

A Hybrid Simulated Annealing Algorithm for Mechanism Synthesis with N-Accuracy Points

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Abstract—This paper presents the results of a newly developed hybrid Simulated Annealing (HSA) for the optimization of mechanism for function generation problem. A hybrid optimization method based on the fusion of the Simulated annealing (SA) and Marquardt Search (MS), gradient search based method nonlinear regression algorithm, in which the SA is embedded the MS to enhance its search capability. The paper describes a new hybrid optimization method that combines advantages of both global and local search. A brief overview of HSA is presented and applied to dimensional synthesis of a planer four bar mechanism. The optimization is carried out to minimize the objective function formulated from the structural error at the accuracy points. It is employed to determine the optimal values for the design variables that best satisfy the desired objectives of the problem. Simulation results demonstrate the remarkable advantages of our approach in achieving the diverse optimal solutions and improved converge speed. The applicability of HSA algorithm is illustrated by solving two nonlinear function generation problems. It produces accurate and acceptable solution in all cases.

Index Terms—Function generation, Simulated annealing, Marquardt search, Hybrid algorithm, and four bar mechanism.

I. INTRODUCTION

In the past, a number of different techniques have been employed for the synthesis of mechanisms [1-2]. In the traditional approaches, the solution method is based on the graphical, and /or analytical design methods. Later with the proliferation of high speed computers and their integration into mechanism analysis and synthesis, a wide variety of numerical optimization methods have been developed for the synthesis of mechanisms. The graphical methods can provide a quick and easy method of design. But this approach has accuracy limitations. The analytical methods in practice today are mostly based on algebraic methods [3], displacement matrix method [4] or complex number methods [5]. In this approaches, the mechanism synthesis problem (MSP) is solved and carried out to satisfy accuracy points exactly. A major drawback in using the analytical methods is that there could be significant error in the overall output between the precision points, branching and incorrect sequence of accuracy points and impractical for design and optimization of complex mechanism.

Mechanism involving a finite number of links posses an inherent error and it is the task of the designer to reduce this error to a sufficiently low value. Many numerical optimizations [6, 7] are available at present for design optimization of engineering problems to find optimum design.

Solving MSP can be complex and a time consuming process when there are large numbers of design variables and constraints. Hence there is a need for efficient and

reliable algorithms that solves such problems. It is known that no algorithm can surely find the absolute minimum in a polynomial time with number of variables; some very successful heuristic algorithms have been developed. Amongst these, the SA, method of Kirkpatrick has proven to very successful in a broad class of situations. SA is a stochastic heuristic algorithm in which the solutions are searched in hill climbing process constantly commenced by random moves. Because of its ease use, SA is an extremely popular method for solving large-sized and practical problems. However, for various reasons, like many other search algorithms, SA may become trapped by any local minima, which doesn't allows moving up or down, or take a long time to find a reasonable solution, which sometimes makes the method unreferrable. For this reason, many SA implementations have been done as a part of a hybrid method [8-11].

In this paper, SA has been developed and later on hybridizing externally with MS, the local search provides the fastest solutions at the end of each generation. The Four bar function generator was optimally optimized using SA and also using HSA. On getting satisfactory results from SA, MS is activated and carried out local search to gain accuracy i.e. when SA procedure is in progress, a list of twenty best points is maintained and constantly updated, iff a new point is randomly created. At the termination of SA algorithm, these points are fed to MS algorithm as the starting points for the local search and new list of best points is created.

II. SIMULATED ANNEALING (SA)

A. Fundamental Concept:

SA [12, 13] is a generic probabilistic Meta-algorithm for the global optimization problem, namely locating a good approximation to the global optimum of a given function in a large search space. SA is based on an analogy to the cooling of heated metals. In any heated metal sample the probability of some cluster of atoms at a position, (r_i), exhibiting a specific energy, $E(r_i)$, at some temperature T , is defined by the Boltzmann Probability factor:

$$P(E(r_i)) = e^{-[E(r_i)/k_B T]} \dots \dots \dots (1)$$

Where k_B is Boltzmann's constant. As a metal is slowly cooled, atoms will fluctuate between relatively higher and lower energy levels and allowed to equilibrate at each temperature T . The material will approach a ground state, a highly ordered form in which there is a very little probability for the existence of a high energy state throughout the material. If the energy function of this physical system is replaced by an objective function, $f(X)$, that is dependent on a vector of design variables, X , then the

slow progression towards an ordered ground state is representative of a progression to a global optimum. To achieve this, a control temperature T , analogues to a temperature, and a constant C , analogues to k_B , must be specified for the optimization problem.

In standard iterative improvement methods, a series of trial points is generated until an improvement in the objective function is noted in which case the trial point is accepted. However, this process only allows for downhill movements to be made over the domain. In order to generate the annealing behavior, a secondary criterion is added to the process. If a trial point generates a large value of objective function then the probability of accepting this trial point is determined using the Boltzmann Probability distribution:

$$P[\text{accept } X_t] = e^{-[f(x_t) - f(x_0)]/CT} \dots \dots \dots (2)$$

Where x_0 is the initial starting point. This probability is compared against a randomly generated number over the range [0 1]. If $P[\text{accept } X_t] \geq \text{random}[0....1]$ then the trial point is accepted. This dependence on random numbers makes a SA a stochastic method.

B. SA Algorithm

The algorithm proceeds as follows:

Step1: Starting from the initial point x_0 , the algorithm generates successively improved points x_1, x_2, \dots moving towards the global minimum solutions. The initial value of a control parameter (T) is suitably high and a methodology for decrementing (T) is applied.

Step2: A sequence of design vector is then generated until equilibrium is reached.

Step3: During this phase the step vector (S) is adjusted periodically to better follow the function behavior. The best point is recorded as **Xopt**.

Step4: Once thermal equilibrium is reached, the temperature (T) is reduced and a new sequence of moves is made starting from **Xopt**. Until thermal equilibrium is reached again.

Step5: This process is continued until a sufficiently low temperature is reduced, at which stage no more improvement in the objective function value can be expected.

III. MARQUARDT SEARCH (MS)

MS [6, 12] is one of the concept of non linear optimization. The goal of MS is to search for the minimum of a nonlinear object function. In this search, Steepest Descent (SD) is initially followed. Thereafter Newton's Search (NS) is adopted.

SD is the most straightforward method in optimization. By computing the gradient direction followed by a 1D search, SD iteratively approaches the minimum point of the object function in parameter space. Mathematically, SD can be expressed as follows:

$$x^{(k+1)} = x^{(k)} - \lambda^{(k)} \nabla f(x^{(k)}) \dots \dots \dots (3)$$

Since only the first order derivative information is used, SD suffers from the slow convergence. However, it is relatively robust even initial guess is far away from the true value. NS goes one step further than SD: in the Taylor's

expansion of the object function at the current point, the second order derivative term is now included to compute for the update.

$$x^{(k+1)} = x^{(k)} - \lambda^{(k)} [H^{(k)}]^{-1} \nabla f(x^{(k)}) \dots \dots \dots (4)$$

Where $H^{(k)}$ is the Hessian matrix of function $f(x)$ at $x^{(k)}$ denoting the second order derivative. NS converges faster than SD. The price to pay is the reduction in robustness, i.e. it is much more sensitive to initial guess than SD. Another drawback is the requirement of computing $H^{(k)}$ which could be a big issue in many applications where the analytical form of $f(x)$ is not available. It is usually not known whether the chosen initial point is away from the minimum or close to **Xopt**, but wherever be **Xopt**, MS attempts to take advantage of both the SD and NS.

The gradient - based methods work on the principle of generating new search directions iteratively and performing an unidirectional search along each direction. These methods make use of the gradient vector in finding the search direction. MS modifies the diagonal elements of the Hessian matrix, $H(k)$, as

$$\hat{H}^{(k)} = H^{(k)} + \lambda^{(k)} I \dots \dots \dots (5)$$

Where I is an identity matrix and $\lambda^{(k)}$ is a positive constant that ensure the positive definiteness of $\hat{H}^{(k)}$ when $H^{(k)}$ is not positive definite. It can be noted that when $\lambda^{(k)}$ is sufficiently large (on the order of 10^4), the term $\lambda^{(k)} I$ dominates $H^{(k)}$ and the inverse of the matrix $H^{(k)}$ becomes

$$[\hat{H}^{(k)}]^{-1} = [H^{(k)} + \lambda^{(k)} I]^{-1} \approx [\lambda^{(k)} I]^{-1} = 1/\lambda^{(k)} I \dots \dots \dots (6)$$

Thus if the search Direction $S^{(k)}$ is computed as

$$S^{(k)} = -[\hat{H}^{(k)}]^{-1} \nabla f(x^{(k)}) \dots \dots \dots (7)$$

$S^{(k)}$ becomes a steepest descent direction for large values of $\lambda^{(k)}$

In MS, the value of $\lambda^{(k)}$ is taken to be large at the beginning and then reduced to zero gradually as the iterative process progress. Thus, as the value of $\lambda^{(k)}$ decreases from a large value to zero, the characteristics of the search method change from those of a steepest descent method to those of the Newton method.

A. MS Algorithm

The algorithm proceeds as follows:

Step1: Select the best point vector from SA, **Xopt**, as starting point, the maximum number of iterations, M , and a termination parameter (ϵ). Set $k=0$ and $\lambda^{(0)}=10^4$.

Step2: Calculate $\nabla f(x^{(k)})$.

Step3: If $\|\nabla f(x^{(k)})\| \leq \epsilon$ or $k \geq M$? Terminate;

Else go to Step4

Step4: Calculate $S^{(k)} = -[\hat{H}^{(k)}]^{-1} \nabla f(x^{(k)})$.
 Set $x^{(k+1)} = x^{(k)} + S^{(k)}$.

Step5: Is $f(x^{(k+1)}) < f(x^{(k)})$? If yes, go to step6;
 Else go to step 7

Step6: Set $\lambda^{(k+1)} = \lambda^{(k)}/2$, $k=k+1$, and go to step2

Step7: Set $\lambda^{(k)} = 2\lambda^{(k)}$, and go to step4.

IV. HYBRID OPTIMIZATION ALGORITHM:

Any one algorithm may not be sufficient to provide a satisfactory solution for the given problem at hand.

In recent years there has been a great deal of interest in the development of optimization algorithms that deal with the problem of finding a global minimum of a given continuous function. These algorithms were innovated to confront the rapid growth of many optimization problems in engineering.

Meta-heuristics methods are considered to be acceptably good solvers of this problem. The power of meta-heuristic methods come from the fact they are robust and can deal successfully with a wide range of problem areas.

However, these methods, especially when they are applied to complex problems, suffer from the high computational cost due to their slow convergence. The main reason for this slow convergence is that these methods explore the global search space by creating random movements without using much local information about promising search direction. In contrast, local search methods have faster convergence due to their using local information to determine the most promising search direction by creating logical movements. The local search methods can easily be entrapped in local minima.

One approach that recently has drawn much attention is to combine meta-heuristic methods with local search methods to design more efficient methods with relatively faster convergence than the pure meta-heuristic methods. Moreover, these hybrid methods are not easily entrapped in local minima because they still maintain the merits of the meta-heuristic methods.

The reason for studying and developing a hybrid algorithm can be summarized as follows:

1. To improve the performance of the Non-traditional search computation (NTSC).

(Example: fast convergence speed and robustness) i.e. SA method usually suffers from slow convergence due its random nature of movements. Moreover, it also suffers from the difficulty in obtaining some required accuracy although it may quickly approach the neighborhood of the global minimum.

Hence we have focused on the importance of creating the hybrid approach and importance of accelerating the final stage of SA by using the faster convergent method.

2. To improve the quality of the solutions obtained by the NTSC.

3. It achieves the searching not only globally but also locally.

4. To prevent the premature convergence.

In order to prevent the premature convergence, the coupling of SA and local search methods, to form hybrid SA can be advantageous.

5. It searches larger regions of the solution space effectively using both SA-based global search capability with traditional search-based local search feature.

6. The goal of developing a Hybrid simulated Annealing approach is to use this algorithm to help in solving the problem of optimal synthesis of a mechanism for the function generation problem.

Achieving this goal will help in providing the best set of

design variables, reducing the structural error and satisfying number of design constraints.

7. Modern meta-heuristics manage to combine exploration and exploitation search.

The exploration search seeks for new regions, and once it finds a good region, the exploitation search kicks in. However, since the two strategies are usually inter-wound, the search may be conducted to other regions before it reaches the local optima. As a result, many researchers suggest employing a hybrid strategy, which embeds a local optimizer in between the iterations of the meta-heuristics. The diagram of HSA scheme is shown in Fig.1

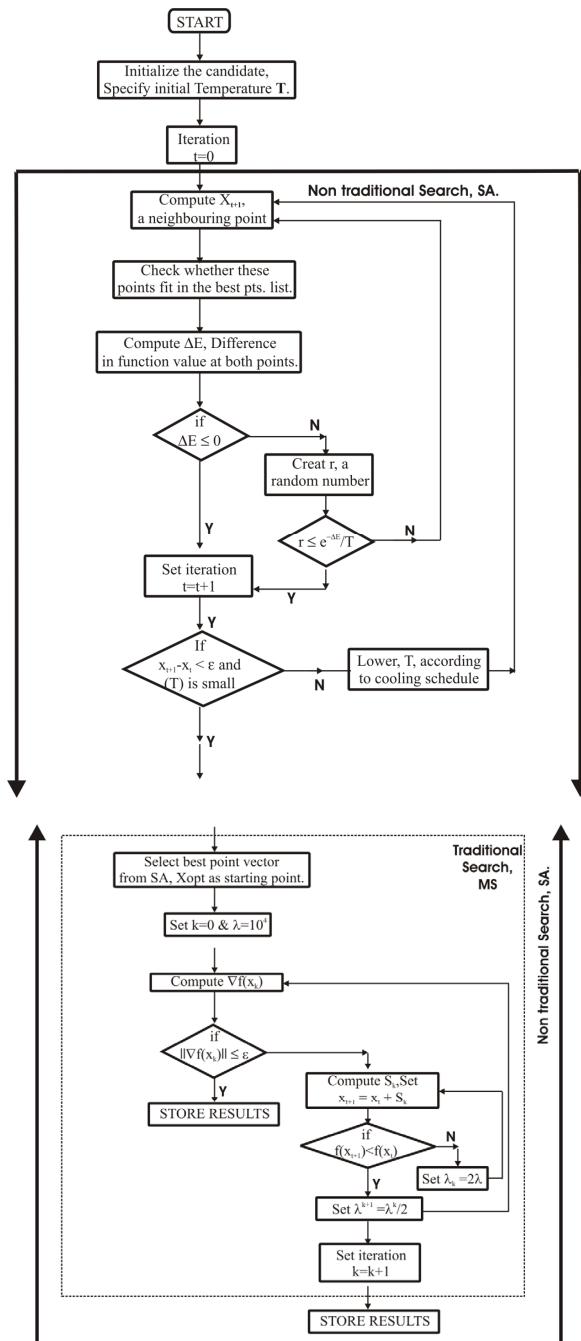


Fig. 5.2 flow chart of SAG

Fig. 1 : Flow Chart of HSA

In this section, we developed a hybrid optimization algorithm based on the principles of both the aforementioned SA and MS. The SA method occasionally

24	20.5	23.3	-2.8	58.1	53.0	5.1
36	21.1	22.3	-1.2(min.)	61.3	57.7	3.6
48	23.5	18.8	4.7	64.5	59.7	4.8
60	23.9	19.6	4.3	67.6	63.5	4.1
72	23.4	19.8	3.6	70.8	74.7	-3.9
84	24.3	20.2	4.1	74.0	71.5	2.5
96	25.2	20.9	4.3	77.3	72.2	4.9
108	26.2	30.4	-4.2	80.5	86.1	-5.6(max.)
120	27.3	30.9	-3.6	83.8	89.1	-5.3
132	28.4	32.7	-4.3	87.0	82.3	4.7
144	29.5	24.7	4.8(max.)	90.3	88.0	2.3(min.)
156	30.8	26.7	4.1	93.7	97.0	-3.3
168	32.1	35.8	-3.7	97.0	92.4	4.6
180	33.4	31.8	1.6	100. 4	96.7	3.6

TABLE4 OBJECTIVE FUNCTION (RAD²), MAXIMUM AND MINIMUM ERROR (DEGREES) FOR 20 BEST POINTS.

Best Points	FunType1			FunType2		
	Objecti ve Functi on	Maxim um error	Minim um error	Objecti ve Functi on	Maxim um error	Minim um error
1	0.0036 4	4.8	-1.2	0.0177	-5.6	2.3
2	0.0039 6	4.8	1.6	0.0397	6.1	2.8
3	0.0039 8	4.5	1.6	0.0460	6.3	2.3
4	0.0046 3	-5.3	-1.3	0.0471	6.3	-2.6
5	0.0048 9	5.0	-1.8	0.0487	5.9	2.6
6	0.0051 5	5.6	1.6	0.0491	-5.2	-2.5
7	0.0054 1	4.6	1.7	0.0496	5.2	-2.6
8	0.0054 3	4.6	-1.8	0.0533	6.7	2.6
9	0.0054 9	-4.8	1.8	0.0541	-5.9	3.3
10	0.0055 6	5.2	1.8	0.0554	-5.9	3.3
11	0.0055 9	5.4	-1.4	0.0570	5.9	-3.6
12	0.0056 3	5.4	-1.8	0.0588	-5.6	-3.2
13	0.0057 7	5.3	1.6	0.0593	5.6	-3.1
14	0.0059 2	5.4	1.6	0.0598	5.3	-3.1

15	0.0063 0	4.8	1.6	0.0615	5.8	-3.1
16	0.0064 2	4.6	-2.1	0.0621	5.8	3.5
17	0.0064 7	4.1	1.9	0.0649	-5.8	3.5
18	0.0065 5	-4.6	1.8	0.0651	5.7	3.5
19	0.0065 8	4.8	1.8	0.0666	-5.5	-2.6
20	0.0067 9	4.8	1.8	0.0675	5.8	2.8

B. Results obtained by Hybrid iteration (HSA)

TABLE 6 STRUCTURAL ERROR ($\Phi_{\text{EXP}} - \Phi_{\text{GEN}}$) IN DEGREES FOR THE BEST SOLUTION OUT OF 20 BEST POINTS.

Θ	FunType1			FunType2		
	Φ_{EXP}	Φ_{GEN}	$(\Phi_{\text{EXP}} - \Phi_{\text{GEN}})$	Φ_{EXP}	Φ_{GEN}	$(\Phi_{\text{EXP}} - \Phi_{\text{GEN}})$
12	20.2	22.7	-2.59(max.)	58.1	61.2	-3.1
24	20.7	21.9	-1.2	61.2	57.9	3.3(max.)
36	21.3	21.6	-0.3(min.)	64.4	62.7	1.7
48	21.9	21.6	0.3	67.6	65.3	2.3
60	22.6	21.9	0.7	70.7	68.6	2.1
72	23.3	22.4	0.9	74.0	75.8	-1.8
84	24.0	23.2	0.8	77.2	76.7	0.5(min.)
96	24.8	24.0	0.8	80.4	77.9	2.5
108	25.6	25.0	0.6	83.7	86.3	-2.6
120	26.4	26.0	0.4	87.0	90.4	-3.4
132	27.3	27.0	0.3	90.0	87.1	2.9
144	28.3	27.9	0.4	93.0	92.3	0.7
156	29.3	28.9	0.4	96.0	97.3	-1.3
168	30.3	29.7	0.6	101. 3	98.5	2.8
180	31.4	30.1	1.3	103. 6	102. 1	1.5

TABLE7 OBJECTIVE FUNCTION (RAD²), MAXIMUM AND MINIMUM ERROR (DEGREES) FOR 20 BEST SOLUTIONS

Best Poi	FunType1			FunType2		
	Objecti ve	Maxim um	Minimu m error	Objecti ve	Maxim um error	Minimu m error
1	0.0021 3	2.3	-0.3	0.0172	3.3	0.5
2	0.0038 6	2.3	0.5	0.0396	3.8	-0.8
3	0.0026 3	2.8	0.5	0.0393	-3.6	0.8
4	0.0033 2	-3.3	-0.7	0.0305	3.6	-0.8

5	0.00489	-3.1	-0.7	0.0339	3.8	-0.5
6	0.00232	3.3	0.7	0.0215	-3.8	-0.6
7	0.00336	-2.5	0.3	0.0434	-3.8	-0.8
8	0.00258	2.3	-0.8	0.0399	3.4	0.8
9	0.00448	-2.8	0.7	0.0326	3.3	0.6
10	0.00283	3.5	0.7	0.0347	4.1	-1.3
11	0.00301	3.4	-0.6	0.0332	3.8	-1.9
12	0.00553	3.4	-0.5	0.0588	3.6	-0.9
13	0.00549	3.2	0.5	0.0506	3.7	-1.6
14	0.00592	3.3	0.3	0.0448	3.5	-1.6
15	0.00382	-2.7	0.3	0.0297	3.5	-2.1
16	0.00598	2.3	0.8	0.0361	-3.8	1.7
17	0.00383	-2.1	0.6	0.0536	3.8	0.8
18	0.00560	-2.6	0.6	0.0348	3.8	0.8
19	0.00518	2.6	0.6	0.0666	3.5	-1.6
	0.00332	2.8	0.5	0.0622	3.5	20

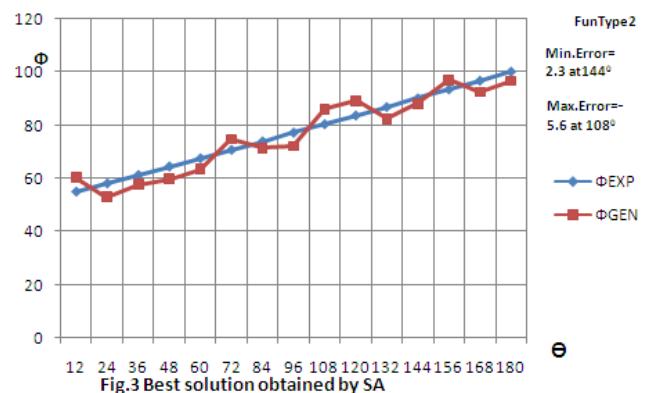


Fig.3 Best solution obtained by SA for FunType2

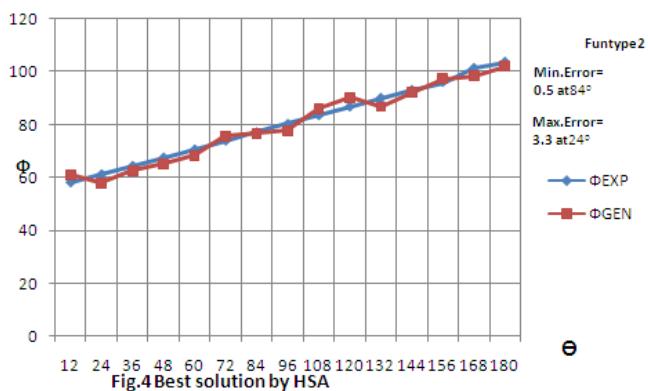


Fig.4 Best solution obtained by HSA for FunType2

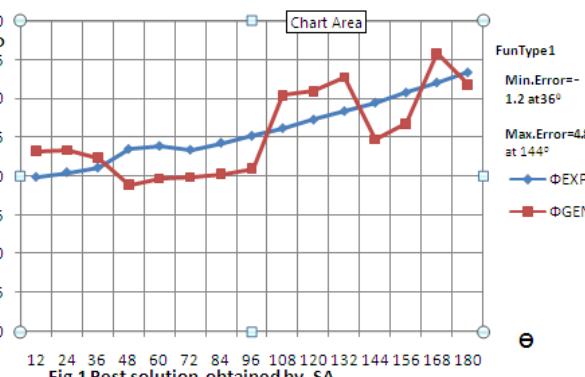


Fig.1 Best solution obtained by SA for FunType1

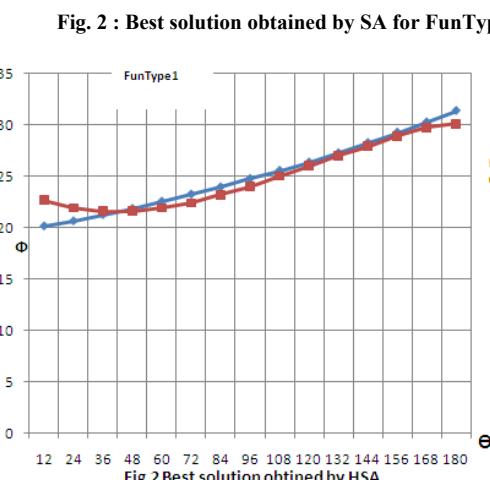


Fig.2 Best solution obtained by HSA for FunType1

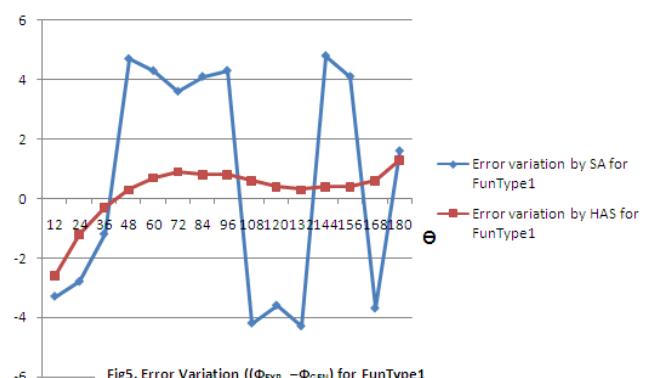


Fig.5. Error Variation ($\Phi_{\text{EXP}} - \Phi_{\text{GEN}}$) for FunType1

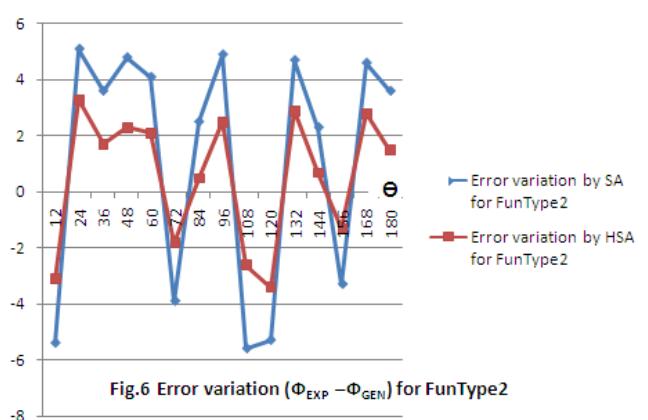


Fig.6 Error variation ($\Phi_{\text{EXP}} - \Phi_{\text{GEN}}$) for FunType2

Fig.3 Best solution obtained by HSA for FunType2

VII. CONCLUDING REMARKS:

The paper may be concluded with the following observations:

1. A hybrid optimization method based on the fusion of the SA and MS is developed and applied on MSP. A newly developed HSA is effective and fast in the solution of MSP. It also shows closer convergence properties.
2. The results are encouraging and suggest that HSA can be used effectively and efficiently in other complex and realistic design often encountered in engineering applications.
3. It can conclude from computer simulation results (Fig.3 and Fig5.) that Minimum and Maximum error is greatly reduced by Hybrid iteration (HSA). Therefore HSA is an effective tool for MSP.

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TABLE 5 BEST DESIGN VARIABLES OUT OF 20 BEST POINTS.

Best Points	X1	X2	X3	X4	X5	X6	X7
FunType1	3.9890	3.3502	3.5578	6.7595	4.5895	0.3569	0.2600
FunType2	3.6442	2.5728	3.5581	11.7742	4.6921	0.2682	0.7101