

Electronic commerce network modeling of randomly broken multi-channel servers with human factor

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Abstract — The electronic commerce (e-commerce) has been a big issue not only in the technology but also in the business. This paper presents the stochastic management methodology to determine the decision points in the electronic commerce setting that focused on network management methodologies. The unreliable network architecture is considered with human behaviors. This network architecture falls into the category of multi-channel queues with unreliable repair facilities and be more realistic than basic model. The *shopping time* (or *human factor*) that spent by the customer who wants to buy something online is considered. Analytic tractable results are obtained by using various mathematical techniques and demonstrated in the framework of optimized network server allocation with human factor in e-commerce ecosystem.

I. INTRODUCTION

Electronic commerce consists of the buying and selling of products or services over electronic systems such as the Internet (mostly World Wide Web) and other computer networks [5]. It has been one of major topics in modern IT (Information Technology) industries. Current IT providers build the network architecture for e-commerce by using multi-server oriented because of salability and flexibility issues. A single-server system is very difficult and expensive to upgrade or repair servers for further customer dimensions. For instant, all of business must be stopped if the server in a single-server system is broken. Contrarily, we do not need to stop all of our business to repair broken servers in the multi-server system because the multi-server system can keep doing their jobs even during upgrading servers. Even though this theoretical model is complicated, it is very practical to design the network management system. In addition, another uncertain factor is added to make the current theoretical model even more practical but the system is getting very complicated. The following assumptions are given for compromising the system complexity.

- (1) All network servers hooked to the Internet that supports the web-based shopping transactions.
- (2) These servers are randomly broken with certain distributions.
- (3) Each server hook up open transaction access channels for the customer and the number of channels is equal for all servers.

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- (4) Total service time of each channel includes *human factor*.
- (5) This system has the deterministic *checkout* process .

The last assumption (5) is the key factor to adopt the human factor into the equation. The checkout process that the network server possesses all customer information and the checkout time is relatively fixed but the checkout spend depends on how many channels are opened and available by the network server. In the other hand, the "shopping period" consists of the sum of the periods between random moments of choosing items and each on of these periods is exponentially distributed. More over, the count of the event, which has to do with choosing items, is yet another random phenomenon. This implies that the "shopping times" is not only a random process within the random event of the entire shopping transaction but also the process that includes the time spent of customers on the each network server. Several mathematical techniques and stochastic optimization methods are applied to solve this system. All these functions need to be evaluated to improve their performances such as service time of servers, waiting times of customers and so on. One of practical dimension of this modeling, which is called "*human factor*". This has triggered interest on part of researches to model network servers and evaluate their performances. A heavy practical dimension to this modeling, effort, has been ignored. And that is inclusion of customer shopping time. The research attempts to model customer shopping time as part of the network server performance evaluation. This paper also attempts to the model with unreliable multi-server as a more practical model. It means that any network server can be broken any time. These assumptions make my model even more realistic.

II. SIMPLIFIED MATHEMATICAL DESIGN OF HUMAN FACTOR

In this section, the enhanced service time that represents human factor in the same customers influence the total service time is considered. Customers access the single network server. The service time begins as a customer connects to one of available channels until the checkout is done. We will calculate the service time component that represents only the shopping time. Typically, a customer takes a look at the merchandise displayed on the net, clicks on the item of invest until he/she decides to checkout. The residual time between choosing items by a customer is exponentially distributed because the action for choosing during shopping are practically not related with the previous

actions. And it continues until the checkout is completed. When the shopping portion finishes, the "checkout process" begins for dealing with authorization, credit card validation and so on. Since the checkout process is more mechanical than the shopping process, This process can be reasonably deterministic in nature a fixed within amount of time. However, this period depends on the availability of channels in the network server ($\rho(m)$). This process takes more time when more channels are available (i.e., $\rho(m) \propto m$) since the channels share their same resource in each server because of the performance limitations.

Let g_k be the k -th period between choosing items assumed to follow exponential distribution with parameter μ and iid (independently identically distributed). Let Λ be the random transaction per customer with a geometric distribution and parameter p . Then the total shopping time by a customer (G) and total service time of the network server (S) are as follows :

$$G = g_1 + g_2 + \dots + g_\Lambda, S = G + \rho(m). \quad (2.1)$$

Since a past event (e.g., previous shopping) does not any effect on a present event (such as new shopping, surfing or inquiry), we can reasonably assume g_k to be exponential because of its memoryless properties [8]. The transaction event Λ has the geometric characteristic because it is same as independent trial until first success [10]. Let $s(t)$ be the distribution of the total service time as follows:

$$s(t) = P\{S \leq t\} = P\{G \leq t - \rho(m)\}. \quad (2.2)$$

The distribution of the total service time G needs to be determined and it is the main target of this session. The moment generating function (mgf) [10] is considered to find the distribution of the G . Since $g_k, k = 1, \dots, \Lambda$ are exponentially distributed ($\frac{1}{\mu} = \mathbb{E}[g_1] < \infty$) and Λ has the geometric distribution with the parameter p :

$$\mathbb{E}\left[\left(\frac{\mu}{\mu - \theta}\right)^\Lambda\right] = \frac{p\mu}{p\mu - \theta}. \quad (2.3)$$

According to (2.2), the random variable G is exponentially distributed with parameter $p\mu$ and the distribution function of the total service time is:

$$s(t) = 1 - e^{-p\mu(t-\rho(m))} \quad (2.4)$$

where $t \geq \rho(m)$. This simplified modelling approach approximately explain the human factors such as preferences of customers, statistical data of merchandise. In addition, the analyzing the shopping time is one of practical factors for the online shopping. The further section will give the analysis of the network architecture for e-commerce by using the multi-channel queueing models.

III. MODIFIED MULTI-CHANNEL QUEUEING SYSTEM WITH RANDOMLY BROKEN CHANNELS

The modification of conventional $GI/M/m/0$ (multi-channel) queueing system is considered. Recall that such a system is characterized by the general-independent input (i.e. a renewal process), m parallel channels without buffer in the system [1]. Even though we start with the service time of this system to be general, we will end up with the exponential

distribution with the parameter $p\mu$ and shifting the constant $\rho(m)$, i.e., the PDF (Probability Distribution Function) of the service time S is $s(t)$. This applicable queueing type becomes $GI/\tilde{M}/\tilde{m}/0$ type instead of the $GI/M/m/0$ type because the total number of the system is the combination of the number of channels (m) and the random number of servers (ξ_t):

$$\tilde{m} = m \cdot \xi_t. \quad (3.1)$$

And inter-arrival times are distributed in accordance with the PDF $A(x)$ (3.2). The latter is a classical system investigated by Takacs [11] and extended by some other queueing experts [4, 7]. Let ξ_t is the random number of servers at time t which serve customers in the system and $\xi_n := \xi_{\tau_n-}, n = 0, 1, \dots$ are iid and the stationary probabilities are exist:

$$q_i := \lim_{t \rightarrow \infty} q(i, t), i = 0, 1, \dots \quad (3.2)$$

where $q(i, t) = P\{\xi_t = i\}$. Denote the number of customers in the system at time t by Q_t . $Q_n := Q_{\tau_n-}, n = 0, 1, \dots$ are the number of customers immediately before the arrival of the n -th customer. Let $\tau_0 (= 0), \tau_1, \dots$ be the inter-arrival time which is sequences of inter-arrival moments of customers and these sequences are iid (identically independently distributed) with a given number of servers (i).

$$(\Omega, \mathcal{U}, (P^x)_{x \in E}, (Q_t; t \geq 0)) \rightarrow E = \{0, 1, \dots, v\},$$

$$v = m \cdot i$$

respectively. The process is a semi-regenerative one relative to these sequences [6]. Let assign $Q_n := Q_{\tau_n-}, n = 0, 1, 2, \dots$. Their limiting probabilities are expressed through the common invariant probability measure $P^i = (P_0^i, P_1^i, \dots, P_v^i)$ with the given number of servers (i). The random variable $\sigma_n := \tau_{n+1} - \tau_n$ is supposed to have a PDF

$$A(x) := P\{\sigma_n \leq x\}, x \geq 0, \quad (3.3)$$

with the mean $a = \mathbb{E}[\tau_1] = \mathbb{E}[\sigma_n] < \infty$. There are m channels which support customers on Internet. If channels are not available, the system loses its customers. Simplified human factor from the previous section is applied to determine the service time (2.4). Let $C_n(r)$ be the number of customers served during $[\tau_n, \tau_{n+1})$ with initial condition r and $C_n(r)$ has the binomial properties [7]. Then Q_{n+1} yields:

$$\begin{cases} Q_n + 1 - C_n(Q_n + 1), & Q_n = 0, \dots, v - 1, \\ m\xi - C_n(v), & Q_n = v. \end{cases} \quad (3.4)$$

Let

$$p_{jk}^i := P\{Q_{n+1} = k | Q_n = j, \xi_{n+1} = i\},$$

$$j = 0, 1, \dots, m \cdot i \text{ and } k = 0, 1, \dots, mi$$

then

$$p_{jk}^i = \binom{j+1}{k} \int_{\mathbb{R}} (1 - e^{-\mu(x-\rho(m))})^{j+1-k} \cdot (e^{-\mu(x-\rho(m))}) \mathbf{1}_{\{x \geq \rho(m)\}} dA(x),$$

$$j = 0, 1, \dots, v - 1,$$

and

$$p_{m,i,k} = \binom{m-i}{k} \int_{\mathbb{R}} (1 - e^{-\mu(x-\rho(m))})^{mi-k} \cdot (e^{-\mu(x-\rho(m))})^k \mathbf{1}_{\{x \geq \rho(m)\}} dA(x), \quad j = v.$$

The conditional stationary probabilities $\mathbf{P}^i = (P_0^i, P_1^i, \dots, P_v^i)$ for the embedded process with the given number of servers i , (i.e., $P_k^i = \lim_{n \rightarrow \infty} P\{Q_n = k | \xi_n = i\}$), are known to satisfy the following formulas from Kleinrock (1975)

$$\mathbf{P}(i, z) = \sum_{j=0}^{mi} P_j^i z^j = \sum_{j=0}^{mi} A_j(z) P_j^i, \quad (3.5)$$

where

$$A_j(z) = z^{j+1} \mathbb{E} \left[\left[(1 - e^{-\mu(\sigma_n - \rho(m))}) z^{-1} + e^{-\mu(\sigma_n - \rho(m))} \right]^{j+1} \right];$$

$$j = 0, 1, \dots, v-1,$$

and

$$A_v(z) = \mathbb{E} \left[z^v \left[(1 - e^{-\mu(\sigma_n - \rho(m))}) z^{-1} + e^{-\mu(\sigma_n - \rho(m))} \right]^v \right].$$

The stationary probabilities $\mathbf{P} = (P_0, P_1, \dots)$ and $P_k = \lim_{n \rightarrow \infty} P\{Q_n = k\}$ yields

$$P_k = \lim_{n \rightarrow \infty} \sum_{i=0}^{\infty} P\{Q_n = k | \xi_n = i\} P\{\xi_n = i\}$$

$$= \sum_{i=0}^{\infty} P_k^i q_i. \quad (3.6)$$

Let $h(z, \sigma_n) := (1 - e^{-\mu(\sigma_n - \rho(m))}) z^{-1} + e^{-\mu(\sigma_n - \rho(m))}$ and (3.5) yields:

$$\mathbf{P}(i, z) = \sum_{j=0}^{mi} \mathbb{E} \left[(1 - e^{-\mu(\sigma_n - \rho(m))} + z e^{-\mu(\sigma_n - \rho(m))})^{j+1} \right] P_j^i$$

$$- \mathbb{E} \left[(1 - e^{-\mu(\sigma_n - \rho(m))})^{mi} e^{-\mu(\sigma_n - \rho(m))} (1 - z) \right] P_v^i$$

Define the operators R_r and B_r which are developed by Takacs [11]. R_r and B_r have following properties:

$$(i) R_r f = \lim_{z \rightarrow 1} \frac{1}{z^r} f^{(r)}(z), \quad (3.7)$$

$$(ii) B_r^i = R_r \mathbf{P}(i, z) = \sum_{k=r}^{mi} \binom{k}{r} P_k^i, \quad (3.8)$$

$$(iii) P_k^i = \sum_{r=k}^{mi} B_r^i \binom{r}{k} (-1)^{r-k}. \quad (3.9)$$

Using (3.7), we can have

$$R_r (1 - e^{-\mu(\sigma_n - \rho(m))} + z e^{-\mu(\sigma_n - \rho(m))})^{j+1}$$

$$= \begin{cases} \binom{j+1}{r} e^{-\mu(\sigma_n - \rho(m))}, & j+1 \geq r, \\ 0, & j+1 < r \end{cases} \quad (3.10)$$

and

$$R_r \left((1 - e^{-\mu(\sigma_n - \rho(m))})^m (1 - z) \right)$$

$$= \begin{cases} - \binom{m}{r-1} e^{-\mu(r-1)(\sigma_n - \rho(m))}, & m \geq r, \\ 0, & m < r. \end{cases} \quad (3.11)$$

Let $\hat{\alpha}(\theta) := \mathbb{E}[e^{-\theta(\sigma_n - \rho(m))}]$. Using the R_r operator to (3.7), we have:

$$R_r \mathbf{P}(i, z) = \sum_{j=r-1}^{mi} \binom{j+1}{r} \hat{\alpha}_r P_j^i - \binom{mi}{r-1} \hat{\alpha}_r P_v^i,$$

$$\hat{\alpha}_r = \hat{\alpha}(\mu r).$$

From (3.8),

$$B_r^i = \sum_{j=r-1}^{mi} \binom{j+1}{r} \hat{\alpha}_r P_j^i - \binom{mi}{r-1} \hat{\alpha}_r P_v^i. \quad (3.12)$$

Since we know that $\binom{j+1}{r} = \binom{j}{r} + \binom{j}{r-1}$ and (3.8):

$$B_r^i = B_r^i \hat{\alpha}_r + B_{r-1}^i \hat{\alpha}_r - \binom{v}{r-1} B_v^i \hat{\alpha}_r. \quad (3.13)$$

Let

$$a_r := \begin{cases} 1, & r = 0, \\ \prod_{i=1}^r \frac{\hat{\alpha}_i}{e^{\rho(m)} - \hat{\alpha}_i}, & r > 0, \end{cases} \quad (3.14)$$

then (3.13) yields

$$B_n^i = a_n B_v^i \sum_{r=n}^v \binom{v}{r} / a_r, \quad n = 0, 1, \dots, v. \quad (3.15)$$

From (3.8), $B_0^i = 1$ and we can find B_v^i from (3.15)

$$a_n B_v^i \sum_{r=n}^v \binom{v}{r} / a_r = 1.$$

So,

$$B_v^i = \left[a_0 \sum_{l=0}^v \binom{v}{l} / a_l \right]^{-1} = P_v^i. \quad (3.16)$$

From (3.15)-(3.16)

$$B_n^i = a_n \left[a_0 \sum_{l=0}^v \binom{v}{l} / a_l \right]^{-1} \sum_{r=n}^v \binom{v}{r} / a_r,$$

$$n = 0, 1, \dots, v. \quad (3.17)$$

Then we can find P_k , $k = 0, \dots, m$ as follows from (3.9):

$$P_k^i = \sum_{n=k}^v \left[a_n \left(a_0 \sum_{l=0}^v \binom{v}{l} / a_l \right)^{-1} \cdot \sum_{r=n}^v \binom{v}{r} / a_r \right] \binom{n}{k} (-1)^{n-k}, \quad (3.18)$$

$$k = 0, \dots, v.$$

From (3.6) and (3.18), The stationary probabilities $\mathbf{P} = (P_0, P_1, \dots)$ are

$$P_k = \sum_{i=0}^{\infty} q_i \left[\sum_{n=k}^{mi} \left[a_n B_{mi}^i \cdot \sum_{r=n}^{mi} \binom{mi}{r} / a_r \right] \binom{n}{k} (-1)^{n-k} \right], \quad k = 0, 1, \dots \quad (3.19)$$

We now turn to the continuous time parameter queueing process of $GI/\tilde{M}/m/0$. The treatment is same as that of Kim and Dshalalow [6]. To find the limiting distribution of the process (Q_t) , we use the Kolmogorov differential equations [8] and semi-regenerative techniques to determine the initial conditions. The limiting probabilities $\boldsymbol{\pi} = (\pi_0, \pi_1, \dots)$ for the process with the continuous time parameters are known to satisfy the following formulas:

$$\begin{cases} \pi_0 = 1 - \sum_{k \geq 1} \pi_k, & k = 0 \\ \pi_k = \frac{P_{k-1}}{k a \mu}, & k = 1, 2, \dots \end{cases} \quad (3.20)$$

For the process Q_t , the corresponding formulas yield

$$\pi_k = \frac{1}{k a \mu} \sum_{i=0}^{\infty} q_i \left[\sum_{n=k-1}^{mi} \left[a_n B_{mi}^i \sum_{r=n}^{mi} \binom{mi}{r} / a_r \right] \binom{n}{k-1} (-1)^{n-k+1} \right], \quad k = 1, 2, \dots \quad (3.21)$$

and

$$\pi_0 = 1 - \sum_{l \geq 0} \frac{P_l}{(l+1)a\mu} \quad (3.22)$$

along with (3.20).

IV. OPTIMALITY OF THE NETWORK MANAGEMENT MODEL

In this section, we deal with a class of optimization problems that arise in stochastic management. Commonly used stochastic optimization method [3, 4, 6] is applied. Let a strategy Σ is a set of action we impose on the system (e.g., the choice of customer distribution, the number of channels in the network server and so on). For instance, $A(x)$ and μ in turn can be functions of m . On the other hand, a system can be subject to a set, say C , of cost functions. Denote by $\phi(\Sigma, C, t)$ the expected costs within $[0, t]$, due to the strategy Σ costs C and define the expected cumulative cost rate over an infinite horizon [2]:

$$\phi(\Sigma, C) := \lim_{t \rightarrow \infty} \frac{1}{t} \phi(\Sigma, C, t). \quad (4.1)$$

Since that Q_t is the number of available channels in the given number of network servers (ξ_t) . The number of unoccupied channels at time t is assigned as $M_t = m \cdot \xi_t - Q_t$. Let $f(n)$ be the cost rate for maintaining the n unoccupied channels of the system. Then the expected cost for all empty channel in a network sever during $[0, t]$ is:

$$Uf(t) = \sum_{i \geq 0} q(i, t) \quad (4.2)$$

$$\cdot \left[\sum_{k=0}^{mi} f(mi-k) \int_{s=0}^t P^i \{Q_s = k\} ds \right].$$

If $h(n)$ is the cost rate (it can be reward rate, too.) for servicing n customers who are served, then the expected cost for served customers in the interval $[0, t]$ is

$$Uh(t) = \sum_{i \geq 0} q(i, t) \left[\sum_{k=0}^{mi} h(k) \int_{s=0}^t P^i \{Q_s = k\} ds \right]. \quad (4.3)$$

Let $o(n)$ be the cost for using the network servers (n) in the interval $[0, t]$ and . So average cost rate for usage of servers is

$$O(t) = \sum_{i \geq 0} o(i) \int_{s=0}^t q(i, s) ds. \quad (4.4)$$

The Markov renewal function $R^i(t) = \mathbb{E}^i[N_t] = \mathbb{E}^i \left[\sum_{n=0}^{\infty} \mathbf{1}_{[0,t]}(\tau_n) \right]$ gives the total expected number of customers in the time interval $[0, t]$. So, the functional $UR^i(t) = rR^i(t)$ gives the total reward for all channels are completely occupied in $[0, t]$ if r is the common reward for each customer. The cumulative cost of the entire procedures involved in the multi-channel system of the network server on the interval $[0, t]$ yields:

$$\phi(\Sigma, C, t) = Uf(t) + Uh(t) + O(t) + UR^i(t).$$

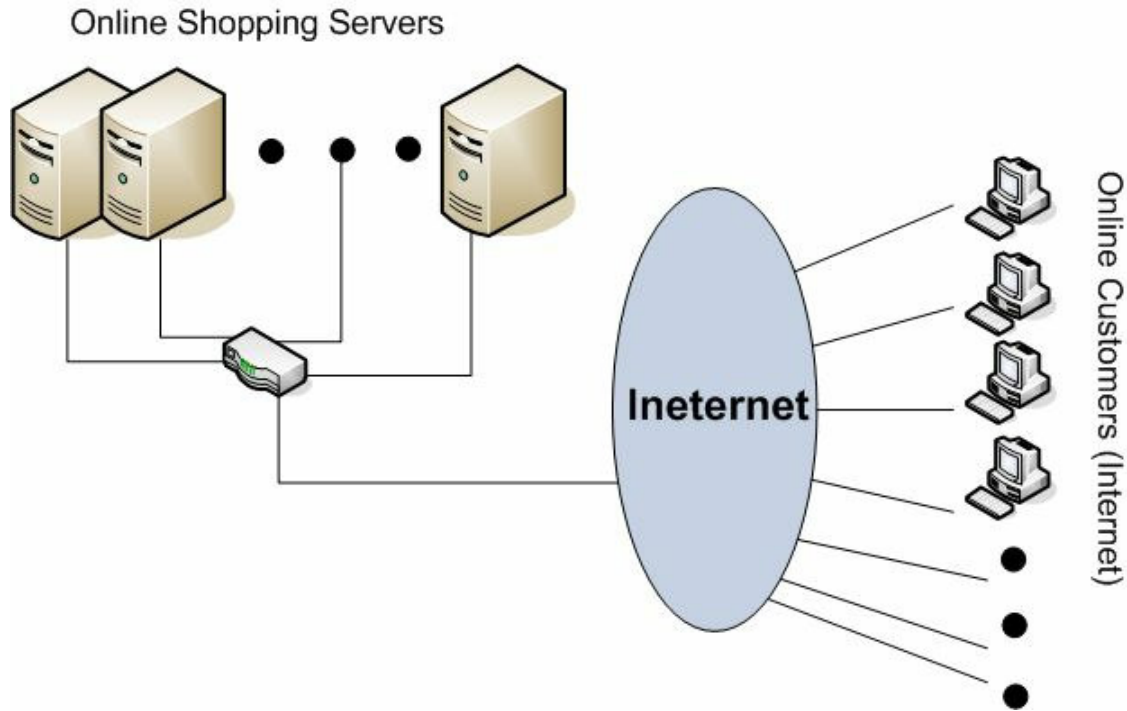
Now we turn to convergence theorems for regenerative, semi-regenerative, and semi-Markov processes [2], to arrive at the objective function $\phi(\Sigma, C)$, which gives the total expected rate of all processes over an infinite horizon. In light of equations [2], we have

$$\phi(\Sigma, C) = \frac{r}{a} + \sum_{i \geq 0} q_i \left[\sum_{k=0}^{mi} (f(mi-k) + h(k)) \pi_k + o(i) \right] \quad (4.5)$$

V. APPLICATION FOR ELECTRONIC COMMERCE ECOSYSTEM

The electronic commerce (e-commerce) has been a big issue not only in the technology but also in the business. Because of the heavy population of the Internet, more people want to live in better and more convenient place. The e-commerce ecosystem makes these people potential customers. In business way, companies try to make actual customer, who really purchases, from these potential customers via Internet [9]. The culmination of e-commerce where customers can stop on the computer without actually traveling any distance to the shopping has been a great influence of the technology and business.

The business model of e-commerce includes all major functions that can handle e-business from purchasing inquires to actual purchases. The paper attempts to model customer shopping time as part of the network server performance evaluation. Once again, we allow the exponential distribution for customer input.



[Figure 1. e-Commerce Architecture: Online Shopping]

5.1. M/M/m/0 Queueing System

The pertinent special formulas under these assumptions are as follows:

$$A(x) = 1 - e^{-\frac{1}{a}x}, \quad (5.1)$$

$$\alpha(\theta) = \int_{\mathbb{R}_+} e^{-\theta x} dA(x) = \int_{x=0}^{\infty} \left(\frac{1}{a}\right) e^{-(\theta + \frac{1}{a})x} dx = \frac{1}{1 + a\theta},$$

$$\hat{\alpha}(\theta) = \int_{\mathbb{R}_+} e^{-\theta(x - \rho(m))} \mathbf{1}_{\{x \geq \rho(m)\}} dA(x) = \frac{e^{\rho(m)\theta}}{1 + a\theta},$$

$$\rho(m) = \hat{\rho} \cdot m \text{ where } \hat{\rho} \text{ is a ratio of checkout process.} \quad (5.2)$$

We assume that the limiting probabilities of available servers are the binomial random variables with parameter p that is a probability of

$$q_i = \binom{n}{i} p^i (1-p)^{n-i}, \quad i = 0, 1, \dots, n$$

Now, we can calculate a_r from (3.14) and B_n from (3.15) to get P_k (from (3.18)). Once we get P_k , π_k ($k = 0, 1, \dots$) can be found by using the formula (4.9) and (4.14).

In the other hand, we specify the remaining of the three primary cost functions are as follows:

$$f(n) = G \cdot n, h(n) = H \cdot n, O(n) = O \cdot n. \quad (5.3)$$

Finally, we arrive at the following expression for the objective function:

$$\phi(\Sigma(n), C) = \frac{r}{a} + \sum_{i \geq 0} q_i \sum_{k=0}^{mi} (G - H) \pi_k + (G \cdot m + O) np \quad (5.4)$$

Here we use formulas (3.20) and (5.4). Notice that we have only one parameters m vary. Comparing (5.4) with (3.20), we identify $d = G$ and $l = G - H$. We restrict the initial strategy of this model to a combination of the number of channels and servers. In other words, we need to find the number of servers n such that

$$\phi(\Sigma(n), C) = \min\{\phi(\Sigma(n), C)\}. \quad (5.5)$$

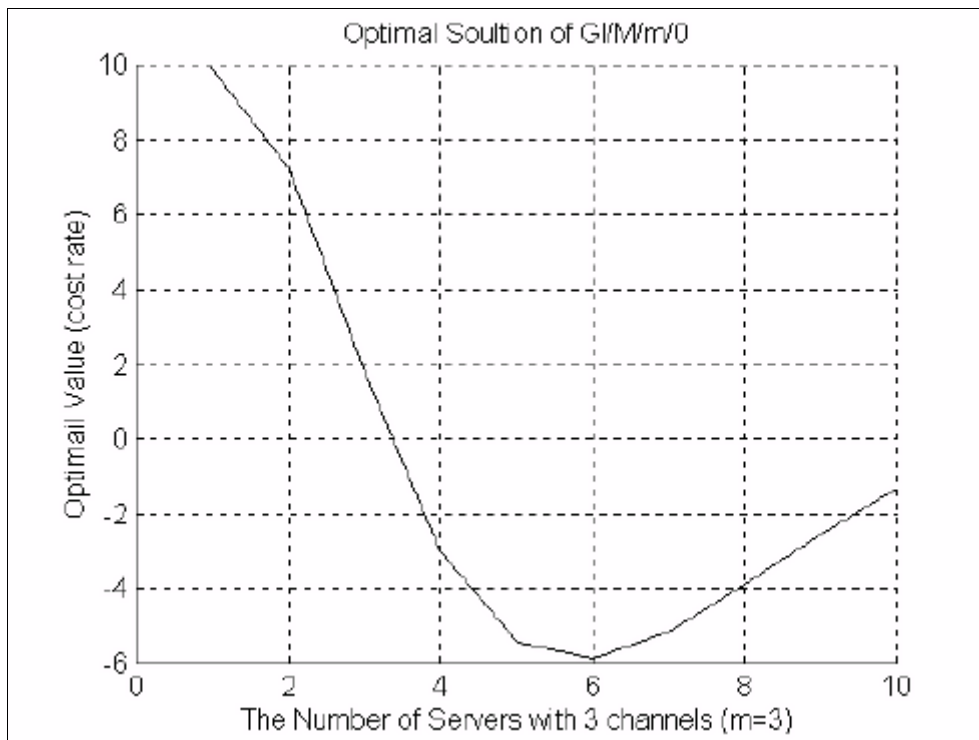
As an illustration, we take $r = 5$, $G = 4$ and $H = -9$. Inter-arrival time distribution is exponential with mean $a = 1.9$, the parameter $p\mu$ is 0.41. Take the reliability of each server (p) as 0.5 and $\hat{\rho} = 1$ from (5.2). Now, we calculate $\phi(S(n), C)$ and (n_0) that gives a minimum for $\phi(S(n), C)$. Recall that the n_0 stands for the number of servers which minimizes the total cost of this system when 3 channels are available for each server. Below is a plot of $\phi(S(n), C)$ for $n \in \{n : n = 1, \dots, 10\}$.

The calculation yields that $n = 6$ resulting making the minimum cost equal -5.874 . It means that we $n_0 = 6$ channels from a network server to minimize the cost of server maintenance.

VI. CONCLUSION

This paper shows the full analytic solutions of the enhanced e-commerce management model. The mathematical tools to solve the system include the embedded Markov process, semi-Markov process, semi-regenerative techniques. The practical setup which is called "human factor" is added to provide the more realistic performance measuring. These analytic solutions, which are stationary probabilities of embedded process and continuous time process, give all of performance measures and the optimization solution gives the critical decision points for better performances. This

research gives the guideline to evaluate the simulation and real data for e-commerce network management design.



[Figure 2. Optimal Solution of the Application]

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