

Predication of Steering Geometry of Front Suspension using Experimental Data Based Model

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Abstract—Automobile Industry is one of the key industries in the business sector. Automobile industry has the largest growth because of changes in consumer preferences, business policies and changing economic conditions. Almost all modern cars have independent front suspension, which means that each front wheel of your auto is linked separately to the automobile frame. The front wheels must steer as well as respond to the road surfaces because of bounces up and down when either wheel hits a bump or pothole. Automobile should drive down on the road in comfort and safety.

The performance of front suspension of automobile is based on the steering geometry of the suspension. The steering geometry parameters are kingpin angle, camber angle, caster angle, scrub radius, toe in and toe out. In this paper model equations are formulated based on which steering geometry can be predicated and vehicle performance can be determined.

Index Terms—Front Suspension, Steering Geometry, Automobile, Modeling.

I. INTRODUCTION

The Front suspension comprises of a linkage which is a 3 Dimensional mechanism RSSR (Revolute, Spherical, Spherical, Revolute paired). Steering performance depends on the appropriateness of spherical bushes located at various joints. Steering performance depends on the position of kingpin axis. Depending on the position of kingpin axis, kingpin angle, caster angle, camber angle, toe angle and scrub radius of a vehicle are decided.

Presently paper details the steering geometry parameters such as kingpin inclination angle, caster angle, camber angle, toe angle are calculated using model equations derived using experimental data based model theory.

II. FRONT SUSPENSION MECHANISM

A. Working of 3D Front Suspension

On the basis of six included angles of the 3D front suspension mechanism, one at each revolute joints and two at

each spherical joints of this four bar chain, position of kingpin axis is determined. Steering performance depends on the position of kingpin axis. Depending on the position of Kingpin axis, Caster angle, Camber angle, Kingpin angle and toe angle of four wheel vehicle are decided. Position of kingpin axis is determined using modeling techniques describe in the paper using dimensionless parameters involved in the experimentation [1].

B. RSSR Mechanism

Joint O_1 and O_2 are revolute joints and joints A and B are Spherical joints as shown in Figure 1. The relative orientation of two links connected at joint can be decided in terms of magnitudes of included angles which in turn can be measured by potentiometer and associated electronic instrumentation. Six potentiometers are located at four joints (two spherical and two revolute) of the RSSR mechanism. At revolute joints O_1 & O_2 the one included angle each of these joints and at spherical joints A & B the two included angles at each of these joints [2].

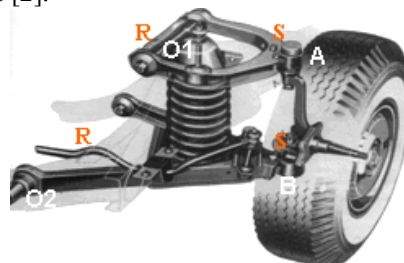


Fig 1: Front Suspension of an Automobile

III. DESIGN OF AN EXPERIMENTAL SET UP

The planning of experimentation is carried out by using the classical plan of experimentation [3]

A. Identification of various Physical Quantities affecting Front Suspension Geometry

The variables affecting the vehicle performance in the context of phenomena of steering are given below.

Independent Variables

1. Length of Upper control arm
2. Length of Lower control arm
3. Length of Knuckle arm
4. Length of Fixed link
5. Diameter of wheel
6. Mass of wheel
7. Road surface roughness in terms of Braker Height
8. Road surface roughness in terms of Braker Width
9. Wheel linear velocity

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10. Operational time
11. Acceleration due to gravity
12. Clearance at spherical joint A
13. Clearance at spherical joint B
14. Clearance at revolute joint O1
15. Clearance at revolute joint O2
16. Lateral displacement
17. Spindle length

Dependent Variables

1. Kingpin angle
2. Camber angle
3. Caster angle
4. Toe angle
5. Toe in
6. Toe out
7. Scrub radius

Dimensional Analysis

The process variables, their symbols and dimensions are listed in Table 1

TABLE I: PROCESS VARIABLES

Sr No	Description	Symbol	Dimension
01	Upper control arm	Ua	[M ⁰ L T ⁰]
02	Lower control arm	La	[M ⁰ L T ⁰]
03	Knuckle arm	Ka	[M ⁰ L T ⁰]
04	Fixed link	Fi	[M ⁰ L T ⁰]
05	Diameter of wheel	Dw	[M ⁰ L T ⁰]
06	Weight of wheel	Wt	[MLT ⁻²]
07	Braker Height	Bh	[M ⁰ LT ⁰]
08	Braker Width	Bw	[M ⁰ LT ⁰]
09	Wheel velocity	Vt	[M ⁰ L T ⁻¹]
10	Operational time	t	[M ⁰ L ⁰ T]
11	Acceleration due to gravity	g	[M ⁰ LT ⁻²]
12	Clearance spherical joint A	Ca	[M ⁰ LT ⁰]
13	Clearance spherical joint B	Cb	[M ⁰ LT ⁰]
14	Clearance revolute joint O1	Co1	[M ⁰ LT ⁰]
15	Clearance revolute joint O2	Co2	[M ⁰ LT ⁰]
16	Lateral displacement	Ld	[M ⁰ LT ⁰]
17	Spindle length	Sl	[M ⁰ LT ⁰]
18	Kingpin angle	Kga	[M ⁰ L ⁰ T ⁰]
19	Camber angle	Cm	[M ⁰ L ⁰ T ⁰]
20	Caster angle	Cs	[M ⁰ L ⁰ T ⁰]
21	Toe angle	Ta	[M ⁰ L ⁰ T ⁰]
22	Toe in	Ti	[M ⁰ LT ⁰]
23	Toe out	To	[M ⁰ LT ⁰]
24	Scrub radius	Sr	[M ⁰ LT ⁰]

M, L and T are the symbols for mass, length and time respectively.

Kingpin angle

$$Kga = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (1)$$

$$Cs = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (2)$$

$$Cm = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (3)$$

$$Ta = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (4)$$

$$Ti = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (5)$$

$$To = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (6)$$

$$Sr = f(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl) \quad (7)$$

General form can be defined as

$$\phi(Ua, La, Ka, Fi, Dw, Wt, Bh, Bw, Vt, t, g, Co1, Co2, Ca, Cb, Ld, Sl, Kgp, Cs, Cm, Ta, Ti, To, Sr) = 0$$

Total number of variables = 24

All these variables can be expressed in terms of three primary dimensions i.e. mass (M), Length (L) and Time (T).

According to Buckingham's theorem

One should get = 24 - 03 = 21 dimensionless terms

$$\phi(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8, \pi_9, \pi_{10}, \pi_{11}, \pi_{12}, \pi_{13}, \pi_{14}, \pi_{15}, \pi_{16}, \pi_{17}, \pi_{18}, \pi_{19}, \pi_{20}, \pi_{21}) = 0$$

Choosing Ua, Vt and Wt as repeating variables

$$\pi_1 = Ua^{a_1} Vt^{b_1} Wt^{c_1} La^{d_1}$$

$$M^0 L^0 T^0 = L^{a_1} (LT^{-1})^{b_1} (MLT^{-2})^{c_1} L$$

$$\text{For M, } 0 = c_1 \quad c_1 = 0$$

$$\text{For L, } 0 = a_1 + b_1 + c_1 + 1 \quad a_1 = -1$$

$$\text{For T, } 0 = -b_1 - 2c_1 \quad b_1 = 0$$

$$\pi_1 = La / Ua$$

Similarly for other π terms are evaluate and dimensionless equations are formed

$$Kgp = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$Cs = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$Cm = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$Ta = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$Ti / Ua = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$To / Ua = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

$$Sr / Ua = f(La \times Ka \times Fi / Ua^3, Dw / Ua, Bh \times Bw / Ua^2, Vt \times t / Ua, Ua \times g / Vt^2, Co1 \times Co2 \times Ca \times Cb / Ua^4, Ld \times Sl / Ua^2)$$

B. Deduction of Generalized Experimental Data Based Models:

Classical experimentation was planned and carried out on the front suspension of an automobile for predicating steering behavior [4], to establish empirical relationships among the dependent π terms and independent π terms. The dependent and independent variables are defined in the Table 1. The dependent dimensionless ratios and Independent dimensionless ratios are as shown below.

TABLE II : INDEPENDENT DIMENSIONLESS PI TERMS

Sr No	Independent Dimensionless ratios	Nature of basic Physical Quantities
01	$\pi_1 = La \times Ka \times Fi / Ua^3$	Front suspension Link Lengths
02	$\pi_2 = Dw / Ua$	Wheel Diameter
03	$\pi_3 = Bh \times Bw / Ua^2$	Braker width and height
04	$\pi_4 = Vt \times t / Ua$	Time of operation
05	$\pi_5 = Ua \times g / Vt^2$	Vehicle Speed
06	$\pi_6 = Co1 \times C02 \times CaxCb / Ua^4$	Clearances at joints
07	$\pi_7 = Ld \times sl / Ua^2$	Displacement

TABLE III : DEPENDENT DIMENSIONLESS RATIOS

Sr No	Dependent Dimensionless ratios or Pi terms	Nature of basic Physical Quantities
01	$\pi_{D1} = Kga$	Kingpin angle
02	$\pi_{D2} = Cs$	Caster angle
03	$\pi_{D3} = Cm$	Camber angle
04	$\pi_{D4} = Ta$	Toe angle
05	$\pi_{D5} = Sr / Ua$	Ratio scrub radius/upper arm
06	$\pi_{D6} = Ti / Ua$	Ratio Toe in / upper arm
07	$\pi_{D7} = To / Ua$	Ratio Toe out / upper arm

C. Formulation of Experimental Data Based Model:

Seven independent pi terms ($\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$) and seven dependent pi terms ($\pi_{D1}, \pi_{D2}, \pi_{D3}, \pi_{D4}, \pi_{D5}, \pi_{D6}, \pi_{D7}$) are decided during experimentation and hence are available for the model formulation.

Each dependent π term is the function of the available independent π terms,

$$\begin{aligned}
 Kga &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 Cs &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 Cm &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 Ta &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 Sr / Ua &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 Ti / Ua &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) \\
 To / Ua &= f(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7)
 \end{aligned}$$

A probable exact mathematical form for the dimensional equations of the phenomenon could be relationships assumed to be of exponential form [5].

For example, the model representing the behavior of dependent pi term Kga with respect to various independent pi terms can be obtained as under.

$$\text{Kingpin angle, } Kga = k_1 \times (\pi_1)^{a_1} \times (\pi_2)^{b_1} \times (\pi_3)^{c_1} \times (\pi_4)^{d_1} \times (\pi_5)^{e_1} \times (\pi_6)^{f_1} \times (\pi_7)^{g_1}$$

The values of exponent's $a_1, b_1, c_1, d_1, e_1, f_1, g_1$ are established independently at a time, on the basic of data collected through classical experimentation.

$$\text{Camber angle, } Cm = k_2 \times (\pi_1)^{a_2} \times (\pi_2)^{b_2} \times (\pi_3)^{c_2} \times (\pi_4)^{d_2} \times (\pi_5)^{e_2} \times (\pi_6)^{f_2} \times (\pi_7)^{g_2}$$

$$\text{Caster angle, } Cs = k_3 \times (\pi_1)^{a_3} \times (\pi_2)^{b_3} \times (\pi_3)^{c_3} \times (\pi_4)^{d_3} \times (\pi_5)^{e_3} \times (\pi_6)^{f_3} \times (\pi_7)^{g_3}$$

$$\text{Toe angle, } Ta = k_4 \times (\pi_1)^{a_4} \times (\pi_2)^{b_4} \times (\pi_3)^{c_4} \times (\pi_4)^{d_4} \times (\pi_5)^{e_4} \times (\pi_6)^{f_4} \times (\pi_7)^{g_4}$$

$$\text{Scrub radius, } Sr / Ua = k_5 \times (\pi_1)^{a_5} \times (\pi_2)^{b_5} \times (\pi_3)^{c_5} \times (\pi_4)^{d_5} \times (\pi_5)^{e_5} \times (\pi_6)^{f_5} \times (\pi_7)^{g_5}$$

$$\text{Toe in, } Ti / Ua = k_6 \times (\pi_1)^{a_6} \times (\pi_2)^{b_6} \times (\pi_3)^{c_6} \times (\pi_4)^{d_6} \times (\pi_5)^{e_6} \times (\pi_6)^{f_6} \times (\pi_7)^{g_6}$$

$$\text{Toe in, } To / Ua = k_7 \times (\pi_1)^{a_7} \times (\pi_2)^{b_7} \times (\pi_3)^{c_7} \times (\pi_4)^{d_7} \times (\pi_5)^{e_7} \times (\pi_6)^{f_7} \times (\pi_7)^{g_7}$$

From these models values of all dependent pi terms are computed.

$$Kga = k_1 \times (\pi_1)^{a_1} \times (\pi_2)^{b_1} \times (\pi_3)^{c_1} \times (\pi_4)^{d_1} \times (\pi_5)^{e_1} \times (\pi_6)^{f_1} \times (\pi_7)^{g_1} \tag{8}$$

There are eight unknown terms in the equation 8 curve fitting constant K_1 and indices $a_1, b_1, c_1, d_1, e_1, f_1, g_1$ to get the values of these unknowns we need minimum a set of eight values of ($\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7$ & π_{D1}).

$$Z = a + bX + cY + dZ + \dots \tag{9}$$

The equation 8 can be brought in the form of equation 9 by taking log on both sides.

D. Model Formulation:

Model of dependent pi term π_{D1} for kingpin angle

$$\begin{aligned}
 Kga &= k_1 \times (\pi_1)^{a_1} \times (\pi_2)^{b_1} \times (\pi_3)^{c_1} \times (\pi_4)^{d_1} \times (\pi_5)^{e_1} \times (\pi_6)^{f_1} \times (\pi_7)^{g_1} \\
 \pi_{D1} &= k_1 \times (\pi_1)^{a_1} \times (\pi_2)^{b_1} \times (\pi_3)^{c_1} \times (\pi_4)^{d_1} \times (\pi_5)^{e_1} \times (\pi_6)^{f_1} \times (\pi_7)^{g_1}
 \end{aligned}$$

Taking log on the both sides of equation for π_{D1} , getting eight unknown terms in the equations,

$$\begin{aligned}
 \text{Log } \pi_{D1} &= \text{log } k_1 + a_1 \text{log } \pi_1 + b_1 \text{log } \pi_2 + c_1 \text{log } \pi_3 + d_1 \text{log } \pi_4 + \\
 &e_1 \text{log } \pi_5 + f_1 \text{log } \pi_6 + g_1 \text{log } \pi_7
 \end{aligned}$$

Let,

$$\begin{aligned}
 Z_1 &= \text{log } \pi_{D1}, \quad K_1 = \text{log } k_1, \quad A = \text{log } \pi_1, \quad B = \text{log } \pi_2, \quad C = \text{log } \pi_3, \\
 D &= \text{log } \pi_4, \quad E = \text{log } \pi_5, \quad F = \text{log } \pi_6, \quad G = \text{log } \pi_7
 \end{aligned}$$

Putting the values in equations 3, the same can be written as

$$Z_1 = K_1 + a_1 A + b_1 B + c_1 C + d_1 D + e_1 E + f_1 F + g_1 G \tag{10}$$

Equation 10 is a regression equation of Z on A, B, C, D, E, F and G. in an n dimensional co-ordinate system..

$$\begin{aligned}
 \Sigma Z_1 &= nK_1 + a_1 \Sigma A + b_1 \Sigma B + c_1 \Sigma C + d_1 \Sigma D + e_1 \Sigma E + f_1 \Sigma F + g_1 \Sigma G \\
 \Sigma Z_1 * A &= K_1 \Sigma A + a_1 \Sigma A * A + b_1 \Sigma B * A + c_1 \Sigma C * A + d_1 \Sigma D * A + e_1 \Sigma E * A + f_1 \Sigma F * A + g_1 \Sigma G * A \\
 \Sigma Z_1 * B &= K_1 \Sigma B + a_1 \Sigma A * B + b_1 \Sigma B * B + c_1 \Sigma C * B + d_1 \Sigma D * B + e_1 \Sigma E * B + f_1 \Sigma F * B + g_1 \Sigma G * B \\
 \Sigma Z_1 * C &= K_1 \Sigma C + a_1 \Sigma A * C + b_1 \Sigma B * C + c_1 \Sigma C * C + d_1 \Sigma D * C + e_1 \Sigma E * C + f_1 \Sigma F * C + g_1 \Sigma G * C \\
 \Sigma Z_1 * D &= K_1 \Sigma D + a_1 \Sigma A * D + b_1 \Sigma B * D + c_1 \Sigma C * D + d_1 \Sigma D * D + e_1 \Sigma E * D + f_1 \Sigma F * D + g_1 \Sigma G * D \\
 \Sigma Z_1 * E &= K_1 \Sigma E + a_1 \Sigma A * E + b_1 \Sigma B * E + c_1 \Sigma C * E + d_1 \Sigma D * E + e_1 \Sigma E * E + f_1 \Sigma F * E + g_1 \Sigma G * E \\
 \Sigma Z_1 * F &= K_1 \Sigma F + a_1 \Sigma A * F + b_1 \Sigma B * F + c_1 \Sigma C * F + d_1 \Sigma D * F + e_1 \Sigma E * F + f_1 \Sigma F * F + g_1 \Sigma G * F \\
 \Sigma Z_1 * G &= K_1 \Sigma G + a_1 \Sigma A * G + b_1 \Sigma B * G + c_1 \Sigma C * G + d_1 \Sigma D * G + e_1 \Sigma E * G + f_1 \Sigma F * G + g_1 \Sigma G * G
 \end{aligned} \tag{11}$$

In the above set of equations the values of the multipliers $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1 are substituted to compute the values of the unknowns (viz. $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1). The values of the terms on L H S and the multipliers of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1 in the set of equations are calculated. After substituting these values in the equations 11 one will get a set of 8 equations, which are to be solved simultaneously to get the values of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1 . The above equations can be verified in the matrix form and further values of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1 can be obtained by using matrix analysis.

$$X_1 = \text{inv}(W) \times P_1$$

The matrix method of solving these equations using 'MATLAB' is given below.

$W = 8 \times 8$ matrix multipliers of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1

$P_1 = 8 \times 1$ matrix of the terms on L H S and

$X_1 = 8 \times 1$ matrix of values of $K_1, a_1, b_1, c_1, d_1, e_1, f_1$ and g_1

Then, The matrix obtained is given by,

$$Z_1 \times \begin{bmatrix} 1 \\ A \\ B \\ C \\ D \\ E \\ F \\ G \end{bmatrix} = \begin{bmatrix} n & A & B & C & D & E & F & G \\ A & A^2 & BA & CA & DA & EA & FA & GA \\ B & AB & B^2 & CB & DB & EB & FB & GB \\ C & AC & BC & C^2 & DC & EC & FC & GC \\ D & AD & BD & CD & D^2 & ED & FD & GD \\ E & AE & BE & CE & DE & E^2 & FE & GE \\ F & AF & BF & CF & DF & EF & F^2 & GF \\ G & AG & BG & CG & DG & EG & FG & G^2 \end{bmatrix} \times \begin{bmatrix} K_1 \\ a_1 \\ b_1 \\ c_1 \\ d_1 \\ e_1 \\ f_1 \\ g_1 \end{bmatrix}$$

$$K_1 = 3.597 \times 10^{29} \quad a_1 = 45.898 \quad b_1 = -52.468 \quad c_1 = 0.080123 \\ d_1 = -11.161 \quad e_1 = -0.074352 \quad f_1 = 3.1647 \quad g_1 = 0.011455$$

Hence the model

$$Kga = 3.597 \times 10^{29} (\pi_1)^{45.898} (\pi_2)^{-52.468} (\pi_3)^{0.080123} (\pi_4)^{-11.161} \\ (\pi_5)^{-0.074352} (\pi_6)^{3.1647} (\pi_7)^{0.011455}$$

Similarly other model are worked out as

$$Cm = 2.76694 \times 10^{18} (\pi_1)^{69.988} (\pi_2)^{52.274} (\pi_3)^{-0.003} (\pi_4) \\ 0.388 (\pi_5)^{0.001} (\pi_6)^{6.174} (\pi_7)^{1.000}$$

$$Cs = 1.020939 \times 10^9 (\pi_1)^{104.05} \\ (\pi_2)^{61.813} (\pi_3)^{0.038466} (\pi_4)^{65.836} (\pi_5)^{-0.236} (\pi_6)^{7.302} (\pi_7)^{0.88782}$$

$$Ta = 2.1086 \times 10^{-71} (\pi_1)^{-42.654} \\ (\pi_2)^{-10.792} (\pi_3)^{-0.095902} (\pi_4)^{92.256} (\pi_5)^{-0.07582} (\pi_6)^{8.1917} (\pi_7)^{0.67123}$$

$$Sr / Ua = 0.99197 (\pi_1)^{2.9186} (\pi_2)^{0.5047} (\pi_3)^{-0.0039} (\pi_4)^{0.1107} (\pi_5) \\ 0.0034 (\pi_6)^{0.0695} (\pi_7)^{-0.0037}$$

$$Ti / Ua = 0.88307 (\pi_1)^{24.305} (\pi_2)^{-15.244} (\pi_3)^{-0.002} (\pi_4)^{46.614} (\pi_5) \\ 0.047 (\pi_6)^{0.801} (\pi_7)^{-0.007}$$

$$To / Ua = 0.89536 (\pi_1)^{26.526} (\pi_2)^{-10.083} (\pi_3)^{0.011} (\pi_4)^{43.541} (\pi_5) \\ 0.036 (\pi_6)^{0.991} (\pi_7)^{-0.006}$$

Thus corresponding to the seven dependent pi terms, seven models are formulated from the set of data for the response of kingpin angle, camber angle, caster angle, toe angle, toe in and toe out.

IV. ANALYSIS OF PERFORMANCE OF DATA

The models have been formulated mathematically. An approximate generalized experimental data based models are evolved for predicating the Steering Behavior [7].

This includes application of Dimensional Analysis is quite simple way in which a given test can be made compact in operating plan. In this experimentation we may not be able to recognize all the variables that influence a test, but we should realize that they and their dimensional equation have reality whether or not it is apparent.

The indices of the model are the indicators of how the phenomenon is getting affected because of the interaction of various independent pi terms in the models. The influence of indices of the various independent pi terms on each dependent pi term is discussed below.

The model for the dependent pi term π_{D1} is as under
 $\pi_{D1} = 3.597 \times 10^{29} (\pi_1)^{45.898} (\pi_2)^{-52.468} (\pi_3)^{0.080123} (\pi_4)^{-11.161} (\pi_5)^{-0.074352} (\pi_6)^{3.1647} (\pi_7)^{0.011455}$

It can be seen from the equation that this model of pi terms containing kingpin angle as response variable [6]. The following primary conclusions appear to be justified from the above model.

1. The absolute index of π_1 is highest viz. 45.898. Thus in π_1 the terms related to the front suspension link lengths of

suspension mechanism are the most influencing factors in this phenomenon. The value of this index is positive indicating π_{D1} is directly varying with respect to π_1 .

2. The absolute index of π_2 is lowest viz. -52.468, then π_2 term related to wheel diameter is the least influencing pi term in the model. The value of the index is negative indicating π_{D1} is inversely varying with respect to π_2 .

3. The sequence of influence of the other independent pi terms present in the model is $\pi_6, \pi_3, \pi_7, \pi_5, \pi_4$ having absolute indices 3.1647, 0.080123, 0.011455, -0.074352, -11.161 respectively. The index of π_5, π_4 negative indicating that π_{D1} inversely proportional with respect to π_5, π_4 .

4. The curve fitting constant in the model is 3.597×10^{29} . This value represents the effect of clearances and other factors which affect the phenomena.

TABLE IV: ERROR OBTAINED FROM EXPERIMENTAL CALCULATED VALUE AND MATHEMATICAL MODEL VALUE FOR KINGPIN ANGLE

Kingpin angle Experimentation	Kingpin angle Mathematical Model	Error calculated
6.3154	6.0180	0.2973
6.6566	6.4047	0.2519
6.8082	6.6689	0.1392
6.9931	7.1868	-0.1937
7.2404	7.3929	-0.1887

V. CONCLUSION

The models values of the dependent pi terms are computed. The observed and computed values of the dependent pi terms are compared by calculating their mean values. In order to check the accuracy of the predicted / computed values of the dependent pi terms, error is worked out (13)

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REFERENCES

- [1] T. D. Gillespie, "Fundamentals of Vehicle Dynamics", Society of Automotive Engineers. Inc Warrendale, PA – 1992.
- [2] Suh and Redcliff, "Kinematics Design of Mechanisms", John Wiley and Sons, New York, 1978.
- [3] M. Modak et al, "Electronic Measurements System for Torsionally Flexible Clutch", Vibro engineering '98', IFToMM Speciality Symposium University of Kanvaus, Lithuania.
- [4] Denavit and Hertenberg, "A Kinematic Notation for Lower Pair Mechanisms Based on Matrices", A.S.M.E. transaction, Journal of Applied Mechanics June 1955, PP 215-221.
- [5] H Sharek. Jr., "Theory of Engineering Experimentation", Mc Graw Hill, 1961, First Edition.
- [6] Prof. Dr. Georg Rill, "Vehicle Dynamics", Fachhochschule Regensburg, University of Applied Sciences, Hochschule for Technik Wirtschaft Soziales.
- [7] K. S. Fu, R. C. Gonzalez, C. C. G. Lee, "Robotics: Control Sensing, Vision and Intelligence", Mc Graw Hill Internation Edition, Singapore.

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