Multi-rate Ripple-Free Deadbeat Control for Nonlinear Systems Using Diophantine Equations

Hatem Elaydi and Mohammed Elamassie

Abstract—**Despite previous efforts to solve linear and nonlinear deadbeat control systems, a need still exists for better methodology in terms of performance and stability. This paper proposes a new design methodology for deadbeat control of nonlinear systems in discrete-time. The proposed methodology is based on partitioning the solution into two components; each with different sampling time. The proposed control can be divided into two sub-controllers: one uses state feedback and the other uses the Diophantine equations. The complete nonlinear design guarantees the convergence to a neighborhood of origin from any initial state in finite time; thus, providing a stable deadbeat performance. Results shows that the ripple-free deadbeat controller is able to track the input signal and the error decays to zero in a finite number of sampling times.**

Index Terms—**Deadbeat control, diophantine equations, multi-rate, output-feedback linearization.**

I. Introduction

Digital deadbeat controller offers the fastest settling time in control theory. Thus, deadbeat controller ensures that the error sequence vanishes at the sampling instants after a finite time [1, 2]. Due to the nonlinearity nature of plants and processes, the deadbeat control technique must be improved in order to overcome the nonlinearity and discretization. All digital deadbeat techniques for linear systems have a common property: all poles of the closed-loop transfer function should are moved to the origin of the z-plane either by using state-feedback [3-4], Diophantine equations design methods [5, 6], or any other technique.

Paz [7] proposed a two-degree-of freedom controller design for a well-known transfer function addressing performance and robustness specifications for linear systems. The controller is given in terms of the solution of two Diophantine equations. Shifting closed loop poles of nonlinear system to the origin may not be acceptable, thus; using full state feedback to deadbeat nonlinear system is not a good technique.

Salgado and Oyarzun [8] proposed two objective optimal multivariate ripple free deadbeat controls with simple parameterization. The designed controller dealt with step input for linear system. Yamada [9] proposed a parameterization of all multivariable ripple-free deadbeat tracking controller that handled various input signals for linear systems. Elaydi and Albatsh [10] and Albatsh [11] solved the multirate ripple-free deadbeat control for linear systems using the Diophantine equations.

In this paper, multi-rate deadbeat control for nonlinear system is proposed based on evaluating the solution of the two independent Diophantine equations for second order approximated model of a linearized nonlinear system. Nonlinear system will be linearized using full state-feedback linearization. This paper will show simulation results of the designed controller on the nonlinear plant.

This paper is organized as follow: section 2 talks about material and methods where it states the problem formulation and talks about state feedback and Diophantine equations control designs, section 3 covers results and discussion by stating the constraints and design steps and solving examples to show the effectiveness of the proposed method, section 4 concludes this paper.

II. Materials and Methods

A. Problem Formulation

Controlling a nonlinear system in a ripple-free deadbeat manner is quite challenging. Typical procedures that are normally followed in linear system are no longer valid here. The problem here is the nonlinearity region and the robustness of the controller around this region. The ripple-free deadbeat controller for nonlinear system, shown in Fig. 1, consists of the following two design steps: First step is concerned with time-domain approach such as state and output feedbacks with integral controller that is used to linearize and stabilize nonlinear system with sampling time T1 to make the response of nonlinear system closely equal the reference signal.

The second step is concerned with polynomial approach namely the Diophantine equations design methods based on the internal model principle are utilized and applied to the linearized and stabilized nonlinear system with sampling time T2 to make the response of the system exactly equal the reference signal and provide some robustness.

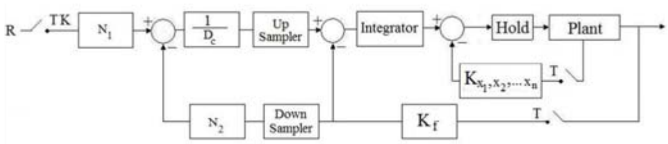

Fig. 1. Multi-rate ripple-free deadbeat controller for nonlinear system

The designed controller is based on using different sampling times in order to ensure that we can use different sampling times in two sub controllers (1-full-state feedback, and 2- Diophantine equations) and to decrease the processing time by decreasing sampling rate of one sub-controller if we can.

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B. Full State Feedback and Diophantine Equations

Feedback linearization shown in Fig. 2 is a popular approach to linearize nonlinear systems. Therefore, linear control techniques can be applied.

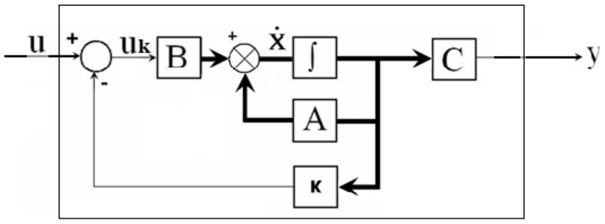

Fig. 2. Block diagram representation of system with feedback linearization

A feedback path from the output is added to form the error, e, which is fed forward to the controlled plant via an integrator as shown in Fig. 3. The integrator increases the system type and reduces the error.

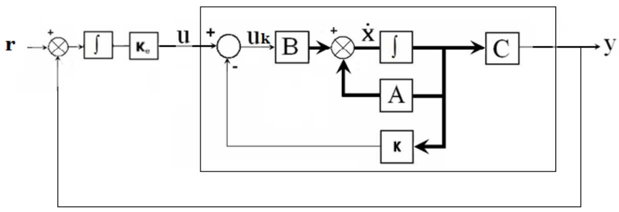

Fig. 3. Plant with state and output feedback with integral control

Second order approximation for linearized model shown in Fig. 3 is evaluated as shown in Fig. 4 using two-parameters: rising time, t_r, (or settling time, t_s) and overshoot of a step response in order to evaluate the deadbeat controller for linearized model with another sampling rate.

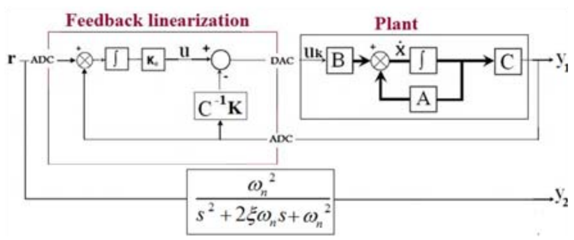

Fig. 4. Second order approximation of maglev with feedback linearization

These metrics are modeled using ω_n and ζ and should be tuned to make the output of approximated model exactly equal the output of the linearized model. A second order approximation will be used to apply Diophantine equations for well known transfer function and to decrease the length of the three polynomials that obtained from minimum order solution of Diophantine equations.

The three polynomials obtained from the two Diophantine equations and feedback linearization is applied to the nonlinear plant as shown in Fig. 5.

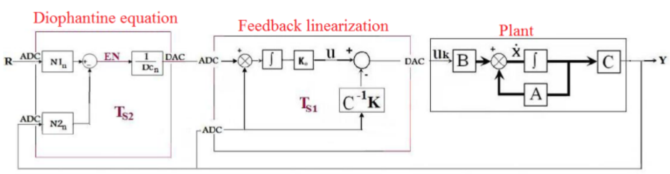

Fig. 5. Deadbeat controller for nonlinear system

The three polynomials N1, N2, and Dc are tuned when applied to nonlinear system until the response exactly equals the reference signal. Saturation may be used to ensure that the nonlinear system works in stable region, region of attraction.

III. Results and Discussion

A. Designing Steps

The following steps are used to evaluate multi-rate deadbeat controller for nonlinear systems:

1) Deriving the input/output relation of nonlinear plant's sub-models (i.e. D/A converter, Power amplifier, ball & coil subsystem, Position sensor, A/D converter).
2) Evaluating state space of nonlinear plant after linearization around operating point.
3) Applying Controllability and Observability tests to check if the following steps can be done.
4) Evaluating feedback linearization with sampling time T1 using MATLAB built-in function "place", and applying it to nonlinear Plant.
5) Evaluating second order approximation to the step response of the plant with feedback linearization.
6) Applying Diophantine equation on approximated model
7) Applying deadbeat controller which consists of two sub-controllers (Full-state feedback and Diophantine equation) on the nonlinear Plant.

B. Constraints

The following assumptions are necessary in order to produce ripple-free deadbeat controller that is able to track the reference signal and has a zero error signal in finite number of steps [11]:

1) The nonlinear system is controllable and observable. Possibility of forcing the system into a particular state by using an appropriate control signal is required; thus, system should be controllable. Possibility of reading all state variables is required in order to apply feedback linearization; thus, system should be observable.
2) Denominator of the reference signal and the numerator of plant are co-prime in discrete-time. Possibility of tracking reference signal requires no common factor between denominator of reference signal and numerator of plant to ensure that, there is no poles zeros cancellation; thus, denominator of the reference signal and the numerator of plant should be co-prime.
3) There is no sinusoidal term in the reference signal with frequency that coincides with an integer multiple of the Nyquist frequency. Possibility of reconstruct the original continuous signal is required to compare between sensed and reference signals; thus, reference signal must not have frequency that coincides with an integer multiple of the Nyquist frequency.

C. Example

The nonlinear system is represented by the magnetic ball levitation, CE152, and its diagram shown in Fig. 6 and Fig. 7, is used as a case study of unstable system.

Fig. 6. CE152 magnetic ball levitation

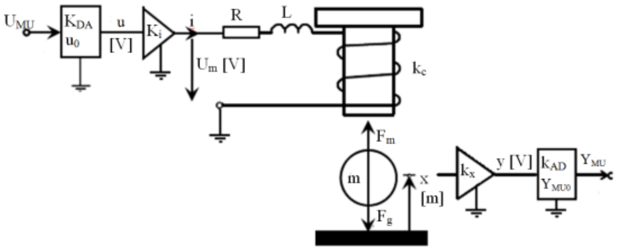

Fig. 7. Principal scheme of the magnetic levitation model

The CE152 model consists of the following sub models:

- D/A converter.

$$U = U_{MU} * K_{DA} + U_0 \tag{1}$$

- Power amplifier.

$$\frac{I}{U} = K_i \frac{1}{T_a s+1} \tag{2}$$

- Ball & coil subsystem.

$$m_k \ddot{x} + k_{fv} \dot{x} = \frac{i^2 k_c}{(x-x_0)^2} - m_k g \tag{3}$$

- Position sensor.

$$Y = k_x x + Y_0 \tag{4}$$

- A/D converter.

$$Y_{MU} = K_{AD} Y + Y_{MU0} \tag{5}$$

where: U_{MU} is the D/A converter input; K_{DA} is the digital to analog converter gain; $U_0$the D/A converter offset; K_i is power amplifier gain; T_a is time constant; F_g is gravity force; F_m is electromagnetic force; F_a is the acceleration force; I is the coil current; k_c is the coil constant; x_0 is the position offset; K_{fv} is the damping constant; x is the ball position; K_x is the position sensor gain; Y_0 is the position sensor offset; Y_{MU} is the model output voltage; Y is the A/D converter input; K_{AD} is the A/D converter gain; and Y_{MU0} is the A/D converter offset.

Fig. 8 shows the final block diagram of the magnetic levitation model in SIMULINK model.

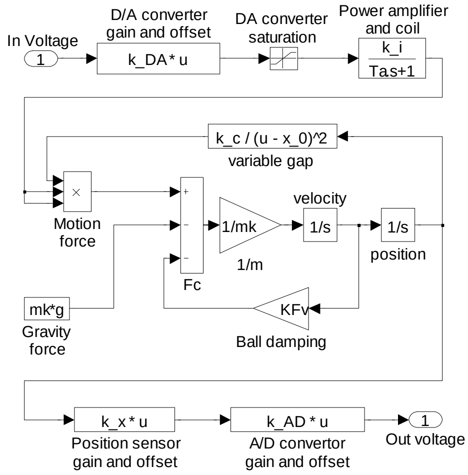

Fig. 8. The complete model of magnetic ball levitation CE152

C. State Space Model

Equations (1), (2), (3), (4), and (5) are used to evaluate the state space model of magnetic ball levitation around an operating point. Taylor series expansion will be used to linearize nonlinear terms in equation (3) around operating point as follow:

$$f(x,i) \approx f(a,b) + \left(\frac{\partial(f(a,b))}{\partial(a)*1!} x(t) + \frac{\partial(f(a,b))}{\partial(b)*1!} i(t) \right) \tag{6}$$

$$\therefore k_c \left[\frac{i}{(x-x_0)} \right]^2 = \frac{b^2 k_c}{(a-x_0)^2} + \left[\frac{-2*k_c*b^2}{(a-x_0)^3} \right] x(t) + \left[\frac{2*k_c*b}{(a-x_0)^2} \right] i(t)$$

The state space model is as follow:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-2*k_c*b^2}{m_k(a-x_0)^3} & \frac{-k_{fv}}{m_k} & \frac{2*k_c*b}{m_k(a-x_0)^2} \\ 0 & 0 & -\frac{1}{T_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{K_i K_{DA}}{T_a} \end{bmatrix} u(t) \tag{7}$$

$$y = \begin{bmatrix} K_x K_{AD} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{8}$$

Evaluating ball position and coil current around midpoint as shown in Fig. 9. (7(

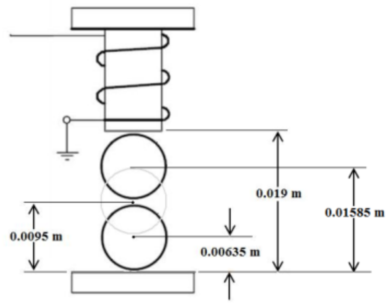

Fig. 9. Position of ball at midpoint a=0.0095 m , b=0.13568 A

Table I shows the magnetic ball parameters and their values that were used to develop the state model in the next equations.

TABLE I: THE PARAMETERS OF MAGNETIC BALL LEVITATION

value	Parameter	Symbol
12.7e-3 m	ball diameter	Dk
0.0084 kg	ball mass	mk
0.019 m	distance from the ground and the edge of the magnetic coil	Td
0.0063 m	distance of limits= 0.019 - Dk	L
9.81 m.s^-2	gravity acceleration constant	g
5 V	maximum DA converter output voltage	U_DAm
3.5 Ω	coil resistance	Rc
$30e^{-3}$ H	coil inductance	Lc
0.25 Ω	current sensor resistance	Rs
13.33	current sensor gain	Ks
100	power amplifier gain	K_am
1.2 A	maximum power amplifier output current	I_am
1.8694e-005 s	amplifier time constant= Lc/((Rc+Rs)+Rs*Ks*K_am)	Ta
0.2967	amplifier gain= K_am / ((Rc+Rs)+Rs*Ks*K_am)	k_i
0.02 N.s/m	viscose friction	KFv
10	converter gain	k_DA
0 V	Digital to Analog converter offset	u_0
0.2	Analog to Digital converter gain	k_AD
0 V	Analog to Digital converter offset	y_MU0
797.4603	position sensor constant	k_x
8.26e-3 m	coil bias	$x_{\_0}$
0.606e-6 N/V	Aggregated coil constant	k_f
6.8823e-6 N/V	coil constant =k_f/(k_i)^2	k_c

The state space model of CE152 around mid point is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -15821.62 & -2.381 & 144.596 \\ 0 & 0 & -53493.1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 79357.013 \end{bmatrix} U_{MU}(t) \quad (8)$$

$$Y_{MU} = \begin{bmatrix} 159.49206 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (10)$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -15821.62 & -2.381 & 144.596 \\ 0 & 0 & -53493.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 79357.013 \end{bmatrix} \quad (11)$$

$$C = [\ 159.49206 \quad 0 \quad 0\ ], \quad D = [0] \quad (12)$$

D. The Controllability and Observability Tests

In order to satisfy assumption (1) we need to check the controllability of the system by obtaining the controllability matrix such as:

$$Q_c = [B\ AB\ A^2B]$$

$$\therefore\ Q_c = \begin{bmatrix} 0 & 0 & 1.1476e7 \\ 0 & 1.1476e7 & -6.139e11 \\ 7.9357e4 & -4.2454e9 & 2.270953e14 \end{bmatrix} \quad (13)$$

since Q_c has full rank, then system is controllable

In order to satisfy assumption (1) we need to check the observability of the system by obtaining the observability matrix such as:

$$O_\upsilon = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} \quad (14)$$

$$O_\upsilon = \begin{bmatrix} 159.49206 & 0 & 0 \\ 0 & 159.49206 & 0 \\ -2523422.76 & -379.750594860 & 23061.913907760 \end{bmatrix}$$

since Q_c has full rank, then system is observable.

Transfer function of the liberalized model around mid point.

$$G(s) = \frac{(1.83e9)}{s^3 + (5.349e4\ s^2) + (1.432e5\ s) + (8.464e8)} = \frac{N(s)}{D(s)}$$

E. State and Output Feedback Linearization

The MATLAB function "place" is used to evaluate state feedback gains as follows:

Position 'vertical displacement', and coil current
Position's feedback Gain = 350.8825
Velocity's feedback Gain = 3.18984
Coil current's feedback Gain= -0.2475

Applying position sensor constant "Kx" , analog to digital converter gain "KAD" and sampling time ="0.001 sec" then:
Position's feedback Gain = 2.2
Velocity's feedback Gain = 20
Coil current's feedback Gain= -0.0016

Applying integral controller, then evaluating integral gain and converting integrator from analog to digital using first order hold gives: Integrator Gain = 25.

The magnetic ball levitation with full state feedback and integral controller is shown in Fig. 10 and its step response is shown in Fig. 11.

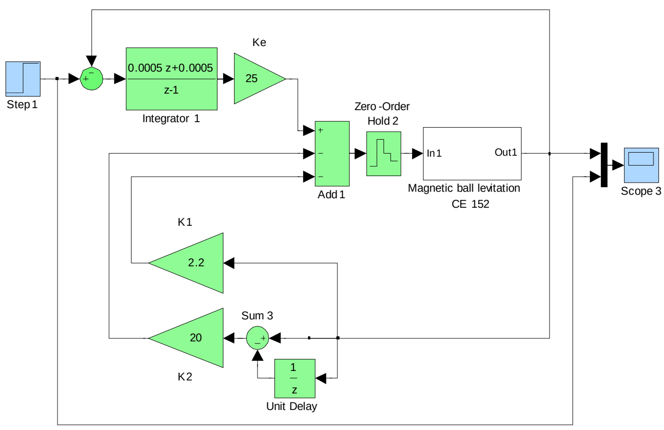

Fig. 10. Magnetic ball levitation with full state feedback and integral controller

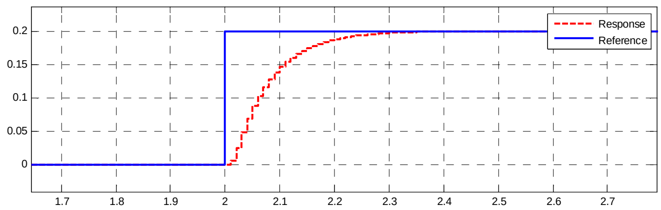

Fig. 11. Step response of magnetic ball levitation with feedback linearization

The step response shouldn't have any overshoot, if not redesign the full state feedback with integral controller.

The second order approximation of CE152 with full-state feedback second order specifications is shown in Fig. 12 and its step response is shown in Fig. 13.

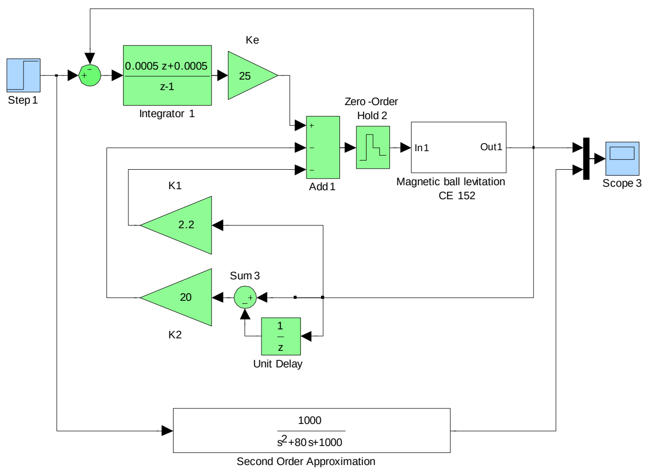

Fig. 12. Second order approximation of linearized maglev CE152

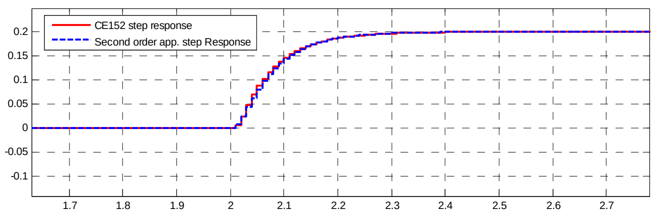

Fig. 13. Step response of linearized maglev and approximated model

Converting approximated model into digital with sampling time = 0.01 sec and applying Diophantine equations to evaluate the minimum order solution.

F. The Diophantine Equations Design Approach

The Plant in q-domain where q represent a delay of one sampling time:

$$\frac{1000}{s^2+80s+1000} \Rightarrow \frac{0.038652\ q\ (1+0.7662q)}{(1\text{-}0.8564q)\ (1\text{-}0.5247q)}$$

$$N_P = 0.038652*q+0.029616*q^2$$

$$D_p = 1\ \text{-}1.381*q + 0.44932896*q^2$$

The reference signal in q-domain:

$$\frac{6}{(s^2+4)} \Rightarrow \frac{0.0003q(1+q)}{(1-2q+q^2)}$$

$$N_R = 0.0003*q + 0.0003*q^2$$

$$D_R = 1\ \text{-}2*q + q^2$$

Minimum order of N1 and Q1

Minimum order of N1 and Q1= max(order(Np), order(Dr))-1=max(2,2)-1 =0

Let N1= a*q+b and Q1= c*q+d where a,b,c, and d are unknowns

FDE:

(0.02961*a+c)*q3+(0.02961*b+0.0386522*a+d-2*c)*q2+(c+0.0386522*b-2*d)*q+d=1

Then: N1 = 35.64 – 21*q and Q1= 1 + 0.622*q

Minimum order of N2 and Dc

Minimum order of N2 and Dc=max(order(Np), order(DP))-1=max(2,2)-1 = 1

Let N2= a*q+b and Dc= c*q+d where a,b,c, and d are unknowns

SDE:

(0.02961*a+0.44933*c)*q3+(0.02961*b+0.038652*a+0.44933*d-1.38106*c)*q2+(c+0.0386522*b-1.38106*d)*q+

Then: N2 = 21.46 – 8.367*q and Dc= 1 + 0.5515*q

The full-state feedback and the Diophantine equations design approach with the three polynomials N1, N2, and Dc are applied to the second order approximation system shown in Fig. 12 and to the magnetic ball levitation CE152 shown in Fig. 14 and its detailed model as shown in Fig. 15.

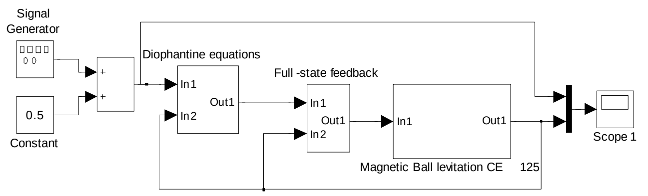

Fig. 14. Multi-rate ripple-free deadbeat controller for magnetic ball levitation

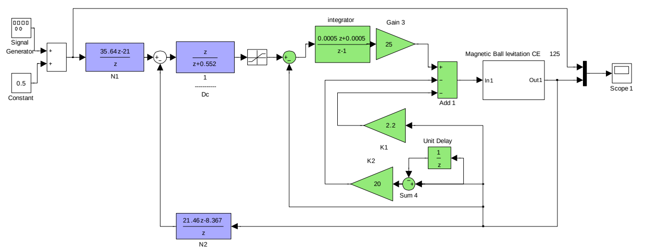
Fig. 15. Multi-rate ripple-free deadbeat controller for magnetic ball levitation (detailed diagram)

Fig. 16 shows the step response of the CE152 with full state feedback and Diophantine equations approach.

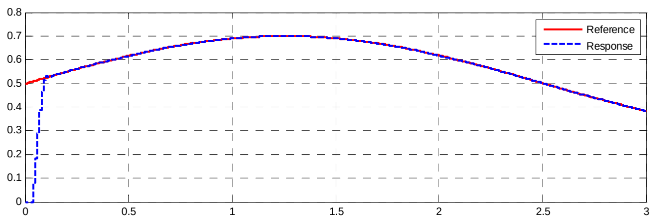

Fig. 16. Response of multi-rate ripple-free deadbeat controller for CE152

Fig. 16 shows that the response of magnetic ball levitation with feedback linearization and with deadbeat controller. As shown, the response (dashed line) is able to track and follows the reference signal (soled line) after small finite time with steady state error equal zero. This validates the design approach and shows that ripple-free deadbeat control was achieved on a nonlinear unstable system such as the magnetic ball levitation.

IV. Conclusions

Ripple-free deadbeat control is a challenging task in nonlinear systems due to the nonlinearity nature in them. This paper proposed a new approach to design a ripple-free deadbeat controller for unstable nonlinear system. The magnetic ball levitation was selected as the unstable nonlinear system. The proposed approach consisted of two steps: obtaining a linearized model by using state feedback linearization; then, applying the Diophantine equations design approach. Solving the Diophantine equations provides degrees of freedom in the design and the degrees of the three polynomials N1, N2, and Dc decided the settling time. The degrees of numerator and denominator of plant, and denominator of reference signal played a big role in deciding the degrees of N!, N2, and Dc. The results showed that the proposed approach was able to produce a ripple-free deadbeat controller that was able to track a step input reference signal in finite number of time with zero steady state error.

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