Enhancement of Performance of Cognitive Radio Network with Incorporation of MRC Scheme at Secondary Receiver

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Abstract—The faithful detection of presence of a primary user (PU) is the most essential requirement of a cognitive radio network. Otherwise the PU will experience jamming from a secondary user (SU) which will eventually lead to reduction in throughput of the PU. Similarly, the false detection of a PU will abstain a SU from its transmission opportunity hence reduce the throughput of the SU. Under this situation we propose a cognitive receiver equipped with multiple antenna and maximal ratio combining scheme (MRC) to detect the presence of a PU. The rest of the communication links like PU to PU or SU to PU uses single antenna. In this paper the concept of test statistics of fusion center from a previous literature is applied in the derivation of the probability of false alarm, probability of detection, channel capacity and symbol error rate of the network. The performance of a cognitive radio network under MRC scheme at receiving mode of SU is found better than the case of a single antenna.

Index Terms—MRC diversity, probability of detection, probability of false alarm, primary user, secondary user.

I. INTRODUCTION

The ubiquitous and diversified use of wireless technology has made the electromagnetic spectrum an invaluable resource for wireless communication and there exists no alternative to its proper utilization and management. However, recent studies show that licensed spectrum remains grossly inactive or is rarely utilized continuously across time and space thereby resulting in spectrum wastage. This problem of inefficient use of spectrum has been addressed by the FCC through the introduction of cognitive radio (CR). To utilize licensed spectrum effectively, CR user must perform spectrum sensing to extract inactive licensed band [1]. Spectrum sensing plays a critical role in CR network and is challenged by uncertainties like channel randomness, channel fading or shadowing, aggregate interference, noise uncertainties etc.

In [1], fundamental issues specific to CR have been investigated while primary focus of [2] was on signal processing in context of spectrum sensing implementation in CR networks. A survey of various spectrum sensing techniques and the associated challenges has been studied in [3] and [4]. In [5] a statistical model for interference aggregation has been developed while the probabilities of detection and false alarm over fading channel have been addressed in [6]. Beacon transmitter placement effects on aggregate interference and capacity outage performance were examined in [7] and [8]. In [9], the concept of fusion centre and global test statistics has been used to derive probabilities of false alarm on detection and then these have been deployed to investigate the spectrum sensing and throughput trade off in CR under outage constraints. Collaborative sensing to handle problems of deep shadowing and fading effects for opportunistic access were investigated in [10].

Application of the maximum ratio combining (MRC) diversity i.e. equipping the receiver with multiple antennas is known to improve system performance significantly. It has been shown that capacity performance can be improved considerably through MRC diversity by mitigating severe fading [11] and channel estimation error [12] in interference channel. However, a major short coming in [11] and [12] is that the effect of transmit power constraint has not been taken into consideration.

In this paper we have made an effort to deploy MRC diversity for sensing spectrum by a secondary user (SU) and accordingly make decision on the presence or absence of the primary user (PU). The derived cumulative distribution function (cdf) of the received signal-to-noise ratio (SNR) has been utilized to evaluate the spectrum sensing system performance with MRC diversity scheme in Nakagami channel and the probability of false alarm and the detection probability are then derived. Probability of outage is calculated taking into consideration the interference power constraint and the transmit power constraints on the transmit power of the cognitive user.

The paper is organized as follows: section II outlines the system model and spectrum sensing scheme. Results and analysis are presented in section III. Finally, section IV concludes the paper.

II. SYSTEM MODEL

Let us consider a system comprised of a primary transmitter-receiver pair and a secondary/cognitive
transmitter-receiver pair. The cognitive users use $K$ number of antennas only during receiving operation and utilizes MRC scheme for decision making. All the other conditions (PU to PU and SU transmitter to PU) at both ends single antenna is used. The goal of the spectrum sensing scheme at any antenna of the cognitive user is to test two hypotheses where hypothesis $H_0$ and $H_1$ denote respectively the absence and presence of the PU and is defined as follow:

In the case of absence of the PU,

$$H_0: \ x[n] = w[n], \ n = 0, 1, \ldots, N-1;$$  \hspace{1cm} (1a)

while in the case of presence of the PU,

$$H_1: \ x[n] = h_k e^{j\theta} s[n] + w[n], \ n = 0, 1, \ldots, N-1;$$  \hspace{1cm} (1b)

where $w[n]$ is the circularly symmetric noise signal at the wireless channel and considered to be a complex Gaussian random variable, with zero-mean and variance $\sigma^2_n$; $s[n]$ is the $n^{th}$ sample of the PU signal and $h_k$ is the channel gain experienced by the $k^{th}$ antenna of the SU during the receiving mode; $N$ is the total number of samples.

Energy detection spectrum sensing technique with MRC diversity has been used because of its relatively low computational complexity and the fact that prior knowledge of the PU signal is not required.

If each antenna of the SU uses $N$ samples for sensing, then the average energy detected by the $i^{th}$ antenna is

$$E_i = \frac{1}{N} \sum_{n=0}^{N-1} |x[n]|^2,$$  \hspace{1cm} (2)

where $x[n]$ is the cognitive user’s received samples used for sensing and let this be denoted by

$$x[n] = [x[0], x[1], \ldots, x[N-1]]^T.$$  \hspace{1cm} (3)

Therefore, the final average energy of a receiver of the SU is:

$$E_{av} = \frac{1}{K} \sum_{i=0}^{K-1} E_i,$$  \hspace{1cm} (4)

where $E_{av}$ is a random variable which follows normal distribution with mean $\mu_a = \sigma^2_n$ and variance $\sigma^2_n = \frac{\sigma^4_n}{KN}$ in the case of absence of PU. Here $\sigma^2_n$ is the variance of noise voltage of zero average. It can be easily shown that the probability of false alarm is

$$P_{FA} = Q\left(\frac{\tau}{\sigma^2_n} - 1\right) \sqrt{KN},$$  \hspace{1cm} (5)

where $\tau$ is the threshold of detection. If it is found that $E_{av} > \tau$, then the SU decides that hypothesis $H_1$ is true and the channel is being used by the PU. Consequently, if the opposite is true that is if $E_{av} < \tau$, then the SU decides that the frequency band of interest is not in use by the PU, and therefore, the SU can take this opportunity to transmit data.

In the case of presence of the PU:

$$\mu_p = \sigma^2_n + S_r,$$  \hspace{1cm} (6)

where $S_r$ is the primary signal power received by the SU which can be shown to be $S_r = \frac{(\sigma^2_n)^2}{KN}$. After substituting this value of $S_r$, the expression for $\mu_p$ becomes

$$\mu_p = \sigma^2_n + \frac{\sigma^2_n}{K} \sum_{k} h_k^2,$$  \hspace{1cm} (7)

where $\sigma^2_n$ is the variance of the PU’s signal $s[n]$ and $h_k$ is the channel gain of the $k^{th}$ received antenna. The expression for the variance in the case of presence of the PU is

$$\sigma^2_p = \frac{1}{KN} (S_r + \sigma^2_n)^2 = \frac{S_r^2 + 2S_r \sigma^2_n + \sigma^4_n}{KN} = \frac{\sigma^4_n}{KN} + \frac{2\sigma^2_n}{K^2 N} \sum_{k} h_k^2 + \frac{\sigma^4_n}{K^3 N} \left(\sum_{k} h_k^2\right)^2$$  \hspace{1cm} (8)

Under this condition, the probability of detection is shown to be

$$P_D = Q\left(\frac{\tau}{\sigma^2_n} - 1 - \frac{\gamma}{K}\right),$$  \hspace{1cm} (9)

where $\gamma = \left(\sigma^2_s / \sigma^2_n\right) \sum_{k} h_k^2$ is a random variable which represents the instantaneous SNR.

Probability of detecting or sensing signal on the carrier frequency $f_c$, where a PU is truly present, is called the probability of detection, $P_D$, as given by Eq. (9). If the SU detects the presence of a PU on its test statistics at the carrier frequency $f_c$ of a PU but actually there is no PU in the transmission mode - this is known as the probability of false alarm or misdetection.

In MRC, the instantaneous SNR is given by the following expression

$$\gamma_{MRC} = \sum_{i=1}^{K} \gamma_i = P \frac{1}{N_0} \sum_{i=1}^{K} h_i^2,$$  \hspace{1cm} (10)

where $P$ is the transmit power constraint and $N_0$ is the noise power spectral density.

For the case of the Nakagami ($m, \Omega$) fading channel, the pdf of the instantaneous SNR $\gamma_{MRC}$ is expressed as [9]

$$f_{\gamma_{MRC}}(x) = \frac{\Omega}{\Gamma(Km)} x^{Km-1} e^{-\frac{\Omega x}{\Gamma(Km)}} u(x),$$  \hspace{1cm} (11)
where $x$ is a random variable which represents the SNR of the received signal and $u(x)$ is the unit step function. The cdf of $\gamma_{\text{MRC}}$ is given by the following expression

$$F_{\gamma_{\text{MRC}}} = \Pr \left\{ \gamma_{\text{MRC}} \leq \frac{R}{N_0} \right\}, \quad (12)$$

which is the outage probability, $P_{\text{out}}$, of the SU [13]. Here $R$ is the power constraint of interference. By using the expression for $\gamma_{\text{MRC}}$ from Eq. (10), Eq. (12) becomes

$$P_{\text{out}} = F_{\gamma_{\text{MRC}}} = \Pr \left\{ P \sum_{i=1}^{K} h_i^2 \leq \frac{R}{N_0} \right\} = \Pr \left\{ \sum_{i=1}^{K} h_i^2 \leq \frac{R N_0}{PN_0} \right\}. \quad (13)$$

After simplification, the above expression for $P_{\text{out}}$ becomes

$$P_{\text{out}} = \Pr \left\{ \sum_{i=1}^{K} h_i^2 \leq \frac{\text{INR}}{\text{SNR}} \right\}, \quad (14)$$

where $\text{INR}$ represents the interference-to-noise ratio. Equation (14) can be further simplified, and finally becomes

$$P_{\text{out}} = 1 - \frac{\Gamma \left( Km, \frac{\text{INR} / \text{SNR}}{\Omega} \right)}{\Gamma(Km)}, \quad (15)$$

where $\Omega$ is the mean value of random variable $h_i^2$.

The normalized channel capacity of the wireless link can be written as

$$C = \int_0^\infty \frac{1}{1+z} \frac{1 - F_{\gamma_{\text{MRC}}}(z)}{dz} = \int_0^\infty \frac{1}{1+z} \frac{1}{\Gamma(Km)} \Gamma \left( Km, \frac{z}{\Omega} \right) dz, \quad (16)$$

where $F_{\gamma_{\text{MRC}}}(z)$ is the cdf of the Nakagami $(m, \Omega)$ fading channel. The variable $z$ in (16) is the instantaneous SNR of the received signal. According to Eq. (14) $z$ may be replaced by $\text{INR}/\text{SNR}$ as well.

After obtaining the expression for $F_{\gamma_{\text{MRC}}} = P_{\text{out}}$ from Eq. (15), the probability of symbol error rate (SER) in the generalized form can be calculated by the following standard formula [13]:

$$P_s = \mathbb{E} \left[ a Q \left( \sqrt{2b} \gamma_{\text{MRC}} \right) \right] = \frac{a}{2} \sqrt{\frac{b}{\pi}} \int_0^\infty e^{-bz} F_{\gamma_{\text{MRC}}}(z) dz, \quad (17)$$

where the parameters $a$ and $b$ depend on the particular modulation schemes used. It is to be noted here that Eq. (17) will have to be evaluated numerically to calculate the SER for the model presented in this paper.

### III. Results

Fig. 1 shows the variation of the outage probability against the SNR taking the number of received antennas as a parameter. The parameters considered for the profile of the outage probability are: $m=2$, $\Omega=0.3$, $\text{INR}=1.5$ and $N=50$. The outage probability decreases with the increase in both the SNR and the number of receiving antennas, $K$. The separation between any two curves is found to be wider at higher SNR. The variation of channel capacity/Hz against SNR is shown in Fig. 2. It is seen that the channel capacity increases with both the increase in SNR and $K$. Again the separation between any two curves is found to be little wider at higher SNR i.e. the impact of number of antennas is more prominent at higher SNR. The profile of symbol error rate (SER) is shown in Fig. 3 against the same parameters. Here SER is determined for QPSK scheme ($a=2$, $b=0.5$); changing the parameters $a$ and $b$ of Eq. (16), the SER for other modulation schemes like 8-PSK and 16-QAM can also be determined; which are suitable for wireless link with proper throughput and SER. It is visualized from Figs. 1-3 that the performance of the network increases nonlinearly with the increment of the number of antennas at the receiving end.
from Fig. 4 that the probability of false alarm decreases
different values of threshold of detection
number of received antennas is shown in Fig. 4 for three

![Fig. 4. Variation of probability of false alarm against the number of antenna](image)

It is observed that there is significant reduction in the outage capacity and enhancement in the channel capacity with increasing in the number of antennas in the SU receiver. There is a nonlinear rise in system performance that can be attributed to the deployment of the MRC diversity. Moreover, it is observed that the value of the threshold of detection \( \tau \) needs to be adjusted to optimize the probabilities of detection and false alarm for maximum possible system performance enhancement. Still there is a scope of using matched filter, adaptive equalizer, zero-forcing equalizer and other combining schemes to combat the sensing of false alarm and the detection of PU precisely.

![Fig. 3. Variation of SER of QPSK against SNR.](image)

![Fig. 5. Variation of probability of detection against SNR for different thresholds.](image)

The profile of the probability of false alarm against the number of received antennas is shown in Fig. 4 for three different values of threshold of detection \( \tau \). It is observed from Fig. 4 that the probability of false alarm decreases rapidly with the increase in both \( K \) and \( \tau \). Figure 5 Shows the variation of the probability of detection, \( P_D \) against SNR for three different values of \( \tau \). It is easily visualized from Fig. 5 that \( P_D \) attains the value 1 for lower values of \( \tau \) which apparently enhances the performance of the network. However, by lowering the value of \( \tau \) the probability of false alarm increases as is visualized from Fig. 4. Therefore, a tradeoff is necessary between Fig. 4 and Fig. 5.

IV. CONCLUSION

This paper proposes a new design scheme in which MRC diversity is deployed in the SU receiver to facilitate efficient spectrum sensing and consequently lead to enhanced system performance. It is observed that there is significant reduction in the outage capacity and enhancement in the channel capacity with increasing in the number of antennas in the SU receiver. There is a nonlinear rise in system performance that can be attributed to the deployment of the MRC diversity. Moreover, it is observed that the value of the threshold of detection \( \tau \) needs to be adjusted to optimize the probabilities of detection and false alarm for maximum possible system performance enhancement. Still there is a scope of using matched filter, adaptive equalizer, zero-forcing equalizer and other combining schemes to combat the sensing of false alarm and the detection of PU precisely.

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