", by varying one parameter within a chosen interval and fixing the remaining parameters. The range of values of the coefficients of variation for each parameter is chosen appropriately, according to data available in literature.

Three examples, of 03 types of soil (cohesive, purely cohesive and granular soils), were considered. Each example represents a special combination between the different coefficients of variation of each parameter. The soil types chosen in this study are summarized in Table IV. Obtained results, for soil type 1 are shown through Figs. 3-7. Relative results for soils type 2 and 3 are placed in appendix, through Figs. 8 -14.

TABLE IV: PARAMETRIC STUDY DATA.

Type de sol	C (kPa)	φ (°)
1- Cohesive soil (C ≠ 0, φ ≠ 0)	9	27
2- Granular soil (C = 0 , φ ≠ 0)	0	37
3- Purely cohesive soil (C ≠ 0, φ = 0)	25	0

C. Results and Discussion

Through parametric study results, it can be assumed that the most significant parameters are cohesion "*C*" and the internal friction angle "φ". Beside that, the following comments can be affirmed:

1) First, we can notice that combinations for soil type 2 and soil type 3 doesn't give any graphical result because $Cv(X_1= C)$ and $Cv\ (X_2 = \varphi)$ are nulls;
2) The safety factor "F_S" monotonously increases according to layer's number "n" and anchorage length "*Le*" (Figs. 3a and 3b), except for soil with a zero internal friction angle "φ" (Figs. 9);
3) For cohesive soil ($\varphi \neq 0$ and $C \neq 0$), probability of failure "P_r" increase when $Cv\ (X_1)$ is bigger than 10; this same probability decrease when $Cv\ (X_2)$ fluctuate between 0 and 15 and increase when $Cv\ (X_2)$ is bigger than 15; we can notice a slight dependence of $Cv\ (X_3)$ (Figs. 4 and 5);
4) For purely cohesive soil ($\varphi = 0$), "P_r" increases when $Cv\ (X_1)$ is bigger than 10 (Fig. 9); it also increases when $Cv\ (X_3)$ is bigger than 10 but with a small values of other remaining parameters (Fig. 10); it becomes constant for all remaining parameters (Fig. 11);

For granular soil ($C = 0$), "P_r" increases when $Cv\ (X_2)$ is bigger than 10 (Fig. 12); it also increases when $Cv\ (X_3)$ is bigger than 10 but with a small values of other remaining parameters (Fig. 13); it becomes constant when depending with $Cv\ (X_4)$ (Fig. 14).

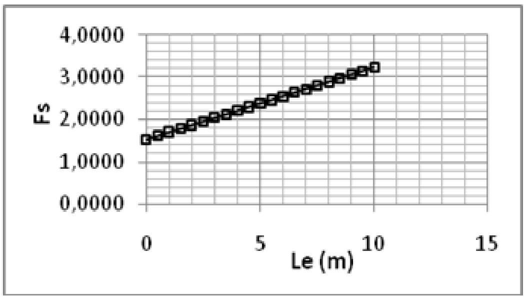

Fig. 3a.Variation of «F_s»: with *anchorage length* "Le".

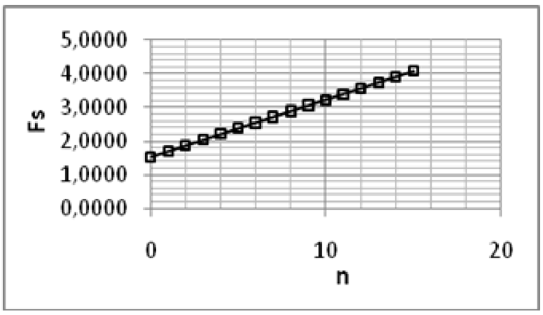

Fig. 3b. Variation of «F_s» *with number of geotextile strips* «n».

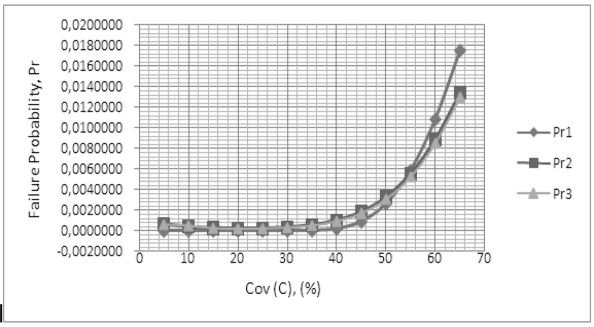

Fig. 4a. Variation of «P_r » with Cv (C).

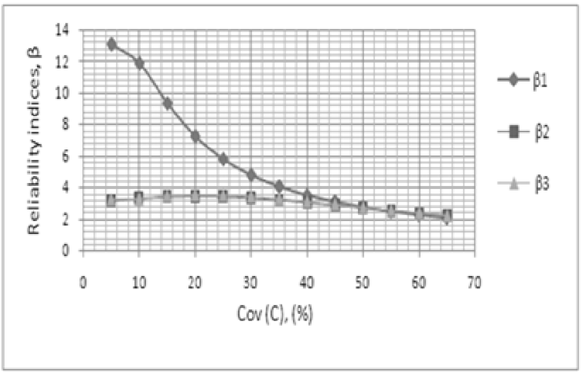

Fig. 4b. *Variation of «β »with* Cv (C).

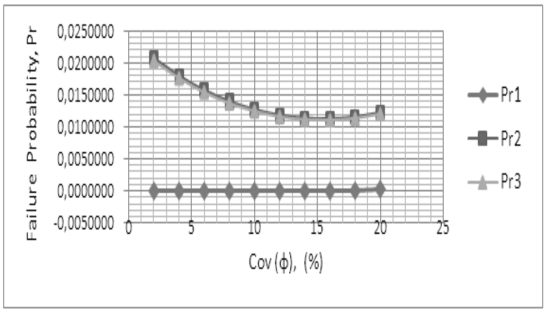

Fig. 5a. Variation of «P_r »with Cv (φ).

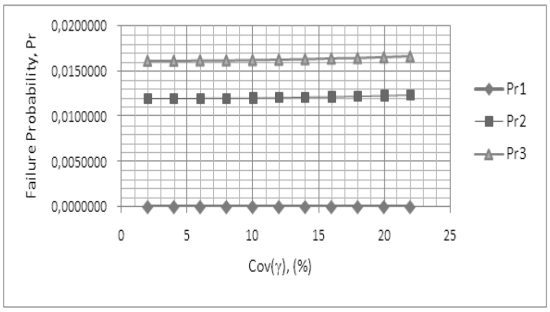

Fig. 5b. Variation *of «β »with* Cv (φ).

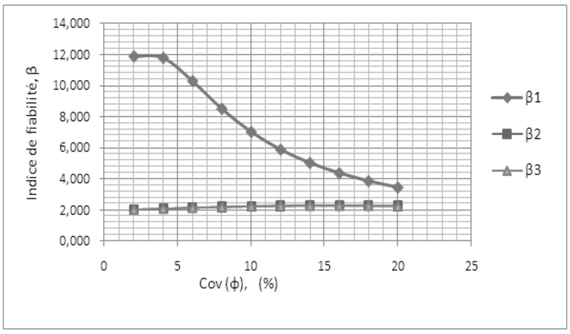

Fig. 6a. Variation of «β »with Cv (γ).

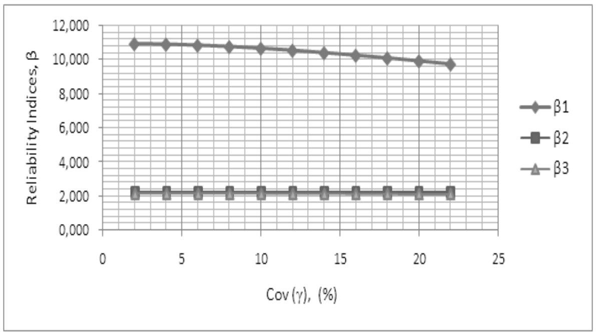

Fig. 6b. Variation *of «P_r »with* Cv (γ).

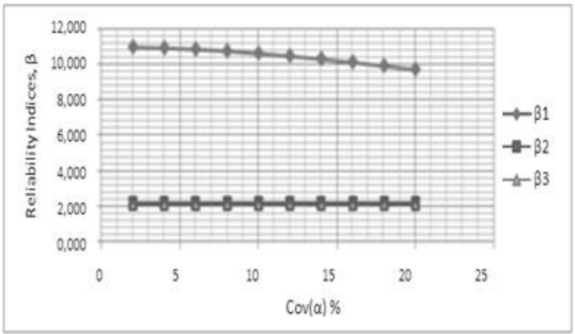

Fig. 7a. Variation of «β » with Cv (α).

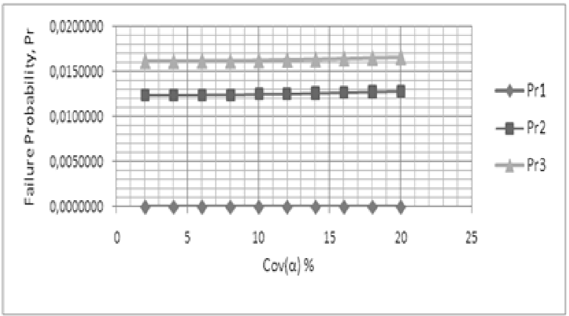

Fig. 7b. Variation *OF «P_R »WITH* CV (A).

V. Appendix

A. *Sol Type 2 (C > 0; φ = 0)*

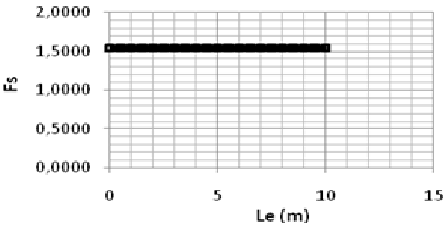

Fig. 8. Variation of «F_s »with *anchorage length "* Le*"*.

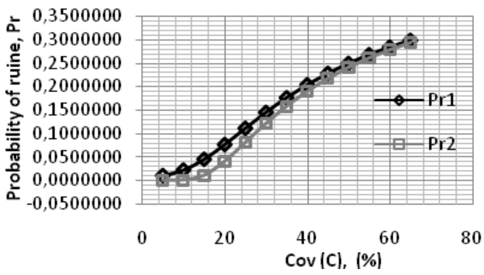

Fig. 9. Variation *of «P_r »with* Cv (C).

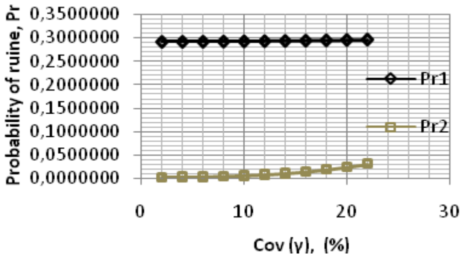

Fig. 10. Variation *of «P_r »with* Cv (γ).

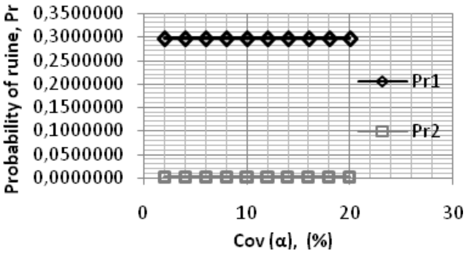

Fig. 11. Variation *of «P_r »with* Cv (α).

B. Sol type 3 (C = 0 ; φ > 0) :

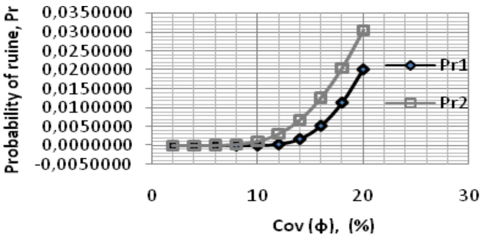

Fig. 12. Variation *of «P_r »with* Cv (φ).

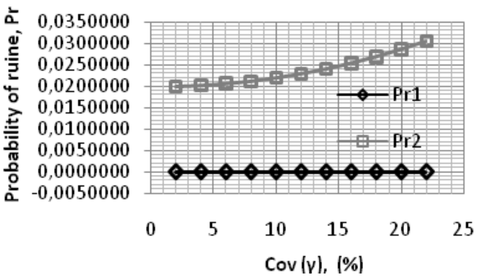

Fig. 13. Variation *of «P_r »with* Cv (γ).

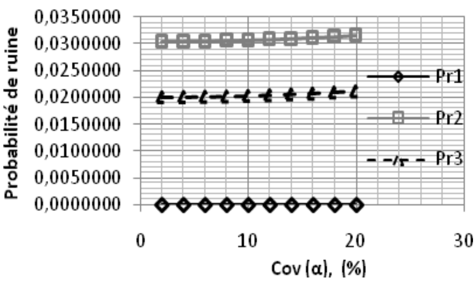

Fig. 14. Variation *OF «P_R »WITH* CV (A).

VI. Conclusion

The calculation of the variance and standard deviation involves many comments, it expresses the weight of each parameter through its variance and the gradient that causes the security equation; it is also noticed that the correlation between two different variables is not negligible. Thanks to the parametric study and the sensitivity of variables analysis, it can be assumed that the significant soil parameters are cohesion "*C*" and the angle of internal friction "φ" especially when dealing with natural soils (*φ* and *C* ≠ 0); when these parameters are nulls reliability index "β" and the ruin or failure probability "*Pr*" show a certain dependence of the two remaining parameters (α and γ).

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