Abstract—The effect of constellation size on energy efficiency of Multi carrier Direct Sequence code-division multiple-access (MC DS/ CDMA) system for M-QAM modulation is studied using a game theoretic approach. A non cooperative game is proposed in which each user seeks to choose its transmit power, transmit symbol rate as well as the constellation size in order to maximize its own utility while satisfying its delay quality of service (QoS) constraint. The utility function used here measures the number of reliable bits transmitted per joule of energy consumed and it is particularly suitable for energy-constrained networks. It is shown that in order to maximize the user’s utility, a user must choose the lowest constellation size that can accommodate the user’s delay constraint. This strategy is different from one that could maximize spectral efficiency. Using this approach, the trade offs among energy efficiency, delay, throughput and constellation size are also studied and quantified.

I. INTRODUCTION

Code division multiple access (CDMA) is a multiple access technique where different users simultaneously access the same frequency band at the same time. Orthogonal frequency division multiplexing (OFDM) is a multicarrier technique that performs serial to parallel conversion of the data and modulates each sub-stream with a different subcarrier. To transmit data at higher rates as well as to accommodate more users with the available resource, a combination of OFDM and CDMA systems is used. This result in a powerful transmission technique called Multi-carrier Direct Sequence CDMA which has the benefits of both the time diversity gain of CDMA and the high data rates of OFDM systems. Multicarrier direct sequence CDMA is technique where the data streams is first serial to parallel converted, spread with a pre assigned code and then the same spreading stream modulates different subcarriers [1]. Due to the wide bandwidth occupied by the future wireless communication systems, the MC DS/CDMA with spreading in the time domain requires high chip rate. This provides an efficient approach to reduce the chip size. Wireless networks are expected to support a variety of applications with diverse QoS requirements. The goal is to use the network resources as efficiently as possible while providing the required QoS to the users. Game-theoretic approaches to power control have recently attracted considerable attention [2-7]. In [4] motivations for using game theory to study communication systems and in particular power control are provided.

In [2], an overview of multicarrier code division multiple access techniques is studied and in [3] the effect of orthogonal multicarrier techniques applied to CDMA. In [4], power control is modeled as a non-cooperative game in which users choose their transmit powers in order to maximize their utilities, where utility is defined as the ratio of throughput to transmit power. A game-theoretic approach to power control for multicarrier systems is studied in [8]. In [9] pricing is used to obtain a more efficient solution for the power control game. Similar approaches are taken in [10-13] for different utility functions. In this work we formulate power control as a non cooperative game where each user of the MC-DS/CDMA network chooses its strategy, which includes the choice of the transmit power, transmit symbol rate and the constellation size in order to maximize its own utility while satisfying its QoS constraints. In this work, we use a game-theoretic approach to study the effects of modulation on energy efficiency of MC-DS/CDMA networks.

II. MULTI CARRIER DS/CDMA

To transmit data at higher rates as well as to accommodate more users with the available resource, a combination of OFDM and CDMA systems is used. This result in a powerful transmission technique called Multi-carrier Direct Sequence CDMA which has the benefits of both the time diversity gain of CDMA and the high data rates of OFDM systems. The Multicarrier DS-CDMA transmitter spreads the S/P-converted data streams using a given spreading code in the time domain so that the resulting spectrum of each subcarrier can satisfy the orthogonality condition with the minimum frequency separation [14]. This scheme is originally proposed for a uplink communication channel. In [9], a multicarrier based DSCDMA scheme with a larger sub-carrier separation is proposed in order to yield both frequency diversity improvement and narrow band interference suppression. In addition a Multicarrier based DS-CDMA scheme, which transmits the same data using several sub-carriers, is proposed in [15-16].

III. NON COOPERATIVE GAME

A non-cooperative game is a one in which players can cooperate, but any cooperation must be self-enforcing. A game in which players can enforce contracts through third parties is a cooperative game. There are 3 basic concepts pertaining to non cooperative game which are as follows.
**Concept 1: Nash equilibrium:**

This is the major non-cooperative game concept where everyone plays their best move as a reply to the opponent's best move. When this is done, no player is benefited alone but as a group.

**Concept 2: Sub-game perfect equilibrium:**

This concept deals with reading the opponents moves and thinking forward but for making our moves, we must study and reason back.

**Concept 3: Expectations equilibrium:**

If we play our best move thinking that the opponent is playing his best, then this result implies that we are heading towards equilibrium and that we do not want to move. This type of equilibrium is called expectations equilibrium. Of all the above conditions, Nash equilibrium is the most sought after for wireless engineering and in our paper too, we make use of Nash equilibrium.

A non-cooperative power control game, in general, can be expressed as $G = \{K, \{A_k\}, \{u_k\}\}$ where $K = \{1, \ldots, K\}$ is the set of users/players, $A_k$ is the strategy set for the $k$th user, and $u_k$ is the utility function given by (3). Each user decides what strategy to choose from its strategy set in order to maximize its own utility. Hence, the best-response (i.e. utility maximizing) strategy of user $k$ is given by the solution of $\max_{a_k \in A_k} u_k = \max_{a_k \in A_k} R_k^*(f(\gamma_k)) P_k$. For this game, a Nash equilibrium is a set of strategies $(a_1^*, \ldots, a_K^*)$ such that no user can unilaterally improve its own utility, that is, $u_k(a_1^*, \ldots, a_k^*, \ldots, a_K^*) \geq u_k(a_1^*, a_k^*, \ldots, a_K^*)$ for all $a_k \in A_k$ and $k = 1, \ldots, K$.

**IV. PROPOSED GAME**

In this work, a non-cooperative games in which the actions open to each user are the choice of transmit power, and possibly transmit symbol rate, as well as the choice of constellation size, choosing which the user’s utility is maximized.

The utility function of a user as the ratio of its throughput to its transmit power is defined as, i.e.

$$u_k = T_k p_k. \quad (1)$$

This utility function is similar to the one used in [5] and [16]. Throughput in (1) is defined as the net number of information bits that are transmitted without error per unit time (it is sometimes referred to as goodput), and is expressed as

$$T_k = R_k f(\gamma_k) \quad (2)$$

where $R_k = b_j R_{j,k}$ is the transmission rate, $\gamma_k$ is the output signal-to-interference-plus-noise ratio (SIR) for user $k$, and $f(\gamma_k)$ is the “efficiency function” which represents the packet success rate (PSR) for the $k$th user. We require that $f(0) = 0$ to ensure that $u_k = 0$ when $p_k = 0$. In general, the efficiency function depends on the modulation, coding and packet size.

An automatic-repeat-request (ARQ) mechanism in which the user keeps retransmitting a packet until the packet is received at the access point without any errors has been assumed. Based on (1) and (2), the utility function for user $k$ can be written as

$$u_k = R_k f(\gamma_k) p_k. \quad (3)$$

This utility function, which has units of bits/joule, measures the number of reliable bits that are transmitted per joule of energy consumed, and is particularly suitable for energy constrained networks.

The efficiency function of the user for MC DS/CDMA is given by

$$f_\delta(\gamma) = ((1 - \alpha_b Q(\sqrt{\beta_\delta} \gamma))^{2L_b} - 2^{-\gamma})^M \quad (4)$$

**V. DELAY-CONSTRAINED GAME**

In this part, the effects of constellation size on delay. It is best for a user to use the lowest-order modulation for the utility function is studied [1]. The case in which the users have delay QoS requirements is considered. A game is considered such that each user seeks to maximize its own utility by choosing its transmit power, symbol rate and constellation size. This is done taking into account the required delay QoS constraint. By delay QoS constraint we take into account the delay to reach the receiver and also the packet’s waiting time in case of a queue.

The delay model considered is a M/G/1 model, where $M$ refers to the arrival rate and $G$ refers to the service rate. $M$ signifies that arrival rate, incoming packets, follows a poission distribution. $G$ signifies that service rate follows an arbitrary distribution. The model considered here is similar to the one considered in [1]. The received packets are assumed to be stored in a queue and transmitted in a first-in-first-out (FIFO) fashion. The packet success probability (per transmission) as before is represented by the efficiency function $f_b(\gamma)$. ARQ mechanism is followed where each packet is retransmitted until the packet is received without error. The packet transmission time takes into account the time taken to receive the acknowledgement. Since it is assumed to be small we neglect the time taken for receiving the acknowledgement. From [1]

$$W = \tau \left( \frac{1 - \lambda \tau}{f_b(\gamma) - \lambda \tau} \right) \quad \text{with } f_b(\gamma) > \lambda \tau \quad (5)$$

We specify the delay QoS constraint of a user by an upper bound on the average packet delay, i.e., we require

$$\bar{W} \leq D. \quad (6)$$

This delay constraint can equivalently be expressed as

$$f_b(\gamma) \geq \eta_b \quad (7)$$

where
Proposition 1: For a fixed $b$, the source rate $\lambda$ and the delay constraint $D$ are feasible if and only if

$$\frac{L\lambda}{bB} + \frac{L}{bR_s D} - \frac{L^2 \lambda}{2b^2R_s^2 D} < 1 \quad (14)$$

where $B$ is the system bandwidth.

Proof: If $\eta b < 1$, only then it is possible for the source rate and the delay constraint to be feasible and $\eta b$ is obtained from (8). From (8) it can also be seen that $\eta b$ is a decreasing function of $R_s$. The maximum value of $R_s$ is achieved at $R_s = B$. Hence the lowest possible value of $\eta b$ is achieved when $R_s = B$. Therefore (14) has to be satisfied so that the source rate $\lambda$ and the delay constraint $D$ are feasible.

The delay constraint however has a minimum value of transmission delay i.e. $D$ cannot be smaller than $\tau$. Hence, it can be shown that the condition $0 \leq \eta b < 1$ is equivalent to $R_s > \Omega^*/b$ where

$$\eta b = \frac{L\lambda}{bR_s} + \frac{L}{bR_s D} - \frac{L^2 \lambda}{2b^2R_s^2 D}. \quad (8)$$

Note that it is equivalent to the condition

$$\gamma \geq \gamma_b \quad (9)$$

Where

$$\gamma_b = f_b^{-1}(\eta b) \quad (10)$$

Based on (4), $\gamma_b$ is given by

$$\gamma_b = \frac{1}{\beta_b} \left[ Q^{-1} \left( \frac{1 - \eta b^2/2L}{\alpha_b} \right) \right]^2 \quad (11)$$

This means that the delay constraint in (6) translates into a lower bound on the output SIR. However the upper bound cannot be less than the time taken for transmission time, i.e., $D \geq \tau$, we must have that $bR_s \geq L/D$. So $L/(b^2R_s D)$ becomes smaller than $1/(bR_s D)$. Hence the negative term in (8) becomes smaller which implies that $\eta b > 0$. Since the efficiency function is limited as $0 \leq \eta b \leq 1$, (7) is possible only if $\eta b < 1$.

We propose a game in which each user chooses its transmit power and symbol rate as well as its constellation size in order to maximize its own utility while satisfying its delay requirement. Fixing the transmit powers and rates of other users, the best-response strategy for the user of interest is obtained from (8). From (8) it can also be seen that $\gamma_b = \gamma_b^*$, i.e.,

$$\Omega_b^* = \left( \frac{L}{D} \right) \frac{1 + D\lambda + \sqrt{1 + D^2\lambda^2}}{2b*} \quad (15)$$

where $f_b^* = f_b(\gamma_b^*)$

Utility is maximum when $\gamma_b = \gamma_b^*$ and this value of the SIR occurs at a symbol rate of $\Omega_b^*$.

Proposition 2: For given values of $\lambda$ and $D$, the best-response strategy for a user (i.e., the solution of (12)) is any combination of $p$ and $R_s$ such that

$$\min \left\{ \Omega_b^*/\tilde{b}, B \right\} \leq R_s \leq B \quad (16)$$

and

$$\gamma = \begin{cases} \hat{\gamma}_b^*, & \text{if } \Omega_b^*/\tilde{b} \leq E \\ \hat{\gamma}_b^*, & \text{if } \Omega_b^*/\tilde{b} > E \end{cases} \quad (17)$$

where $\tilde{b}$ is the lowest constellation size for which $\lambda$ and $D$ are feasible, $\gamma_b^*$ is the solution of $f_b(\gamma) = \gamma f_b(\gamma)$, and $\gamma_b^*$ is given by (10).

Since $\eta b$ represents the packet success probability, $\eta b \leq 2^{-\tilde{b}}$ would correspond to a very poor design. This can be seen from (4) and (7). In fact, in such a case, there would not be any need to transmit the packets since random guessing at the receiver would give a PSR of $2^{-\tilde{b}}$.

Proof: [Proposition 2] The utility is maximized when the user’s SIR is equal to $\gamma_b^*$ which is the solution of $f_b(\gamma) = \gamma f_b(\gamma)$. It is straightforward to show that $\gamma^*$ is a decreasing function of $R_s$ for all $R_s \geq \Omega^*/b$. Including the effect of constellation size, let us assume $b^*/b^*$ and consider the following cases.

- Since $b^*/b^*$, if we have $\Omega_b^*/b^* \leq B$, then this implies that $\Omega_b^*/b^* \leq B$. This means both $\gamma_b^*$ and $\gamma_b^*$ are feasible. However, it is evident that the user’s utility will drop in case the user moves to a higher-order modulation i.e. if the number of bits per symbol increases. The gain in utility which is because of an increase in $b$ is dominated by the increase in the optimum operating SIR Therefore, in this case, the user would choose the smallest $b$.

- If $\Omega_b^*/b^* \leq B$ but $\lambda$ and $D$ are feasible with $b^*$, then the user’s utility is maximized when the symbol rate is equal to $\lambda$ and the SIR is equal to $B$. On the other hand, the user can switch to $b^*/b^*$. In that case, $\gamma_b^*$ is smallest when the symbol rate is equal to $B$. However, with $R_s = B$ and $b^*/b^*$ we have $\gamma_b^* > \gamma_b^*$. Furthermore, the increase in $\gamma_b^*$ is exponential. Since the exponential increase in the SIR would dominate the linear increase in the rate caused by an increase in $b$, it is best for the user to use $b^*$ (i.e., the smaller constellation size).
• If $\Omega_k^*/b" > B$ and $\lambda$ and $D$ are not feasible, the user must switch to a higher constellation size and a similar argument as above would follow.

Therefore, the user must always choose the lowest constellation size for which the user’s QoS constraint can be satisfied.

Proposition 2 implies that, in terms of energy efficiency, choosing the lowest-order modulation (i.e., QPSK) is the best strategy unless the user’s delay constraint is too tight. In other words, the user would jump to a higher-order modulation only when it is transmitting at the highest symbol rate (i.e., $R_s = B$) and still cannot meet the delay requirement. Also, the proposition suggests that if

$$\Omega_k^*/b < B$$

the user has infinitely many best-response strategies. In particular, the user chooses the lowest constellation size that can accommodate the delay constraint. Then, for that constellation, any combination of $p_k$ and $R_s,k$ for which $\gamma_k = \gamma_k^*$ and $R_s,k \geq \Omega_k^*/b$ is a best response strategy.

At Nash equilibrium, the transmit powers, symbol rates and constellation sizes of all the users have to satisfy Proposition 2 simultaneously. There are, therefore, cases where we have infinitely many Nash equilibria. For a matched filter, for example, the best-response transmit power of user $k$ is given by

$$P_k = \frac{\sigma^2}{h_k} \left( \frac{\Phi_k}{1 - \sum_{j=1}^K \Phi_j} \right) \tag{19}$$

where

$$\Phi_k = \left( 1 + \frac{B}{R_s,k/\gamma_k} \right)^{-1} \tag{20}$$

and $\gamma_k$ and $R_s,k$ are determined according to Proposition 2 for $k = 1, \ldots, K$. We refer to $\Phi_k$ as “size” of user $k$. $\Phi_k$ is a measure of the amount of network resources that is consumed by the $k$th user. Note that $\gamma_k$ and $R_s,k$ are feasible if and only if

$$\sum_{j=1}^K \Phi_j < 1. \tag{21}$$

Utility of user $k$ at Nash equilibrium is given by

$$u_k = \frac{B f(\gamma_k) h_k}{\sigma^2 \gamma_k} \left( 1 - \sum_{j\neq k} \Phi_j \right) \tag{22}$$

Therefore, the Nash equilibrium with the smallest $R_s,k$ achieves the largest utility. A higher symbol rate (i.e., smaller processing gain) for a user requires a larger transmit power by that user to achieve the required SIR. This causes more interference for other users in the network and forces them to rise their transmit powers as well. As a result, the level of interference in the system increases and the users’ utilities decrease. This means that the Nash equilibrium given by (19) is the Pareto-dominant Nash equilibrium. As the delay constraint of a user becomes tighter, according to Proposition 2, the user will increase its symbol rate. This results in an increase in the user’s “size”. When the symbol rate becomes equal to the system bandwidth, the user will increase its SIR which again results in an increase in $\Phi$. Finally, when the user’s delay constraint is not feasible anymore, the user will switch to a higher constellation size. This results in an exponential increase in the required SIR, which dominates the linear decrease in the symbol rate. Therefore, $\Phi$ increases again. This shows that the user’s “size” increases as the delay requirement becomes more stringent. The feasibility condition given by (21) determines the maximum number of users that can be accommodated by the network. A tighter delay constraint results in a larger “size” for the user. This, in turn, results in a smaller network capacity.

VI. SIMULATION RESULTS

The effect of constellation size on energy efficiency of a user with a delay QoS constraint is quantified in this section. The effect of constellation size is studied taking into account the delay constraint as well. The number of subcarriers is assumed to be 4. The packet size is assumed to be 100 bits, and the source rate (in bps) for the user is assumed to be equal to 0.01$B$ where $B$ is the system bandwidth. We further assume that a user chooses its constellation size, symbol rate and transmit power according to its best-response strategy corresponding to the Pareto-dominant Nash equilibrium. For the coded system, we assume an 8-state convolutional encoder with rate 2/3. The code rate for QPSK is chosen to be 1/2.

The optimum constellation size, transmit power, throughput, and user’s utility is shown as a function of the delay constraint for both uncoded and coded systems. The packet delay is normalized by the inverse of the system bandwidth. To keep the spectral efficiency of the two systems the same, we assume that the number of information bits transmitted per symbol is the same for both uncoded and coded systems. The throughput corresponds to the transmission rate for the user which is obtained by multiplying the symbol rate by the number of (information) bits per symbol (i.e., $b$), and is normalized by the system bandwidth. The transmit power and user’s utility are also normalized by $h^*$ and $Bh^*$, respectively. Let us for now focus on the uncoded system. When the delay constraint is large, QPSK (which is the most energy efficient M-QAM modulation) can accommodate the delay requirement and hence is chosen by the user. As the delay constraint becomes tighter, the user increases its symbol rate and also rises the transmit power. In this case, the user’s utility stays constant. Once the symbol rate becomes equal to the system bandwidth, the user cannot increase it anymore. Hence, as the delay constraint becomes more stringent, the user is forced to aim for a higher target SIR to meet its delay requirement. In this case, the transmission rate stays constant and the transmit power increases.
Figs. 1 and 2 show the efficiency function and the normalized user utility without the effect of coding as a function of the SIR for different choices of \( b \). It is observed that the user’s utility is maximized when \( b = 2 \) (i.e., QPSK modulation). This is because, as \( b \) increases, the linear increase in the throughput is dominated by the exponential increase in the required transmit power (which results from the exponential increase in SIR). Therefore, it is best for a user to use QPSK modulation. Fig 3 shows the variation of coding gain as a function of SIR. The BER curves are used to estimate the coding gain as a function of SIR for various values of constellation sizes.

\[
G_b(\gamma) = A_b + C_b \tan^{-1}((\gamma - \gamma_b)/D_b)
\]

The above function gives a reasonable estimate of coding gain. \( A_b, C_b, \gamma_b, D_b \) are constants and their values can be determined by trial and error method.

VII. CONCLUSION

In this work we have studied the effects of modulation order on energy efficiency of wireless networks (MC DS CDMA) using a game-theoretic approach. A non-cooperative game has been proposed focusing on M-QAM modulation in which each user chooses its strategy in order to maximize its utility while satisfying its delay QoS constraint. The actions open to the users are the choice of the transmit power, transmit symbol rate and constellation size. The utility function measures the number of reliable bits transmitted per joule of energy consumed and is particularly suitable for energy-constrained networks. The best-response strategies and the Nash equilibrium solution for the proposed game have been derived. It is shown that to maximize its utility (i.e., energy efficiency), the user must choose the lowest modulation level that can accommodate the user’s delay constraint. Using our non-cooperative game-theoretic framework, the tradeoffs among energy efficiency, delay and constellation size have also been
studied and quantified.

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