Multi-Objective Generation and Emission Dispatch Using NSGA-II

Javed Dhillon and Sanjay K. Jain

Abstract— **The multi-objective problem of the generation and emission dispatch is solved to find the generation levels that best compromise the generation cost and the emission level while satisfying the power balance constraint. The solution is attempted using non-dominated sorting genetic algorithms–II (NSGA-II) to find non-dominated solutions of with good diversity. The best compromise solution has been obtained using Fuzzy cardinal priority ranking. The results are presented for a system of 6-generators by neglecting the losses and accounting them for different combinations of Fuel cost, NO_x, CO_x and SO_x emission objectives. The simulated results demonstrate the effectiveness of the proposal formulation.**

Index Terms— **Generation dispatch, Emission dispatch, Multi-objective optimization, Evolutionary algorithm.**

I. Introduction

The cost of power system operation is minimized by economic or generation dispatch, which is the allocation of generation to various units to meet a given load demand. For thermal units, operating cost is mainly due to the fuel cost. The operation of these units also produces large amount of emission like oxides of sodium SO_X, nitrogen NO_X, carbon CO_X etc. These emissions, an environmental concern, have forced the utilities to adopt various practices like use of higher quality fuel, upgrading older plants with new efficient cleaner plants or considering emission-free alternate forms of energy. The economic dispatch with reference to clean air act [2] has been discussed. The clean air act persuades the utilities to change their practices to meet the environmental emission norms. Thus, it becomes important to perform the emission dispatch with generation dispatch.

Many studies have been carried out to solve the generation dispatch with or without emission dispatch. These studies include use of Goal programming techniques [3], Linear programming techniques [4], fuzzy approach [5,6] and Evolutionary Algorithms [7-11].

The generation and emission dispatch problem has been reduced to a single objective problem [12,13] by treating the emission as a constraint with a permissible limit. Alternatively, minimizing the emission has been handled as another objective in addition to usual cost objective. A linear programming based optimization by considering one objective at a time has been presented in [14]. The multi-objective emission and generation dispatch problem has been converted to a single objective problem by linear combination of different objectives as a weighted sum [15,16]. A set of non-inferior (or Pareto-optimal) solutions is obtained by varying the weight and therefore requires multiple runs. Goal programming method was also proposed for multi-objective generation and emission dispatch problem [17]. This method requires a prior knowledge about the shape of the problem search space.

The methods arising from evolutionary computation are fast and effective techniques capable of finding a well-distributed set of diverse trade-off solutions, with little or no more effort than sophisticated single-objective optimizer. Most multi-objective evolutionary algorithms (MOEAs) use the concept of *Pareto domination* to guide the search. A solution is said to dominate another solution, if it is no worse than other in all objectives and better than in at least one objective. A solution is said to be *non dominated* if it is not dominated by any other solution. Various evolutionary algorithms in [18, 19] are reported for multi-objective optimization.

In this paper, an elitist evolutionary non-dominated sorting algorithm (NSGA-II) is used for solving the multi-objective generation and emission dispatch problem. After obtaining various optimal solutions using NSGA-II, the single best compromise solution is obtained using Fuzzy cardinal priority ranking.

II. Multi-objective Generation And Emission dispatch

Multi-objective problems are often characterized by several non commensurable and often competing objectives [6, 7] subjected to a number of equality and inequality constraints. The general structure of multi-objective generation and emission dispatch problem is expressed as-

Find : $[P_G] = [P_{G1}, P_{G2}, \ldots. P_{GNg}]^T$

By Minimizing: $F = [F_{FC}, F_{NX}, F_{CX}, F_{SX}]$

Subjected to: $h(P_{Gi}) = 0$

$$g(P_{Gi}) \leq 0 \tag{1}$$

where, $i = 1, 2, 3 \ldots\ldots Ng$

where Ng is the total no of generation units, P_{Gi} is the real power output of i_{th} generator, $h(P_{Gi})$ is the equality constraints and $g(P_{Gi})$ is the inequality constraints.

The various objective functions for the generation and emission dispatch problem are :

A. Minimization of Fuel Cost (F_{FC})

B. Minimization of NO_X Emission (F_{NX})

C. Minimization of CO_X Emission (F_{CX})

D. Minimization of SO_X Emission (F_{SX})

Manuscript received April 26, 2011; revised August 31, 2011.
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A. Minimization of Fuel Cost (F_{FC})

The minimization of total fuel cost F_{FC} is expressed as,

$$F_{FC} = \min \sum_{i=1}^{Ng} \left(a_i + b_i P_{Gi} + c_i P_{Gi}^2 \right) \quad \$/hr \tag{2}$$

where a_i, b_i and c_i are the fuel cost coefficient

B. Minimization of NO_X emission (F_{NX})

The minimization of NO_X emission F_{NX} is represented as,

$$F_{NX} = \min \sum_{i=1}^{Ng} \left(a_{Ni} + b_{Ni} P_{Gi} + c_{Ni} P_{Gi}^2 \right) \quad kg/hr \tag{3}$$

where a_{Ni}, b_{Ni} and c_{Ni} are the NO_X emission coefficient

C. Minimization of CO_X emission (F_{CX})

The minimization of CO_X emission F_{CX} is expressed as,

$$F_{CX} = \min \sum_{i=1}^{Ng} \left(a_{Ci} + b_{Ci} P_{Gi} + c_{Ci} P_{Gi}^2 \right) \quad kg/hr \tag{4}$$

where a_{Ci}, b_{Ci} and c_{Ci} are the CO_X emission coefficient

D. Minimization of SO_X emission (F_{SX})

The minimization of SO_X emission F_{SX} is represented as,

$$F_{SX} = \min \sum_{i=1}^{Ng} \left(a_{Si} + b_{Si} P_{Gi} + c_{Si} P_{Gi}^2 \right) \quad kg/hr \tag{5}$$

where a_{Si}, b_{Si} and c_{Si} are the SO_X emission coefficient

Power balance constraints

The total power generation must be equal to the total demand P_D and the real transmission loss P_{LOSS}. Hence,

$$\sum_{i=1}^{Ng} (P_{Gi}) - P_D - P_{LOSS} = 0 \tag{6}$$

Where P_{LOSS} is the total power loss given as below

$$P_{LOSS} = \sum_{i=1}^{Ng} \sum_{j=1}^{Ng} P_{Gi} B_{ij} P_{Gj} \tag{7}$$

Limits on generator output P_{Gi}

For stable operation, the generator outputs must be within the limiting values as follows:

$$P_{Gi\,\min} \le P_{Gi} \le P_{Gi\,\max} \tag{8}$$

III. Elitist Multi-objective Evolutionary Algorithm

The main objective of multi-objective evolutionary algorithm is to find multiple Pareto-optimal solutions in one single simulation run [8]. To enhance the convergence properties of multi-objective elitist operator [9] is used. The elitism helps to keep the best solution of the current population and does not allow it to deteriorate in next generation.

The NSGA-II which is known as elitist non-dominated sorting genetic algorithm, has the following features:

1) It uses non dominated sorting techniques to provide the solution as close as possible to the pareto-optimal solution.
2) It uses crowding distance techniques to provide diversity in solution.
3) It uses elitist techniques to preserve the best solution of current population in next generation.

There are two stages in solving multi-objective problem: determination of the set of non-dominated solutions and selection of the best compromise solution.

A. Description of Algorithm based on NSGA-II

1) Initialize the population P_t.
2) Create the offspring population Q_t from the current population P_t.
3) Combine the two populations Q_t and P_t to form R_t where $R_t = P_t \, U \, Q_t$
4) Find non-dominated fronts F_i of R_t.
5) Initiate the new population $P_{t+1} = null$ and the counter of front for inclusion $i = 1$.
6) While $P_{t+1} + F_i \le N_{pop}$, do: $P_{t+1} \leftarrow P_{t+1} \, U \, F_i$, where $i \leftarrow i+1$
7) Sort the last front F_i using the crowding distance in descending order and choose the first $(N_{pop} - P_{t+1})$ elements of F_i
8) Use selection, crossover and mutation operators to create the new offspring population Q_{t+1}.

Initialization

Initialize the population P_t by the randomly generating P_{Gi}'s and satisfying power balance equation (6). After initialization it creates offspring population Q_t from the current population P_t and then combines the two populations to form R_t. Where R_t is define as:

$$R_t = P_t \, U \, Q_t$$

Non-Dominated Sorting

After the initialization the population is sorted on the basis of on non-domination as shown in Fig. 1. The pseudo code for this is -

```
for each (p ∈ P)
   for each (q ∈ P)
      if (p ≺ q) then
         S_p = S_p ∪ {q}
      elseif (q ≺ p) then
         n_p = n_p + 1
      end
   end
   if (n_p == 0) then
      F_1 = F_1 ∪ {p}
   end
end
```

The pseudo code suggests that if p dominates q then add q in the set S_p. If p dominated by q then increment the dominated counter n_p by 1. If there is no any solution which dominate p i.e $(n_p = 0)$, then p belong to first front F_1.

```
while (F_i ≠ null)
   Q = null
   for each (p ∈ F_i)
      for each (q ∈ S_p)
         n_q = n_q − 1
```

If $(n_q = 0)$ then
$Q = Q \cup \{q\}$
end
end
end
$i = i+1$
$F_i = Q$
end

Assign the front to each q in the set S_p according to its domination level given by n_q

where, P_t Current population
Q_t Offspring population
R_t Population after recombination
n_p Number of solutions dominated by solution, p
S_p Solutions which dominate the solution, p
F_i i_{th} front of non-dominated solutions
n_q Number of solution dominated by solution, q
Q Set of non-dominated solutions belongs to q

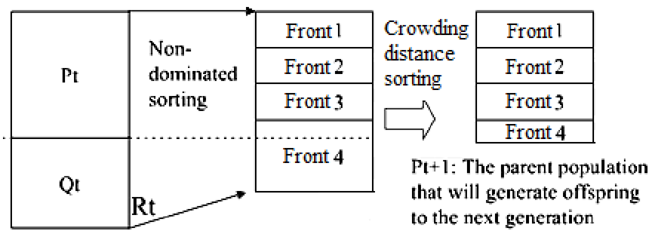

Fig. 1. Non-dominated and Crowding distance sorting

Crowding distance

To provide the diversity in population, the crowding distance is calculated[19]. The following pseudo code is used to calculate the crowding distance of each point in set I.

$l = |I|$
for each i
set $I[i]_{distance} = 0$
end
for each m
$I = sort(I,m)$
$I[1]_{distance} = I[l]_{distance} = \infty$
end
for $(I = 2 \text{ to } (l - 1))$
$I[i]_{distance} = I[i]_{distance} +$
$(I(k+1).m - I(k-1).m)/(f_m^{\max} - f_m^{\min})$
end

Firstly assign the boundary value to infinity and then calculate the crowding distance. Here, $I(k).m$ is the crowding distance for the m_{th} objective function of the k_{th} individual.

Where, I Set of non-dominated solutions
l Total number of solutions in set I
m Number of objective functions
f_m^{max} Maximum fitness value of m_{th} objective function
f_m^{min} Minimum fitness value of m_{th} objective function

Selection

Once the individuals are sorted based on non-domination with the crowding distance assigned, the selection is carried out using a *crowded-comparison-operator* ($>_n$) and best solution is selected. As shown in Fig. 1, it will be used to create Front 4 of small size than obtained after the non-dominated sorting. It assumes that every solution has two attributes:

1) A Non-domination rank (r_i) in population
2) A local Crowding distance ($I[i]_{distance}$)

$i >_n j$
if $(r_i < r_j)$
or
if $((r_i = r_j)$ and $(I[i]_{distance} > I[j]_{distance}))$

The solution i is better than j if rank of i_{th} solution is better than j_{th} or if they have same rank but the crowding distance of i_{th} solution is better than j_{th}

Crossover and Mutation

The real coded genetic algorithm [10] employed in this paper uses Simulated Binary Crossover and Polynomial Mutation to create population Q_t as shown in Fig. 2 .

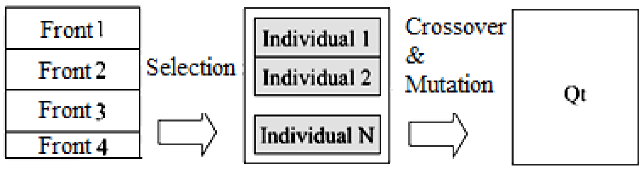

Fig. 2. Crossover and Mutation operation

1) Simulated Binary Crossover (SBX)

To generate the offsprings or child solutions using crossover, randomly select two parents solution ($p_{1,k}$, $p_{2,k}$) from the initial population and then generate the two child solution ($c_{1,k}$, $c_{2,k}$) as per the given pseudo code.

$Npop = |pop|$
for each k
$r_{1,k} = random(1,N_{pop})$
$r_{2,k} = random(1,N_{pop})$
$p_{1,k} = pop(r_{1,k})$
$p_{2,k} = pop(r_{2,k})$
$u_k = \text{random}(0,1)$
if ($u_k > 0.5$)
$\beta_k = (2u_k)^{1/(n_c+1)}$
else
$\beta_k = \dfrac{1}{\{2(u_k - 1)\}^{1/(n_c+1)}}$
end
$c_{1,k} = ½\ [(1-\beta_k).p_{1,k} + (1+\beta_k).p_{2,k}]$
$c_{2,k} = ½\ [(1+\beta_k).p_{1,k} + (1-\beta_k).p_{2,k}]$
$Qt = Qt\ U\ c_{1,k}$
$Qt = Qt\ U\ c_{2,k}$
end

2) Polynomial Mutation

This operator randomly selects one parent solution from the population and applies the mutation operator to generate a single offspring. The pseudo code is given as :

for each k
$r_k = random(1,N_{pop})$
$p_k = pop(r_k)$
$u_k = random(0,1)$

if ($u_k < 0.5$)

$$\delta_k = (2r_k)^{\frac{1}{n_m+1}} - 1$$

else

$$\delta_k = 1 - \{2(r_k - 1)\}^{\frac{1}{n_m+1}}$$

end

$$c_k = p_k + (p_k^u - p_k^l)\delta_k$$

$$Q_t = Q_t \cup c_k$$

end

where, r randome number for selecting the parent solution

- p Parent solution from population, *pop*
- c Child solution
- uk random number
- n_c Crossover distribution index
- n_m Mutation distribution index
- β_k Spread factor
- δ_k Small variation

B. Best Compromise Solution

The optimization of the above-formulated multi-objective formulation using NSGA-II yields set of Pareto optimal solutions [11], in which one objective cannot be improved without sacrificing other objectives. For practical applications, however, we need to select one solution, which will satisfy the different goals to some extent. Such a solution is called best compromise solution. The best compromise solution is obtained using Fuzzy cardinal priority ranking. The pseudo code for this is given as:

for each $(k \in M)$

for each $(i \in N_{obj})$

if $(f_i^k \geq f_{\max}^M)$

$$u_i^k = 0$$

else if $(f_{\min}^M \leq f_i^k \leq f_{\max}^M)$

$$u_i^k = \left(f_{\max}^M - f_i^k\right) / \left(f_{\max}^M - f_{\min}^M\right)$$

else

$$u_i^k = 1$$

end

end

end

for each $(i \in N_{obj})$

$$\beta_i = \sum_{k=1}^{M} u_i^k \Bigg/ \left(\sum_{i=1}^{N_{obj}} \sum_{k=1}^{M} u_i^k \right)$$

end

where β_i is the normalize membership function. The β_i provides the fuzzy cardinal priority ranking of the non-dominated solution. the solution that attains the maximum membership β_i in fuzzy set is considered as best compromise solution.

where, N_{obj} Number of non-dominated solutions

- u_i^k Membership for k_{th} objective and i_{th} solution
- β_i Cardinal pirority ranking

IV. Results and Discussion

The study is carried out for a system of six generators [20] detailed in the Appendix. The results are obtained for multi-objective generation and emission dispatch by using the power balance and generator capacity constraints for the following five cases of optimization formulations:

- Case-A Fuel Cost and NO_X Emission
- Case-B Fuel Cost and CO_X Emission
- Case-C Fuel Cost and SO_X Emission
- Case-D Fuel Cost, NO_X Emission and CO_X Emission
- Case-E Fuel Cost, NO_X Emission and SO_X Emission

The results are obtained by neglecting and considering the losses at the load demand of 1800 MW with the following parameters -

- Population size = 100
- Maximum generation = 20000
- Crossover Distribution index = 20
- Mutation Distribution index = 20
- Crossover Probability = 0.9
- Mutation Probability = 0.1

In typical NSGA-II implementations, the mutation rate is small, typically around 10%. Whereas crossover rate is high, typically around 90%. The proposed study is carried out for two and three objectives functions to yield the relationship between the thermal units operating costs and emission. In all the cases the size of the initial population is 100. The maximum generations are 6000 and 20000 for the optimization of two and three objectives respectively.

A. Multi-Objective optimization when losses are neglected

The results obtained for the multi-objective optimization using the developed algorithm for the above mentioned Cases are summarized in Table I-V. The Table summarizes the solution at the minimum of the respective objective function and the best compromise solution for the respective set of objectives considered. In Case A, B and C two objectives are considered, the pareto optimal front for these are having similar nature. As a sample case the Pareto optimal fronts for case-B, case-D and case-E are shown in Fig. 3, Fig. 4 and Fig. 5 respectively. From the Tables I-V, it is clear that the minimum fuel cost is obtained close to 17520. The marginal difference is due to the solution is being run at different occasion and the convergence is based on evolutionary technique. The cost corresponding to best compromise solution is bound to change as it depends on all the objectives under investigation. The cost in best compromise solution is changing between 17520-17530.

B. Multi-Objective Optimization by Accounting Losses

The losses are now accounted with the help of B-coefficients. Keeping the load demand at the previous level, the generation is bound to increases. The results are obtained for all five cases mentioned earlier and are summarized in Tables VI-X. As expected, the total generation level increases which results into increase in the operation cost and also the emission level. The optimum pareto front for case-A, D and E are shown in Fig. 6, 7 and 8 respectively. The total fuel cost for the best compromise solution changes marginally around Rs. 18900.

TABLE I: RESULT FOR FUEL COST AND NO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Best Compromise Solution
PG1	222.9989	166.5557	169.7719
PG2	229.9978	194.2240	486.3092
PG3	437.9832	486.3091	467.1544
PG4	265.0000	264.9796	265.0000
PG5	442.2199	486.1314	466.2963
PG6	200.0001	200.0001	200.0000
F_{FC} ($/hr)	**17520.3429**	17582.3111	**17529.3159**
F_{NX} (kg/hr)	1847.2957	**1805.3127**	**1816.1465**

TABLE II: RESULT FOR FUEL COST AND CO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{CX}	Best Compromise Solution
PG1	221.8656	250.0000	238.9509
PG2	229.9998	229.9999	229.9994
PG3	437.0433	402.1071	419.3458
PG4	264.9987	264.9987	264.9961
PG5	441.7217	405.8301	424.0666
PG6	202.5707	245.2628	220.84114
F_{FC} ($/hr)	**17520.2842**	17537.8799	**17524.0739**
F_{CX} (kg/hr)	53260.5444	**51912.1721**	**52308.2522**

TABLE III: RESULT FOR FUEL COST AND SO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{SX}	Best Compromise Solution
PG1	221.8976	208.5284	214.7362
PG2	230.0000	230.0000	230.0000
PG3	436.8933	445.9464	441.9424
PG4	265.0000	264.9999	264.9999
PG5	441.0918	445.9877	443.7248
PG6	203.3173	202.7375	202.7965
F_{FC} ($/hr)	**17520.2825**	17520.8838	**17520.4607**
F_{SX} (kg/hr)	10510.5739	**10510.2146**	**10510.2916**

TABLE IV: RESULT FOR FUEL COST, NO_X AND CO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Solution at minimum F_{CX}	Best Compromise Solution
PG1	222.3661	166.8411	250.0000	197.9152
PG2	230.0000	194.2191	229.9987	229.7943
PG3	438.5525	486.0322	404.5769	454.4389
PG4	264.9977	264.9365	264.9993	264.9926
PG5	438.8981	486.1711	403.4438	448.8072
PG6	203.3856	200.0000	245.1813	202.2704
F_{FC} ($/hr)	**17520.2987**	17582.2152	17537.8536	**17522.6806**
F_{NX}(kg/hr)	1848.9139	**1805.3370**	1929.2176	**1828.2927**
F_{CX}(kg/hr)	53212.0024	58053.8654	**51911.6072**	54114.1258

TABLE V: RESULT FOR FUEL COST, NO_X AND SO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Solution at minimum F_{SX}	Best Compromise Solution
PG1	222.9468	166.4123	209.1126	173.5119
PG2	229.9999	194.1320	229.9999	229.6355
PG3	438.1437	486.0273	446.8517	465.7849
PG4	264.9942	264.9999	264.9982	264.9944
PG5	442.1101	486.6284	447.0476	464.2745
PG6	200.0000	200.0000	200.1898	200.0000
F_{FC} ($/hr)	**17520.3408**	17582.5052	17520.9603	**17528.5431**
F_{NX}(kg/hr)	1847.2415	**1805.3021**	1834.9453	**1816.6648**
F_{SX}(kg/hr)	10510.6022	10544.2067	**10510.2455**	**10512.8993**

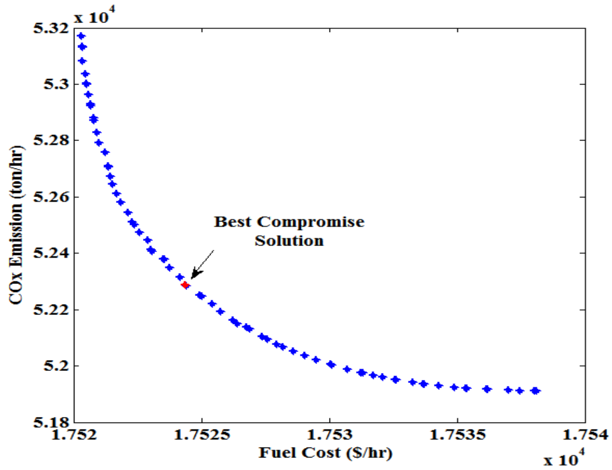

Fig. 3. Pareto optimal solution for fuel cost and CO_X optimization

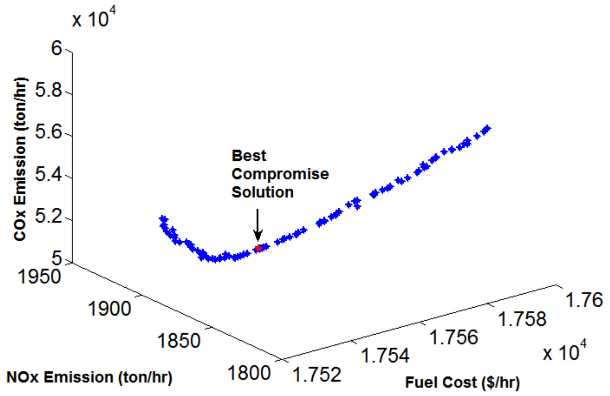

Fig. 4. Pareto optimal solution for fuel cost NO_X and CO_X optimization

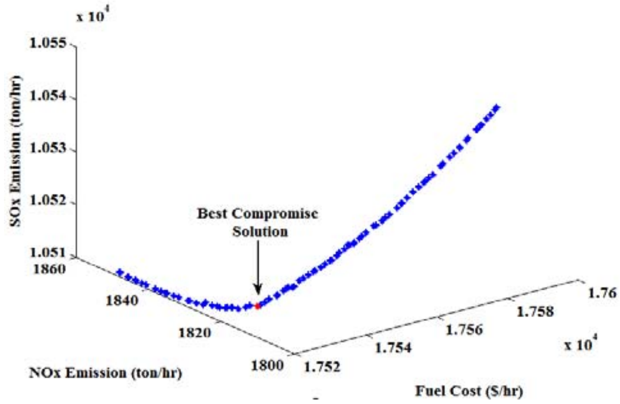

Fig. 5. Pareto optimal solution for fuel cost NO_X and SO_X optimization

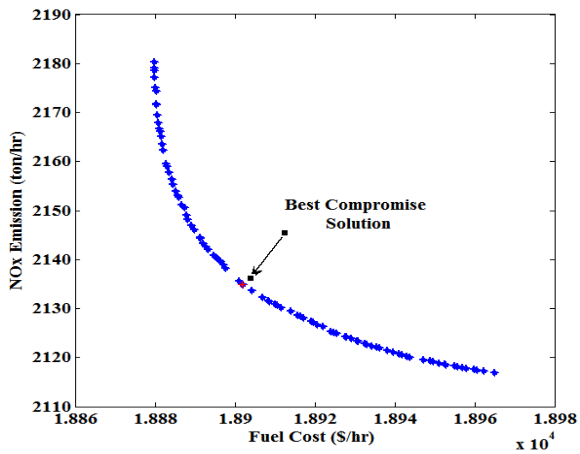

Fig. 6. Pareto optimal solution for fuel cost and NO_X optimization

TABLE VI: RESULT FOR FUEL COST AND NO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Best Compromise Solution
PG1	249.9764	228.9257	249.9608
PG2	230.0000	229.9999	229.9988
PG3	499.9998	499.9997	499.9987
PG4	264.9999	264.9998	264.9999
PG5	420.9971	500.0000	458.5955
PG6	273.5104	226.9316	240.3168
F_{FC} ($/hr)	**18880.1011**	18965.0972	**18899.4910**
F_{NX} (kg/hr)	2175.0972	**2116.9552**	**2136.9552**

TABLE VII: RESULT FOR FUEL COST AND CO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{CX}	Best Compromise Solution
PG1	250.0000	250.0000	250.0000
PG2	229.9992	229.9934	229.9999
PG3	500.0000	499.9999	500.0000
PG4	265.0000	265.0000	265.0000
PG5	417.3858	430.3211	423.9427
PG6	276.7428	265.1376	270.8374
F_{FC} ($/hr)	**18879.9067**	18881.9047	**18880.4279**
F_{CX} (kg/hr)	64013.6629	**63931.8286**	**63952.1125**

TABLE VIII: RESULT FOR FUEL COST AND SO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{SX}	Best Compromise Solution
PG1	250.0000	250.0000	250.0000
PG2	229.9999	229.9999	229.9999
PG3	499.9998	499.9998	499.9998
PG4	265.0000	265.0000	265.0000
PG5	421.3522	421.3522	421.3522
PG6	273.1897	273.1897	273.1897
F_{FC} ($/hr)	**18880.0995**	18880.0995	**18880.0995**
F_{SX} (kg/hr)	11326.0965	**11326.0965**	**11326.0965**

TABLE IX: RESULT FOR FUEL COST, NO_X AND CO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Solution at minimum F_{CX}	Best Compromise Solution
PG1	250.0000	229.6664	250.0000	249.4301
PG2	229.9998	229.9634	229.9972	229.9874
PG3	499.9998	500.0000	499.9999	499.9865
PG4	265.0000	264.9983	264.9999	264.9866
PG5	417.1908	500.0000	430.1076	456.5024
PG6	276.9186	226.1944	265.3245	242.7227
F_{FC} ($/hr)	**18879.9055**	18964.7177	18881.8366	**18898.0244**
F_{NX}(kg/hr)	2180.4788	**2116.9621**	2163.3075	**2137.8856**
F_{CX}(kg/hr)	64016.0833	65823.4113	**63931.7113**	**64243.3887**

TABLE X: RESULT FOR FUEL COST, NO_X AND SO_X OPTIMIZATION

Units (in MW)	Solution at minimum F_{FC}	Solution at minimum F_{NX}	Solution at minimum F_{SX}	Best Compromise Solution
PG1	250.0000	230.2076	250.0000	249.4956
PG2	229.9866	229.9999	229.9866	229.9999
PG3	500.0000	500.0000	500.0000	499.9452
PG4	264.9986	264.9999	264.9986	264.9983
PG5	421.7120	500.0000	421.7121	460.1639
PG6	272.8569	225.5907	272.8567	239.4992
F_{FC} ($/hr)	**18880.1700**	18964.2999	18880.1700	**18901.3563**
F_{NX}(kg/hr)	2174.1202	**2116.9615**	2174.1202	**2135.3562**
F_{SX}(kg/hr)	11326.1342	11374.9355	**11326.1342**	**11338.3420**

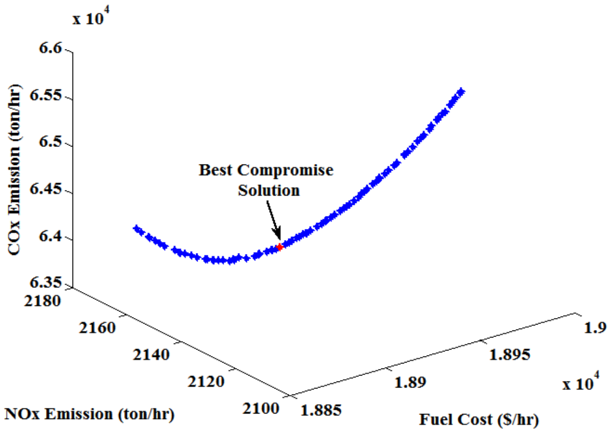

Fig. 7. Pareto optimal solution for fuel cost, NO_X and CO_X optimization

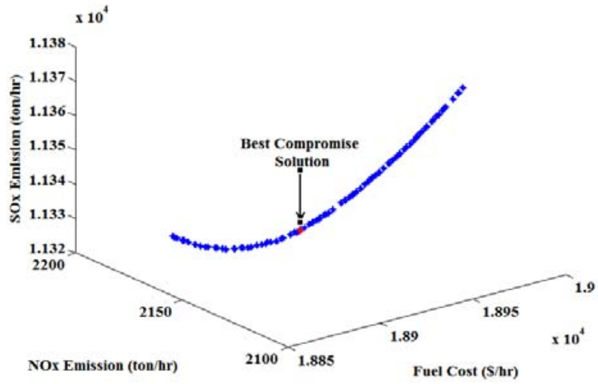

Fig. 8. Pareto optimal solution for fuel cost, NO_X and SO_X optimization

V. CONCLUSION

The multi-objective Generation and emission dispatch problem has been solved using the elitist Non-dominated Sorting Genetic Algorithm. The algorithm has been run on the six generator system by considering the system with or without loss. The study has been extended to two and three objectives problems. The following conclusions are made

- The developed algorithm provides pareto optimal solution with good diversity and best compromise solution.
- In the minimum fuel cost for neglecting the losses (or consider the losses) for different cases is same whereas the cost for best compromise solution in different because it depends on all the objectives considered, however the variation is small.

APPENDIX

The fuel cost, emission and loss coefficients for six generator system are given in Table A1, A2, A3, A4 and A5 respectively.

TABLE A1: FUEL COST COEFFICIENTS

Units	c_i	b_i	a_i	P_{min}	P_{max}
1	0.002035	8.43205	85.6348	100	250
2	0.003866	6.41031	303.7780	50	230
3	0.002182	7.42890	847.1484	200	500
4	0.001345	8.31540	274.2241	85	265
5	0.002162	7.42289	847.1484	200	500
6	0.005963	6.91559	202.0258	200	490

TABLE A2: NO_X EMISSION COEFFICIENTS

Units	c_{Ni}	b_{Ni}	a_{Ni}
1	0.006323	-0.38128	80.9019
2	0.006483	-0.79027	28.8249
3	0.003174	-1.36061	324.1775
4	0.006732	-2.39928	610.2535
5	0.003174	-1.36061	324.1775
6	0.006181	-0.39077	50.3808

TABLE A3: CO_X EMISSION COEFFICIENTS

Units	c_{Ci}	b_{Ci}	a_{Ci}
1	0.265110	-61.01945	5080.148
2	0.140053	-29.95221	3824.770
3	0.105929	-9.552794	1342.851
4	0.106409	-12.73642	1819.625
5	0.105929	-9.552794	1342.851
6	0.403144	-121.9812	11381.070

TABLE A4: SO_X EMISSION COEFFICIENTS

Units	c_{Si}	b_{Si}	a_{Si}
1	0.001206	5.09928	51.3778
2	0.002320	3.84654	182.2605
3	0.001284	4.45647	508.5207
4	0.000813	4.97641	165.3433
5	0.001284	4.45647	508.5207
6	0.003578	4.14938	121.2133

TABLE A5: B- COEFFICIENTS

2.0e-4	1.0e-5	1.5e-5	5.0e-6	0.0	-3.0e-5
1.0e-5	3.0e-4	-2.0e-5	1.0e-6	1.2e-5	1.0e-5
1.5e-5	-2.0e-5	1.0e-4	-1.0e-5	1.0e-5	8.0e-6
5.0e-6	1.0e-6	-1.0e-5	1.5e-4	6.0e-6	5.0e-5
0.0	1.2e-5	1.0e-5	6.0e-6	2.5e-4	2.0e-5
-3.0e-5	1.0e-5	8.0e-6	5.0e-5	2.0e-5	2.1e-4

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