A Fuzzy System for VSI X-Bar Control Chart

Jing-Er Chiu and Bing-Tai Chen

***Abstract*—Statistical quality control charts are widely used in monitoring production process. In this study, a fuzzy control chart with variable sampling interval (VSI) is proposed by combining Fuzzy set theory and VSI method to improve the efficiency of detecting a signal. A fuzzy control chart is constructed using the output value of fuzzy logic approach for process monitoring. The VSI method is then applied for the fuzzy control chart to reduce the time for detecting the out of control signal. The simulation results show that the performance of the proposed fuzzy control chart with VSI in detecting the shift of process average, variation and trend is better than that of the traditional $\overline{X}$ control chart or VSI $\overline{X}$ control chart.**

***Index Terms*—Fuzzy set theory, variable sampling interval, control chart.**

I. Introduction

Statistical Quality Control (SPC) is widely employed in industrial environments. The most popular one for monitoring the process mean is $\overline{X}$ control chart, however, it is not sensitive in detecting small shift. Cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts are quicker at detecting small shifts since they use information from past observations.

Many techniques are investigated to improve the performance of $\overline{X}$ control chart. A useful approach improving the ability to detect process changes is to use an adaptive control chart which allowed changing at least one of the three parameters: the sampling interval, the sample size, and the control limit coefficient in the design and operation of control chart. Reynolds *et al*. (1988) proposed variable sampling interval (VSI) $\overline{X}$ chart. Their work has been the inspiration for several other researchers. The properties of variable sample size (VSS) $\overline{X}$ chart were studied by Prabhu, Runger, and Keats (1993) and Costa (1994). Prabhu, Montgomery , and Runger (1994) and Costa (1997) investigated the properties of $\overline{X}$ chart with both variable sample size and sampling interval variable (VSSI). Costa (1999) proposed variable parameter (VP) $\overline{X}$ chart which allowed all three parameters variable.

Fuzzy logic has been successfully used in varied areas of industrial application. A number of approaches to the employment of fuzzy logic for control charts have been investigated. Raz and Wang(1990) proposed two approaches based on fuzzy set theory in construction of control chart for linguistic data. They showed that the control chart for linguistic data performed better than conventional *p* chart. Rowlands and Wang (2000) designed a fuzzy-SPC evaluation and control method which applied the fuzzy set theory to the SPC zone rules. The fuzzy-SPC evaluation and control method can improve the accuracy and consistency of interpretation of the data, where the numeric output from the fuzzy system indicates what specific action should be taken if the process is out of control. Tannock (2003) constructed a fuzzy control chart by using two fuzzy sets: the Centred set and the Random set, to examine three unnatural patterns (shift, trend, and cyclical) for individual X chart.

When using $\overline{X}$ control chart to monitor the process, user can only identify whether or not the process is out of control. The aim of this study is to construct a new fuzzy control chart using three fuzzy sets to transfer the binary concept (in-control/ out-of-control) to a numerical value on the level of process quality. Further, variable sampling interval (VSI) technique was applied to this fuzzy control chart for fast detecting small shift.

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The authors are with the Department of Industrial Engineering and Management, National Yunlin University of Science and Technology (e-mail: chiuje@yuntech.edu.tw).

II. Methodology

A. Fuzzy System and Design

The proposed fuzzy control chart utilizes central tendency, variation, and unnatural patterns of the sampled data as the inputs to determine the level of process quality based on fuzzy theory. The sample mean is applied as the measure of central tendency. The second input is the sample variance, which indicates how the data deviates from the sample mean. Run rules are used to identify the unnatural patterns of the process. To construct the fuzzy control chart, three fuzzy sets, corresponding membership functions, and fuzzy rules must be built.

1) Fuzzy sets and membership functions

a) Centred Fuzzy Set

To realize the proposed fuzzy control chart, the data is normalized so that the mean value is zero and the variance turns to unity. The triangle in this figure denotes the fuzzy set with the boundary of [-3, 3], referring to typical three-sigma control chart. A sample mean close to zero indicates a good central tendency and a larger absolute value of sample mean represents a poor central tendency. If the absolute value of the sample mean is greater than 3, the membership degree is zero.

b) Variation Fuzzy Set

The approach proposed by Tannock (2003) is designed for individual X chart. This method cannot detect the changes in the level of within-sample variation. In order to improve that, the membership function of the variation is defined. Small sample variance indicates a high degree of membership in the set of sample. If a sample variance is greater than 4, the membership degree is zero.

c) Random Fuzzy Set

If there existence of four increasing/decreasing consecutive points or less, the membership degree is one. If there are more than nice increasing/decreasing consecutive points, the membership degree is zero.

2) Output membership function

The output membership function is to describe the stability of the control process using eight linguistic variables: VH (very high), H (high), LH (less high), HM (high medium), LM (low medium), LL (less low), L (low), and VL (very low).

3) Fuzzy rules

Conventional approaches of control charts apply only mean value to evaluate the process stability. In addition to the mean value, we further consider the variation tendency and continuity tendency as auxiliaries for evaluating the stability. The fuzzy rules are built as follows:

- If Center Td. is Good, Var. Td. is Low, and Con. Td. is Low, then stability is VH.
- If Center Td. is Good, Var. Td. is Low, and Con. Td. is High, then stability is H.
- If Center Td. is Good, Var. Td. is High, and Con. Td. is Low, then stability is LH.
- If Center Td. is Good, Var. Td. is High, and Con. Td. is High, then stability is HM.
- If Center Td. is Poor, Var. Td. is Low, and Con. Td. is Low, then stability is LM.
- If Center Td. is Poor, Var. Td. is Low, and Con. Td. is High, then stability is LL.
- If Center Td. is Poor, Var. Td. is High, and Con. Td. is Low, then stability is L.
- If Center Td. is Poor, Var. Td. is High, and Con. Td. is High, then stability is VL.

4) Fuzzify and defuzzify

The fuzzy control chart starts with fuzzifying three inputs based on the membership functions. Then check the fuzzy rules triggered by the fuzzified inputs. The triggered rules are then combined to obtain an output value, or called defuzzification. This value can depict how stable the process is.

Among several methods of defuzzification, center of gravity is one of the most popular approaches and is employed in this study to combine the triggered rules. This approach calculates the position of gravity of the area that covers the triggered fuzzy sets associating membership degrees.

B. Variable Sampling Interval (VSI) $\overline{X}$ Control Chart

VSI $\overline{X}$ Control Chart

Reynolds *et al*. (1988) proposed the variable sampling interval (VSI) $\overline{X}$ chart. Instead of the standard fixed sampling interval, if a sample point falls close to a control limit (waning region), there might be some indication of a change in the process and the time interval until the next sample should be short (d_1) for quickly detecting the change. If the current sample point is close to the central line (middle region) so that there is no indication of signal, a long sampling interval (d_2) could be taken for saving sampling cost. The results of Reynolds' study suggest that d1 should be taken as small as is practical, whereas d_2 should be large if interest is in small shift and small if interest is in large shift. For the most application, they recommended d1 to be 0.1 of the fix sampling interval and d_2 to be in the range between 1 and 2 of the fix sampling interval. In this research, we will use $d_1 = 0.1$ and $d_2 = 1.9$ to reduce the time in detecting small and moderate shifts in the process mean.

1) Evaluation of VSI control chart

Average run length (ARL), the average number of samples taken before the chart signals, is often used to determine the performance of a fixed sampling control chart. The ARL should be large when the process is in control and the ARL should be small when the processes mean shifts. The ability of a VSI control chart to detect the process changes is measured by the average time to signal (ATS), the expected time until the chart signals. Similarly, when the mean is on the target the ATS should be large and when the mean shifts the ATS should be small. In our study, we use ATS as the evaluation of the proposed VSI fuzzy control chart.

III. Simulation Study

A. Data Generation

In order to evaluate the performance of the proposed VSI fuzzy control chart, simulation study was conducted and the VSI fuzzy control chart was compared with the $\overline{X}$ control chart, VSI $\overline{X}$ control chart, and fuzzy control chart. We simulated the quality characteristic, *Y*, having the following properties:

1) *Y* follows a normal distribution with mean μ and variance σ^2. When an assignable cause is shown in the process, one type of shifts in mean, variance and trend pattern appears at one period of time.
2) Assuming the process mean is μ_0 in control status. When an assignable cause by mean shift, the process mean changes from μ_0 to $\mu_0 \pm \delta_1 \sigma_{\bar{x}}$, where δ_1 is the shift parameter of mean.
3) When an assignable cause is shown, the process variance shifts from $\sigma_{\bar{x}}$ shift to $\delta_2 \sigma_{\bar{x}}$, where δ_2^2 is the shift parameter in variance.
4) When an assignable cause is produced by an unusual pattern, the mean shifts from μ_0 to $\mu_0 + \delta_3 t$, where *t* represents the time and δ_3 is the shift parameter of trend.

After generating the quality characteristic, *Y*, *Y* is standardized to standard normal for construction of VSI fuzzy control chart. According to the results of Reynolds *et al*. (1988), the optimal short and long sampling intervals are set to be 0.1 and 1.9 respectively.

The control limits and warning limits of VSI fuzzy control chart were obtained by Monte Carlo method in generating 100,000 data points from standard normal distribution under the condition that the process is in control status. The control limits and warning limits are sown in Table I.

TABLE I: Comparisons of Control Limits

	$\overline{X}$ control chart	Fuzzy control chart
Warning limits	$\pm$ 0.6723	0.7410
Control limits	$\pm$ 3	0.4444

B. The Construction of VSI Fuzzy Control Chart

For illustration of VSI fuzzy control chart, the proposed VSI fuzzy control was constructed for a data set shown in Fig. 1 where d1 and d2 are short and long sampling intervals respectively.

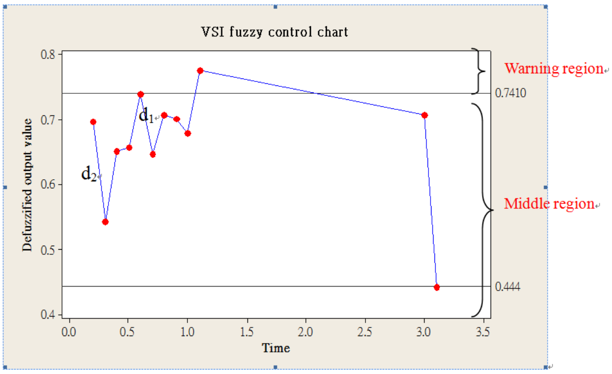

Fig. 1. Illustration of VSI fuzzy control chart.

From Fig. 1, if the output statistic is greater than 0.7410 (i.e. in the middle range), there is no indication of signal and the long sampling interval of 1.9 (d2) was taken for the next time. If the output statistic is between 0.4444 and 0.7410, there is some indication of a change in the process and the short sampling interval of 0.1 (d1) was taken for quickly detecting the change

IV. RESULTS

The simulation results for three conditions with different value of shifts parameters are given in Table II, and Table III respectively. The proposed VSI fuzzy control chart was compared with the $\overline{X}$ control chart, VSI $\overline{X}$ control chart, and fuzzy control chart by evaluating the value of ATS.

TABLE II: THE VALUE OF ATS FOR DIFFERENT VALUES OF Δ1 WHEN PROCESS MEAN SHIFTS.

1	$\overline{X}$ chart	VSI $\overline{X}$ chart	fuzzy chart	VSI fuzzy chart
	370. 40	370.40	370.4 0	370.40
.5	155. 22	**141.43**	169.4 9	151.91
.0	43.9	30.60	41.32	**29.14**
.5	14.9 7	6.95	13.68	**6.18**
.0	6.30	1.82	5.69	**1.40**

From Table II , it is found that when the process is in control (i.e. δ_1=0), the ATS are almost the same for four compared charts which means these control charts have the same false alarm rate in the stable process. When the process mean has shifted, the ATS value for the VSI fuzzy control chart are the smallest, except the case of δ_1=0.5. Thus the VSI fuzzy control chart can quickly detect the moderate and large shift in mean.

Table III shows that the false alarm rates are the same for these four charts. When the process variance shifts, the values of ATS in the proposed VSI fuzzy control chart are smallest among the other three control charts in control status (δ_2=1). Hence, the proposed VSI fuzzy control chart can detect the shift in variance quickly.

TABLE III: THE VALUE OF ATS FOR DIFFERENT VALUES OF Δ2 WHEN PROCESS VARIATION SHIFTS.

δ_2	$\overline{X}$ chart	VSI $\overline{X}$ chart	fuzzy chart	VSI fuzzy chart
1	370. 40	370.40	370.4 0	370.40
1 .25	68.3 0	56.10	38.46	**23.04**
1 .50	40.8 1	29.80	9.91	**3.48**
1 .75	11.8 6	7.57	4.64	**0.98**

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