Horizontal Diffusion in Shallow Coastal Waters of Tamil Nadu

N. Ashok kumar, V. Arutchelvan, and P. Chandramohan

***Abstract*-A series of diffusion experiments using Dye study were performed in the coastal waters of Tamil Nadu by Indomer coastal hydraulics and the data were used and analysed to estimate horizontal mixing coefficients. The data of those studies are used to find out the dispersion coefficients and the oceanic diffusivity is reviewed. The volume of available data allows the effective dispersion coefficient to be estimated as a function of either time or space. Values for this dispersion coefficient vary between 0.60 $\times 10^6$ and 10 $\times 10^8$ m^2/s. The estimated mixing coefficients are compared with those values obtained by earlier studies.**

***Index Terms*—Coastal Waters, Dye, Diffusion coefficient, Horizontal diffusion, Rhoda mine-WT,.**

I. Introduction

Numerical values of dispersion on large scales in coastal waters are relatively difficult to determine and interpret, for a number of different reasons. Different processes are likely responsible for dispersion at different spatial and temporal scales [1]. The Effective horizontal diffusivity observed in both the atmosphere and ocean depends on the scale of phenomenon in question as mentioned [2] and [3]. This result was summarized [4] in a diffusion diagram which that the effective horizontal diffusivity K_h on length scale ranges from l = 0.2m to 100m respectively. Collection of empirical data from dye experiments in the surface mixed layer was compiled [5] and their results ranging in scale from 100m to 100km, which showed that the diffusivity increased as a function of horizontal scale.

Generally there are two methods employed to obtain the horizontal diffusivity of the coastal waters. The first method turbulent velocity component of the current meter data is analysed to obtain the horizontal diffusivity. In the other method is by using tracer experiments. In fairly calm regions of the sea without prominent divergence or convergence, the horizontal diffusion is likely to be dominant and the tracer study gives a good estimate of horizontal diffusivity. Such dispersion studies were more or less limited to the open ocean have the values of 3.06 x 10^8 m^2/sec and 0.54m^2/sec respectively for the Equatorial Indian Ocean.

Rhoda mine dyes are generally preferred to conduct the tracer experiment than other chemicals and radioisotopes for the estimation of diffusivity in the sea, because of (a) the remarkable detectability of the rhodamine family of dyes, especially Rhodamine WT—concentrations smaller than 0.03 μg/kg may be detected in a modern fluorometer, and (b) the simplicity of fluorometric testing procedures— special training is not required and dye concentrations are determined instantaneously without special preparation of the sample. Further Rhodamine WT is considered most accessible, stable, harmless and convenient to use for environmental investigations [7]. Rhoda mine dye tracing is used by government and municipal agencies. national research laboratories, universities, and industrial groups in several applications, which include stream flow measurements, estuarine dispersion studies, herbicide tracing, and ground-water studies.

The number of methods for estimating the magnitude of horizontal diffusion coefficient was summarized [8]. Using the available data on dye diffusion along the coastal areas of Tamil Nadu, India, the horizontal diffusivities of coastal waters are estimated. The overall characteristics of horizontal diffusivity in these coastal regions were reviewed.

II. Rhodamine Dye Release Technique

In order to determine the dispersion and mixing patterns in the study area, a dye tracing experiment was conducted. Rhodamine -WT (20 % solution, density 1190 kg m-3) was released over 30 minutes at a rate of 0.28 L min-1 at the proposed site. The dye was diluted in a drum (Fig. 1) using approximately 40 L of local seawater per minute to produce water with a density approximately 1.1 kg m-3 heavier than local seawater. The dyed water was then pumped at a constant rate into the sea using a pump and hose with a diffuser attached to the base of the hose. In order to fully mix the dye over the water depth the hose with the diffuser was moved up and down in the water column and the dye mixing is released in the water.

The horizontal diffusivity is estimated by instantaneously releasing rhodamine dye from a point in the sea and tracing the spread of the dye cloud .In this experiment, the dye concentration in the vertical direction is assumed to be homogeneous and it is traced continuously using in-situ fluorometer. The time taken to trace the dye concentration over the whole diffusing patch increases as the patch grows. During the final stage of the growth, the patch breaks up into several smaller patches. Patchiness was observed in the early stages of all of the experiments of rhodamine dye studies. They found spatial scales in dye concentrations 6–12 h after injection ranging over 0.5–10 m vertically a few hundred meters to a few kilometers horizontally. An alternative method of dye cloud observation is to take aerial photographs, which give only its area. The area of the dye cloud increases with time attains a maximum and finally vanishes from the

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sea surface. During the 100 hours or so of the experiments the area of the dye patches grew from less than 1 km^2 to more than 50 km^2 [4].

Fig. 1. Dye preparation.

The motion of the patch of the diffusing dye patch in the sea is a sort of irregular meandering of its centre of mass, and we are interested mainly in the distribution of the diffusing substance around the centre of mass. At a time t after the release, the shape of the isolines of dye concentration will be always irregular in an individual experiment and the point of maximum concentration may not necessarily coincide with the centre of mass, while it is reasonable to expect the relative distribution around the respective centres of mass to have the same statistical properties, since eddies with sizes less than or equal to that of the diffusing patch determine the distribution of concentration in the patch. The irregularity in the shape of isolines is caused by the local shear in ocean currents transporting the dye patch and the non-homogeneity of the larger scale turbulence in the sea. Measured horizontal concentration distributions constructed as lines of equal concentration .It is clear that the dye patches show a tendency to elongate in the direction of the mean current with a clockwise curvature from the leading edge to the trailing edge. The leading edge of the elongated dye patch generally contained higher dye concentrations than the trailing edge .These features of dye patch diffusion are no doubt attributed to wind-induced Ekman transport and the associated vertical current shear in the mean flow. Similar observations have been made by others. The horizontal dye distribution usually appears asymmetrical in the sense that the characteristic length is larger in one direction the other

Hence, dye releasing experiments are supposed to be carried out for a large number of times with the same amount of dye solution. When we super impose an infinite number of such dye distributions observed at the same time interval after the beginning of each experiment upon each other, in such a way that the centres of mass coincide, and when we average over all the superimposed distributions, we can expect a rotationally symmetric distribution of substance (owing to a premise that the smaller eddies responsible for the relative distribution are stationary and homogeneous and even isotropic) around the centre of mass; which in other words, the concentration distribution in the mean picture of the diffusing dye patch due to turbulence is only a function of time 't' and distance 'r' from the centre of mass. But the concentration distributions from limited number of dye release experiments will show large irregularities, which require some sort of averaging of the observed instantaneous concentrations. As we seek only the average diffusivity in turbulent diffusion problems in the sea, the measured distribution of dye concentrations is replaced by concentric distributions.

Iso-lines of different concentrations are drawn, and the corresponding equivalent radii are determined. In practice, sometimes horizontal diffusivity is determined from a single dye release experiment. In such cases, the observed concentration distribution will not usually be rotationally symmetric and equivalent radii of surface areas enclosed by different isolines are calculated. The dye cloud, is transported by the riding the ocean current and is dispersed in irregular shape due to in-homogeneity of turbulence and local shear. To analyze such a detailed behavior of the dye cloud would obviously be very difficult to say the least and impossible unless detailed measurement of dye concentration as well as current velocity is made. Frequent practice is then to assume that the dye diffusion can be replaced by the fictitious diffusion into horizontally concentric shape to obtain the measure of the horizontal diffusivity. Radius r is defined as the radius of circle having an area equivalent to the surface enclosed by each isoline. If one imagines in each dye releasing experiment a marked particle of the dye substance is introduced, it can be easily seen that the average concentration distribution of the dye is a measure of the probability density function of the marked particle being found at a distance r from the centre of mass after time t. With this approach a diffusion equation is derived (Okubo, 1962), which gives the space-time distribution of the concentration of diffusing substance in the sea. Such a distribution is expressed by the following equation

$$C = \frac{M}{(kt)^{2n/m}} \exp\left(-\frac{n r^m}{m^2 k t^n}\right) \quad (1)$$

where C is dye concentration, M is the value related to the total dye quantity, r is the radius from centre of mass of the patch, t is time and k, m and n are parameters of the diffusion theory adopted. The parameters and some important values related to different diffusion theory were given by [10]. The lifetime for dye cloud, T, and the time the dye cloud attains the maximum t_m.

III. Dye Dispersion Radius.

From the measured maximum concentration at the centre of the patch, the horizontal diffusion coefficient could easily be computed by dye cloud area Vs time as shown Fig. 2. However, as it cannot be guaranteed that the maximum concentration really occurs at the centre of the patch, this method might give relatively higher value for the coefficient. For example, the plume centre line may not be steady, resulting in a meandering plume (see Kristensen, Jensen & Petersen 1981 for example). Additionally, the dispersion coefficient may not be constant, but instead may vary with location or even with the plume characteristics itself [11].

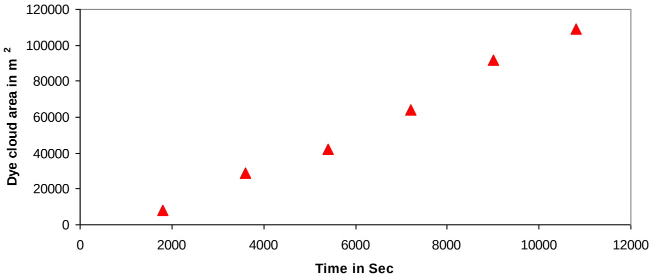

Fig. 2. Dye cloud area vs time.

By fitting the measured distributions of dye release experiments to the theoretical space-time distribution (Eqn. 1) the diffusion parameter k corresponding to different theories can be determined. With the obtained experimental data, the horizontal diffusivity can be calculated as the temporal rate of increase in dye cloud area. Given the dye cloud area S_i at time t_i, i=1, 2,…, the horizontal diffusivity is calculated as follows,

$$k_i = \frac{S_i - S_{i-1}}{t_i - t_{i-1}} \tag{2}$$

The equivalent radius r_i associated with the dye cloud area is given by,

$$r_i = \frac{\sqrt{S_i} - \sqrt{S_{i-1}}}{2\sqrt{\pi}} \tag{3}$$

The other technique to obtain horizontal diffusivity is to regard the peripheral concentration of a dye cloud as constant and the equivalent dye cloud radius r with respect to time t is fitted to the reduced form of Eqn. (1)[11] given by,

$$1 - \frac{(r_1/r_o)^m}{(t_1/t_o)^n} = \frac{2mk\left(t_o^n/r_o^m\right)}{\log e}\log\frac{t_1}{t_o} \tag{4}$$

Based on the method originally proposed by [12] for the atmospheric diffusion, which is applicable for the simple Fickian diffusion law, a generalized method to include various theories of oceanic diffusion is proposed. In this method, the concentric horizontal dye distribution given by Eq. (1) is assumed. A radial symmetrical distributions(t, r_c), thus characterized by the equivalent radius, r_c and the diffusion time *t* is the basic distribution from which the horizontal variance, i.e the mean square distances from the centre of mass may be computed as The dispersion radii for the measured distribution can be estimated. This again provides a method for estimating dispersion parameters by equating the σ_r^2 estimated from the measured distribution and theoretical estimates. The dispersion radii for various theoretical solutions of

$$\frac{\partial S(r,t)}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left\{k\,r^{m+1} f(t)\frac{\partial S}{\partial r}\right\} \tag{5}$$

are given in Table II. Dispersion radius varies, linearly with time for the Fickian dispersion, as the square of the time in the cases of solutions JS, OP & Ok and as the cube of the time in the case of solutions Oz & Ob. In dye release experiment if concentrations distributions were measured, dispersion parameters can be determined by fitting the measured distribution to the theoretical distributions.

Alternatively defining the dispersion radius of the dye concentration as

$$\sigma_r^2 = \int_0^\infty r^2\, C\, 2\pi r\, dr \Big/ \int_0^\infty C\, 2\pi r\, dr \tag{6}$$

Eliminating time dependence from the solution of the diffusion equation (1) using the result of Eq. (6), and noting that the radius r vanishes at the lifetime T, one obtains the expression for dispersion radius, σ_r^2 in σ_r^2 terms of the maximum radius r_m,

$$\sigma_r^2 = a r^2\left[\ln(b r_m^2 e^{2/m}) - \ln\sigma_r^2\right]^{-2/m} \tag{7}$$

where constants a and b are given in Table I. The relationship between dye dispersion and dye cloud radius as given by two typical cases of Okubo-Pritchard (OP) and Obukov (Ob) (two cases give the identical relationship), and Ozmidov (Oz). It shows that the dispersion radius uniformly increases as the dye cloud radius increases, hits the maximum and then decreases to null. The typical application of this method by plotting the dispersion radius calculated from Eq. (6) from Fig. 3.

Comparing with the time dependence of the dispersion radius given in Table II, the theory of either OP or Ob appears to have an edge over that of Ob. Okubo [6] used data from 20 carefully selected instantaneous dye release experiments, with time scales ranging from 2 hr to 1 month, and length scales ranging from 30 m to100 km. He assumed that the dye experiments were radially symmetric, thus computed the variance for a radially symmetric distribution ($\sigma^2 r_c$). If σ^2x and σ^2y denote the variance σ^2_{rc} for a radially symmetrical distribution obtained by taking the equivalent radius is given by $\sigma^2_{rc} = 2\sigma x \sigma y$

Fig. 4 shows the plot of k_i versus r_i calculated from the data .In this particular example, it can be seen that because of the small number of data, Fig. 4 does not show a dependence of k on' r' as expected from the oceanic diffusion theory. Fig. 3 gives the example of the plot of the left-hand-side of Eq. (4) versus t/t０ for OP, Oz and Ob. The diffusion theory is tested whether or not the plotted points follow a straight line. Obukhov best fits the data although this plotting appears to depend on the initial value.

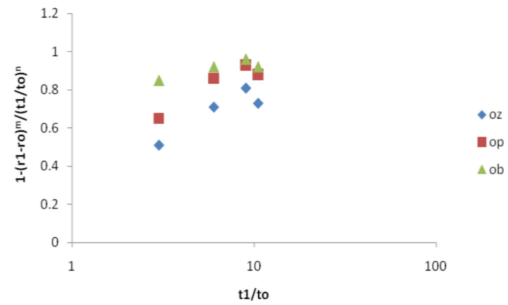

Fig. 3. Fitting of dye cloud area with oceanic diffusion theories.

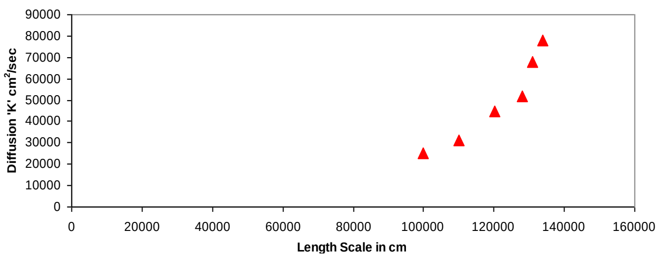

Fig. 4. Horizontal diffusivity calculated with the simple method Eq. (2). At various coastal seas of Tamil Nadu.

TABLE I: CONSTANTS IN EQ. (6)

Theory	a	b
JS	6	3/2
OZ	60	20/9
O-P	1	1
OK	$4/\sqrt{\pi}$	$8\sqrt{2/3\pi}$
Ob	1	1

TABLE II: IMPORTANT FEATURES OF DIFFERENT SOLUTIONS

No	*Solutions*	*Peak concentration S(0, t)/M*	*Spatial distribution {S(r,t)/ S(0,t)}*	*Dispersive radius,* σ_r^2	*m*	*n*
1	Fickian (Fi)	$\frac{1}{4\pi k t}$	$\exp\{-r^2/4kt\}$	$4kt$	2	1
2	Joseph & Sendner (JS)	$\frac{1}{2\pi p^2 t^2}$	$\exp\{-r/pt\}$	$6p^2 t^2$	1	1
3	Okubo & Pritchard (OP)	$\frac{1}{\pi\omega^2 t^2}$	$\exp\{-r^2/\omega^2 t^2\}$	$\omega^2 t^2$	2/3	1
4	Okubo (Ok)	$\frac{1}{0.75\pi^{3/2}\alpha^3 t^3}$	$\exp\{-r^{4/3}/\alpha^2 t^2\}$	$\frac{4}{\sqrt{\pi}}\alpha^2 t^2$	2	2
5	Ozmidov (Oz)	$\frac{1}{6\pi\gamma^3 t^3}$	$\exp\{-r^{2/3}/\gamma t\}$	$6\pi\gamma^3 t^3$	4/3	2
6	Obukhov (Ob)	$\frac{1}{\pi\beta^3 t^3}$	$\exp\{-r^2/\beta^3 t^3\}$	$\beta^3 t^3$	2	3

IV. DISCUSSION

Calculating dispersion radius out of the dye cloud radius is an excellent method since the time dependency of the dispersion radius is simple and straightforward. During the initial stages of diffusion, when the dye patch is small (order of hundreds of meters), marked anisotropy of turbulence, combined with 'shear diffusion' due to vertical shear in the horizontal mean current, gives rise to an apparent increase in the longitudinal diffusion of the patch, whereas for large diffusion times, when the patch grows to the size of kilometers, localized, non-uniformities in the flow do not appear to influence the overall diffusion of the patch.

From the available data, the simple method applied to obtain the horizontal diffusivity versus the length scale as shown in Fig. 4, and it should be regarded as the basic data for the coastal, calm sea. One can draw a conclusion and pleases to apply any of the oceanic diffusion theory given in Table II. Horizontal mixing and dispersion in the region is strongly influenced by the prevailing wind. Dispersion coefficients are found to be the order of $0.6\times10^6$to $10.5\times10^6cm^2$/sec which is the order of magnitude higher compared to the coefficients used for the modelling of surf zone by [13] and also the work done by [14] but they are Clear and logical with the values measured by [15] in similar oceanographic conditions.

The effective horizontal dispersion appeared to be inversely proportional to the cross-sectional area of the estuary, and a $F^{1/3}$ dependence of dispersion on the freshwater flux was observed at all but the lowest flux rates. Estimated horizontal diffusivity was of order $4.5\times10m^2$/sec, comparable with other studies [16]. In the local vicinity of the front the secondary circulation has the potential to supply nutrients from deep water to the level of the photic zone. However, in the stratified region away from the front, the work of [17] suggests that it is vertical diffusive processes that supply the nutrient flux to sustain phytoplankton growth close to the base of the thermo cline.The value of effective eddy diffusivity reported by [18] as 24.7 x 10^2 m^2/sec and 1.2 x 10^2 m^2/sec for Visakhapatnam during March and December (1991) respectively showing a higher value of eddy diffusion by one order under rough weather conditions.

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