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A Dynamic Model for Expansion Planning of Multi Echelon Multi Commodity Supply Chain

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***Abstract*—This paper proposes a dynamic mixed integer linear program (MILP) for design and planning a multi echelon multi product supply chain. The main focus of this paper is the expansion of supply chain from a strategic view point. Here we consider some potential points for establishment of production units and warehouses during planning horizon in which two types of warehouses (private, public) have been considered. Development of the designed supply chain will be planned according to cumulative net income from the first period. This model aim to select suppliers, determine quantity of each raw material to be supplied by each supplier, quantity of each product to be produced in each production unit, quantity of each product to be sent to each warehouse, and quantity of each product to be sent to each market and some other strategic and tactical decisions in order to maximize net profit of the supply chain.**

***Index Terms*—Strategic supply chain planning; dynamic modeling; linear programming.**

I. Introduction

In today's competitive trade world, manufacturers face the continuing challenge to constantly evaluate and configure their production and distribution systems and strategies to provide the desired customer service at the lowest possible cost. Long-range survival for manufacturing firms will be very difficult to attain without highly optimized strategic and tactical logistics systems. Savings in the 5–10% range, which can be achieved by using strategic and tactical logistics models, can dramatically affect the profitability of the corporation [6].

The logistics systems design problem is defined as follows: given a set of potential suppliers, potential manufacturing facilities, and distribution centers with multiple possible configurations, and customers with deterministic demands, determine the configuration of the production–distribution system and the transfer prices between various subsidiaries of the corporation such that seasonal customer demands and service requirements are met and the after tax profit of the corporation is maximized.

The after tax profit is the difference between the sales revenue minus the total system cost and taxes. The total cost is defined as the sum of supply, production, transportation, inventory, and facility costs [6].

A major thrust of recent research in this area is the development of optimization models that integrate different functions (e.g. purchasing, production and distribution) in the supply chain. The basic idea behind this approach is to simultaneously optimize decision variables of different functions that have traditionally been optimized sequentially [9].

One of the most important strategic problems is supply chain optimization. Strategic designing of a supply chain makes the following decisions necessary for logistics engineers and managers:

- Number of production units needed to satisfy customer demands
- Number of warehouses needed for inventories
- Determination capacity of the production units and warehouses
- Supplier selection
- Design of distribution channels
- Quantity of raw materials to be purchased from each supplier
- Quantity of finished products to be produced in each production unit
- Quantity of finished products to be stored in each warehouses

There are several models have been developed to help managers in designing and planning of their supply chain. Arntzen et al. [2] developed a global integrated model based on mixed integer linear programming for production and distribution planning with multiple products and a network of sellers. Amiri [1] proposed a mixed integer linear model to select the optimum numbers, locations and capacities of plants and warehouses to open so that all customer demand is satisfied at minimum total costs of the distribution network in a three echelon, single period and single product. In this paper an efficient heuristic solution procedure for this supply chain system problem has been provided.

Wouda et al. [11] developed a mixed integer linear programming model for optimization of the supply network of Nutricia Hungaryusing. Their model focus was on consolidation and product specialisation of plants, the objective was to find the optimal number of plants, their locations and the allocation of the product portfolio to these plants, when minimizing the sum of production and transportation costs. Cordeau et al. [3] propose a static model for a multi-commodity, multiple facility and single-country network. This model has been developed to determine the

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number of locations, the capacity and technology of manufacturing in plants and warehouses, selection of suppliers, selection of distribution channels, transportation modes and material flows. To solve this problem, the authors present two methods: a simplex-based branch-and bound approach and a Benders decomposition approach.

Noorul Haq and Kannan [8] developed an integrated supplier selection and multi-echelon distribution inventory model in a built-to-order supply chain involving single selected supplier, multiple plants, multiple distributors, multiple wholesalers and multiple retailers. Hamer-Lavoie and Cordeau [4] developed a dynamic model with stochastic demands, which takes inventories into account, including the safety stock. They suppose that the location has already been chosen for plants and the model focuses on warehouse location. The authors propose a linear approximation for the last constraints concerning safety stock and a branch-and-bound method strengthened by valid inequalities. Dias et al. [5] work on the re-engineering of a two-echelon network (facilities and customers). The authors suppose that facilities can be opened, closed and reopened more than once during the planning horizon. They study these conditions within three scenarios: with maximum capacity restrictions; with both maximum and minimum capacity restrictions; and with a maximum capacity that decreases. All of these problems are solved by primal-dual heuristics. In this paper, three linear formulations correspond to the three previous scenarios and their linear dual formulations are presented.

Melo et al. [7] aim at relocating the network with expansion/ reduction capacity scenarios. Capacity can be exchanged between an existing facility and a new one, or between two existing facilities under some conditions. Each change of capacity is penalised by a cost. In this model, closed facilities cannot be reopened, and new facilities will remain in activity until the end of the planning horizon. Thanh et al. [10] proposes a dynamic mixed integer linear programming model for a four echelon supply chain including suppliers, manufacturing firms, distribution centers and customers. Bill of materials and multiple products have been taken into consideration. This paper aims to help strategic and tactical decisions: opening, closing or enlargement of facilities, supplier selection, flows along the supply chain. They make a distinction between a private warehouse (owned by the company) and a public warehouse (hired by the company). The status of a public warehouse can change more than once during the planning horizon.

II. Proposed Model

In this paper we consider a multi-period, multi-commodity multi-facility supply chain problem in which there is a set of potential locations, where a new plant or a new warehouse can be opened. This problem includes four main layers composed of suppliers, production units, distribution centers and customers. The bill of materials is also taken into consideration.

The proposed model aims to make some strategic decisions related to design and planning a supply chain expansion during a planning horizon: selection of suppliers; location and time of facilities opening; planning capacity for existing facilities; production and distribution planning. Inventories also have been taken into consideration.

We assume that there are some potential points that can be selected for establishment of plants and warehouses. Two types of warehouses have been considered in this model: private and public. We assume that the opened plants and private warehouses cannot be closed during the planning horizon. Also hiring a public warehouse for less than *m* periods is not permitted.

The various assumptions involved in this paper are described below:

- Public warehouses have no fixed costs but their variable costs are higher than those of private warehouses.
- Products only can be transferred from each supplier to all plants, from each plant to all distributors and from each distributor to all customer zones.
- Transfers between plants and between warehouses are not permitted.
- Each supplier has a restriction on the available raw materials.
- Each facility has an initial capacity as well as a limited maximal installable capacity.
- For each facility a set of capacity options are available. The capacity of a facility can be increased by adding capacity options.
- Minimal and maximal rates of utilization for each facility have been considered to avoid facilities running at 1% or at 100% of their capacity.
- Transportation cost per product from each supplier to all plants plant, from each plant to all distributors, from each distributor to all customer zones remains fixed for all the periods given.
- Processing cost per product at any plant and inventory cost per product per period at all warehouses remains fixed for all the periods given
- Customer demand is deterministic but it's not necessary to satisfy all customer demands.

The objective function is to maximize total net profit over the time periods computed by subtracting total cost from total revenue. The total revenue is simply the total selling income. The total cost includes the fixed costs of opening facilities, adding facility options, operating facility and variable costs of raw material, production, inventory and transportation.

A. Mixed integer linear programming model

Notation:

The following indices, parameters and decision variables are defined.

INV^t	investment in period t
$\mathcal{T}(t \in \mathcal{T})$	planning horizon
$\mathcal{F}^t$	cumulative net profit from the beginning of the planning to $(t\text{-}1)^{th}$ period
Inc^t	net profit in period *t*
DL^t	cumulative net profit after tax and stakeholders share from the beginning of the planning to $(t\text{-}1)^{th}$ period

TR	tax rate
SH	stakeholders share (in percent)
$\mathcal{S}(s \in \mathcal{S})$	set of suppliers
$\mathcal{M}(i \in \mathcal{M})$	set of plants
$\mathcal{W}(j \in \mathcal{W})$	set of warehouses
$\mathcal{W}p$	set of private (permanent) warehouses
$\mathcal{WH}$	set of public (hired) warehouses
$\mathcal{O}(o \in \mathcal{O})$	set of capacity options
$\mathcal{C}(c \in \mathcal{C})$	set of customers
$\mathcal{p}(p \in \mathcal{p})$	set of products
$\mathcal{p}_r(\mathcal{p}_r \subset \mathcal{p})$	set of raw materials
$\mathcal{p}_f(\mathcal{p}_f \subset \mathcal{p})$	set of finished products
$BigM$	a large number
$\mathcal{F}$	total profit
$\mathcal{R}$	total return after sales
$\mathcal{E}$	total expenses
$R^t_{s,p}$	available capacity of supplier *s* for *p* at *t*
ic_i	initial production capacity at *i*
mc_i	maximal installable production capacity at *i*
iu_i	minimal percentage of utilization of facility *i*
mu_i	maximal percentage of utilization of facility *i*
KT_o	capacity of option *o*
$\bar{S}_j$	initial storage capacity at *j*
$D^t_{c,p}$	demand of customer *c* for product *p* at *t*
$B_{p',p}$	quantity of *p'* necessary to manufacture a unit of *p* (bill of materials)
$LT_{p,i}$	workload for the treatment of a unit *p* at facility *i*
$LS_{p,j}$	workload for the storage of a unit *p* at warehouse *j*
$A_{i,j}$	number of deliveries from plant *i* to warehouse *j* in one period
$\mathcal{PR}_{p,c}$	selling price of item of unit *p* to customer *c*
$\mathcal{PS}_{p,s}$	price of item of unit *p* from supplier *s*
Co_i	fixed cost for opening a facility at a potential location *i*
$Cop_{i,o}$	fixed cost for operating capacity option *o* at facility *i*
CU_i	fixed cost for operating a facility *i*
$CA_{i,o}$	fixed cost for adding capacity option *o* to facility *i*
$CP_{p,i}$	treatment cost of a unit *p* at facility *i*
$CS_{p,j}$	storage cost of a unit of *p* at warehouse *j*
$CT_{p,i,j}$	transportation cost of a unit of *p* from *i* to *j*
$CD_{p,s,i}$	transportation cost of a unit of *p* from *s* to *i*
$CF_{p,j,c}$	transportation cost of a unit of *p* from *j* to *c*
x^t_i	1 if the entity *i* is active at *t*; 0 otherwise
$y^t_{i,o}$	1 if the capacity option *o* is added to *i* at t; 0 otherwise
$z^t_{s,p}$	1 if the supplier *s* is selected for the raw material *p* at *t*; 0 otherwise
$f^t_{p,i,j}$	quantity of product *p* transferred from location *i* to *j* at *t*
$g^t_{p,i}$	quantity of product *p* produced in plant *i* at *t*
$h^t_{p,j}$	quantity of product *p* held in warehouse *j* at the beginning of *t*

Objective Function:

The objective function is to maximize total net profit over the time periods computed by subtracting total cost from total revenue.

$$Maximize\ \mathcal{F} = (\mathcal{R} - \mathcal{E}) \tag{1}$$

The total revenue is simply the total selling income:

$$\mathcal{R} = \sum_{t \in \mathcal{T}} \sum_{j \in \mathcal{W}} \sum_{p \in \mathcal{p}_f} \sum_{c \in \mathcal{C}} \mathcal{PR}_{p,c} \cdot f^t_{p,j,c} \tag{2}$$

The total cost includes the fixed costs of opening facilities, adding facility options, operating facility and variable costs of raw material, production, inventory and transportation.

$$\mathcal{E} = \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M} \cup \mathcal{W}p} Co_i \cdot (x^t_i - x^{t-1}_i) \tag{3}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M} \cup \mathcal{W}p} \sum_{o \in \mathcal{O}} CA_{i,o} \cdot (y^t_{i,o} - y^{t-1}_{i,o}) \tag{4}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{i \in \mathcal{M} \cup \mathcal{W}p} CU_i \cdot x^t_i + \sum_{o \in \mathcal{O}} Cop_{i,o} \cdot y^t_{i,o} \tag{5}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{p}_f} \sum_{i \in \mathcal{M}} CP_{p,i} \cdot g^t_{p,i} \tag{6}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{p}_f} \sum_{j \in \mathcal{W}} CS_{p,j} \cdot \left(h^t_{p,j} + \sum_{i \in \mathcal{M}} \frac{f^t_{p,i,j}}{2A_{i,j}} \right) \tag{7}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{p}_r} \sum_{i \in \mathcal{M}} CD_{p,s,i} \cdot f^t_{p,s,i} \tag{8}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{p}_f} \sum_{j \in \mathcal{W}} \sum_{c \in \mathcal{C}} CF_{p,j,c} \cdot f^t_{p,j,c} \tag{9}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{p \in \mathcal{p}_f} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{W}} CT_{p,i,j} \cdot f^t_{p,i,j} \tag{10}$$

$$+ \sum_{t \in \mathcal{T}} \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{p}_r} \sum_{i \in \mathcal{M}} \mathcal{PS}_{p,s} \cdot f^t_{p,s,i} \tag{11}$$

Equation (3) calculates fixed costs of opening facility including plants and warehouses. (4) calculates costs related to adding capacity options to plants and warehouses. (5) defines the operating facility fixed cost, (6) is related to production variable costs and (7) calculates storage variable costs. Equations (8)-(10) are related to transportation costs respectively from supplier *s* to plant *i*, from warehouse *j* to customer *c* and from plant *i* to warehouse *j*. Equation (11) calculates the raw material supply costs.

Constraints

$$\sum_{j \in \mathcal{W}} f^t_{p,j,c} \le D^t_{c,p} \qquad \forall c \in \mathcal{C}, \forall p \in \mathcal{p}_f \tag{12}$$

Constraint (12) states that all products transferred to costumers should not be more than their demands in any period and it's not necessary to satisfy all customer demands.

$$h^{t-1}_{p,j} + \sum_{i \in \mathcal{M}} f^t_{p,i,j} = \sum_{c \in \mathcal{C}} f^t_{p,j,c} + h^t_{p,j} \qquad \forall j \in \mathcal{W}, \forall p \in \mathcal{p}_f \tag{13}$$

Constraint (13) is related to flow conservation at warehouses.

$$\sum_{s \in \mathcal{S}} f_{p,s,i}^{t} = B_{p',p} \cdot g_{p,i}^{t} \quad \forall i \in \mathcal{M}, \forall p' \in \mathcal{p}_r, \forall p \in \mathcal{p} \qquad (14)$$

Constraint (14) ensures that plants receive enough raw materials in order to produce the required quantity of finished products.

$$g_{p,i}^{t} = \sum_{j \in \mathcal{W}} f_{p,i,j}^{t} \quad \forall i \in \mathcal{M}, \forall p \in \mathcal{p}_f \qquad (15)$$

Constraint (15) states that the quantity of manufactured products at a plant should be equal to its delivered quantity to warehouses.

$$\sum_{p \in \mathcal{p}_f} LT_{p,i} \cdot g_{p,i}^{t} \le mu_i . (ic_i . x_i^t + \sum_{o \in \mathcal{O}} KT_o y_{i,o}^t) \quad \forall i \in \mathcal{M} \qquad (16)$$

$$\sum_{p \in \mathcal{p}_f} LT_{p,i} \cdot g_{p,i}^{t} \ge iu_i . (ic_i . x_i^t + \sum_{o \in \mathcal{O}} KT_o y_{i,o}^t) \quad \forall i \in \mathcal{M} \qquad (17)$$

$$\sum_{p \in \mathcal{p}_f} \sum_{c \in \mathcal{C}} LS_{p,j} \cdot f_{p,j,c}^{t} \le mu_j . (ic_j . x_j^t + \sum_{o \in \mathcal{O}} KT_o y_{j,o}^t) \quad \forall j \in \mathcal{Wp} \qquad (18)$$

$$\sum_{p \in \mathcal{p}_f} \sum_{c \in \mathcal{C}} LS_{p,j} \cdot f_{p,j,c}^{t} \ge iu_j . (ic_j . x_j^t + \sum_{o \in \mathcal{O}} KT_o y_{j,o}^t) \quad \forall j \in \mathcal{Wp} \qquad (19)$$

Constraints (16)-(19) are related to capacity of plants and warehouses. These constraints prevent a facility to function under its minimum rate of utilization and to exceed the maximum rate of utilization of its installed capacity. The installed capacity is the sum of the initial capacity and the capacity of the added options.

$$\sum_{p \in \mathcal{p}_f} LS_{p,j} \cdot \left(h_{p,j}^t + \sum_{i \in \mathcal{M}} \frac{1}{2A_{i,j}} \cdot f_{p,i,j}^t \right) \le \bar{S}_j x_j^t + \sum_{o \in \mathcal{O}} KT_o . y_{j,o}^t \quad \forall j \in \mathcal{Wp} \qquad (20)$$

$$ic_i . x_i^t + \sum_{o \in \mathcal{O}} KT_o . y_{i,o}^t \le mc_i \quad \forall i \in \mathcal{M} \qquad (21)$$

$$ic_j . x_j^t + \sum_{o \in \mathcal{O}} KT_o . y_{j,o}^t \le mc_j \quad \forall j \in \mathcal{Wp} \qquad (22)$$

Warehouses must not store more than their storage capacity (20). Also the installed capacity at any plants and any warehouse must not exceed its maximal installable capacity (21)-(22).

$$\sum_{i \in \mathcal{M}} f_{p,s,i}^t \le z_{s,p}^t . R_{s,p}^t \quad \forall s \in \mathcal{S}, \forall p \in \mathcal{p}_r \qquad (23)$$

$$\sum_{i \in \mathcal{M}} f_{p,s,i}^t \ge \alpha . z_{s,p}^t \quad \forall s \in \mathcal{S}, \forall p \in \mathcal{p}_r \qquad (24)$$

Suppliers deliver a raw material if and only if they are selected for this raw material (23) and their delivery cannot exceed their capacity. Constraint (24) is to avoid purchasing each raw material less than predetermined minimal amount of the delivered quantity of each supplier.

$$Inc^t = \sum_{j \in \mathcal{W}} \sum_{p \in \mathcal{p}_f} \sum_{c \in \mathcal{C}} \mathcal{PR}_{p,c} . f_{p,j,c}^t \qquad (25)$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{Wp}} Co_i . (x_i^t - x_i^{t-1}) \qquad (26)$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{Wp}} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^t - y_{i,o}^{t-1}) \qquad (27)$$

$$- \sum_{i \in \mathcal{M} \cup \mathcal{Wp}} CU_i . x_i^t + \sum_{o \in \mathcal{O}} Cop_{i,o} . y_{i,o}^t \qquad (28)$$

$$- \sum_{p \in \mathcal{p}_f} \sum_{i \in \mathcal{M}} CP_{p,i} . g_{p,i}^t \qquad (29)$$

$$- \sum_{p \in \mathcal{p}_f} \sum_{j \in \mathcal{W}} CS_{p,j} . \left(h_{p,j}^t + \sum_{i \in \mathcal{M}} \frac{f_{p,i,j}^t}{2A_{i,j}} \right) \qquad (30)$$

$$- \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{p}_r} \sum_{i \in \mathcal{M}} CD_{p,s,i} . f_{p,s,i}^t \qquad (31)$$

$$- \sum_{p \in \mathcal{p}_f} \sum_{j \in \mathcal{W}} \sum_{c \in \mathcal{C}} CF_{p,j,c} . f_{p,j,c}^t \qquad (32)$$

$$- \sum_{p \in \mathcal{p}_f} \sum_{i \in \mathcal{M}} \sum_{j \in \mathcal{W}} CT_{p,i,j} . f_{p,i,j}^t \qquad (33)$$

$$- \sum_{s \in \mathcal{S}} \sum_{p \in \mathcal{p}_r} \sum_{i \in \mathcal{M}} \mathcal{PS}_{p,s} . f_{p,s,i}^t \qquad (34)$$

$$\mathcal{F}^t = \sum_{t=1}^{t-1} Inc^t \qquad (35)$$

$$DL^t = (1 - TR).(1 - SH).\mathcal{F}^t \qquad (36)$$

$$\sum_{i \in \mathcal{M} \cup \mathcal{Wp}} Co_i . (x_i^t - x_i^{t-1}) + \sum_{i \in \mathcal{M} \cup \mathcal{Wp}} \sum_{o \in \mathcal{O}} CA_{i,o} . (y_{i,o}^t - y_{i,o}^{t-1}) \le DL^t + INV^t \qquad (36)$$

Constraints (25)-(34) are related to net profit of supply chain in each period. Here we assume that there is an initial investment for the first period and development budget in each period is determined based on net profit computed by subtracting total cost from total revenue in previous periods. Total cost includes fixed costs of opening facility (26), adding capacity options to plants and warehouses (27), operating facility fixed cost (28), production variable costs (29), storage variable costs (30) transportation costs from supplier *s* to plant *i* (31), from warehouse *j* to customer *c* (32) and from plant *p* to warehouse *j* (33) and finally raw material supply costs (34). Constraint (35) calculates the expansion budget which is the net profit after tax and stakeholder share. Constraint (36) prevents the cost of opening facility and adding option to some opened facilities be more than expansion limitation in each period.

$$y_{i,o}^t \le x_i^t \quad \forall i \in \mathcal{M} \cup \mathcal{Wp}, \forall o \in \mathcal{O} \qquad (37)$$

$$x_i^{t-1} \le x_i^t \quad \forall i \in \mathcal{M} \cup \mathcal{Wp} \qquad (38)$$

$$m . x_j^t - \sum_{t=t-1}^{t-m+1} x_j^t - \sum_{t=t+1}^{t+m-1} x_j^t \le 1 \quad \forall j \in \mathcal{WH} \qquad (39)$$

$$y_{i,o}^{t-1} \le y_{i,o}^t \quad \forall i \in \mathcal{M} \cup \mathcal{Wp}, o \in \mathcal{O} \qquad (40)$$

$$\sum_{c \in \mathcal{C}} \sum_{p \in \mathcal{p}_f} f_{p,j,c}^t \le x_j^t . BigM \quad \forall j \in \mathcal{W} \qquad (41)$$

$$\sum_{o \in \mathcal{O}} y_{i,o}^t \le 1 \quad \forall i \in \mathcal{M} \cup \mathcal{Wp} \qquad (42)$$

$$y_{i,o}^t \le 1 - (x_i^t - x_i^{t-1}) \quad \forall i \in \mathcal{M} \cup \mathcal{Wp}, o \in \mathcal{O} \qquad (43)$$

Constraint (37) states that an opened facility can add available capacity options only. Constraint (38) prevents the opened facilities from closing. For the public warehouses, we suppose that they cannot be hired for less than *m* periods, constraint (39) ensures this condition. Constraint (40) states that we can add new capacity options but we cannot remove them. Constraint (41) ensures that only opened warehouses can send product to customers. Equation (42) states that we cannot add more than one capacity option to a facility in one period, and constraint (43) prevents adding any facility option at the first period of opening a facility.

$$x_i^t \in \{0,1\} \quad (44)$$

$$y_{i,o}^t \in \{0,1\} \quad (45)$$

$$z_{s,p}^t \in \{0,1\} \quad (46)$$

$$f_{p,i,i'}^t \geq 0 \quad (47)$$

$$g_{p,i}^t \geq 0 \quad (48)$$

$$h_{p,j}^t \geq 0 \quad (49)$$

The constraints (44)–(46) require that these variables are binary. The constraints (47)–(49) restrict these variables from taking non-negative values.

III. Illustrative Example

In order to illustrate the proposed model, with 3 suppliers, 3 potential locations for plants, 3 potential locations for warehouses and 3 customer zones a hypothetical example is used to show application of the proposed method. Three investment scenarios are evaluated to make our macro decisions assuming that the total available fund is 30,000,000 that should be invested during the planning horizon. Table I shows the scenarios for investment in each period.

The following parameters are considered in this example:

- Number of suppliers=3
- Number of potential location for plant=3
- Number of potential location for warehouse=3
- Number of customer zone=3
- Number of time period=5
- Number of raw material=4
- Number of final product=2
- Number of capacity option=2
- Tax rate=0.1
- Stakeholders' share=0.4

TABLE I. Scenarios for investment in each period

Periods	Scenario I	Scenario II	Scenario III
1	10,000,000	20,000,000	15,000,000
2	10,000,000	10,000,000	15,000,000
3	10,000,000	-	-
4	-	-	-
5	-	-	-

We assume that initial capacity of each plant and warehouse are 20000 and 100,000 respectively. Maximal installable capacity in each plant is 100,000 and in each warehouse is 200,000. Minimal and maximal percentages of utilization of a facility are 0.3 and 0.9. Two capacity options are considered as 25000 and 50000 for facilities. Other parameters including cost elements, demands, available raw material in each supplier, etc. are presented in appendix. Here we should mention that the demands quantity are generated randomly between 16000 and 20000 and assumed that this amounts will be increased in the following periods with the rate of 30 percent. The model has been solved for all scenarios using GAMS 21.7. The results confirm that the second scenario is the best scenario for investment according to net present value (NPV) of the net profit as well as objective function. Tables 2-4 show the results for scenarios.

Results of scenario II:

As mentioned before, the proposed model has been developed for the design of four echelons, multiple products, and multiple periods supply chain. There are a lot of decisions are made using the proposed method such as supplier selection, plant location, warehouse location, production planning and the amount of finished product to be sent to each customer zone. The objective function is maximization of total net profit during the planning horizon. One of the most important features of this model that makes it more legalistic is that development of the supply chain during the planning horizon is limited to cumulative net profit after tax and stakeholders' share. Fig.1 shows the net profit after tax and stakeholders' share in each period.

In the proposed model, variable x_i^t determines the location of facilities and the time of opening facilities in the selected location. These facilities includes plants, public and private warehouses. In the case of warehouses indices 1 and 2 indicate candidate location for the private warehouse and index 3 indicates public warehouse. If a private facility opens, it cannot be closed till the end of planning horizon. Also a public warehouse cannot be hired for less than 3 periods. Table V shows amount of variable x_i^t .

Fig.1. Net profit after tax and stakeholders' share in each period

As it can be seen in this table, two locations 2 and 3 are selected for the establishment of production units. The time of opening these plants is at the first period and they will remain opened till the last period. In the case of warehouses candidate public warehouse will be hired for 3 periods and location 2 has been selected for establishment of a private warehouse at the second period.

TABLE II. RESULTS OF SCENARIO I

Time period	Investment	Net incomes	Net profit after tax and stakeholders' share
1	10,000,000	20,308,160	10,966,406
2	10,000,000	121,455,100	65,585,754
3	10,000,000	141,888,800	76,619,952
4		78,197,350	42,226,569
5		151,065,400	81,575,316
NPV of net profit	182,606,121		
Objective function	512,914,700		

TABLE III. RESULTS OF SCENARIO II

Time period	Investment	Net incomes	Net profit after tax and stakeholders' share
1	20,000,000	40,015,580	21,608,413
2	10,000,000	127,665,100	68,939,154
3	-	150,886,800	81,478,872
4	-	78,197,350	42,226,569
5	-	151,065,400	81,575,316
NPV of net profit	199,416,878		
Objective function	542,769,800		

TABLE IV. RESULTS OF SCENARIO III

Time period	Investment	Net incomes	Net profit after tax and stakeholders' share
1	15,000,000	20,308,160	10,966,406
2	15,000,000	118,665,100	64,079,154
3		150,888,800	81,479,952
4		78,195,350	42,225,489
5		151,065,400	81,575,316
NPV of net profit			
Objective function			

TABLE V. AMOUNT OF VARIABLE x_i^t

	Time periods	1	2	3	4	5
Plant	1	0	0	0	0	0
	2	1	1	1	1	1
	3	1	1	1	1	1
Warehouse	1	0	0	0	0	0
	2	0	1	1	1	1
	3	1	1	1	0	0

We can add capacity to an opened facility instead of establishment of a new facility. The model allows adding capacity only to an opened facility and an added capacity cannot be removed. Also the model prevents adding a capacity option to a facility at the first period of opening. Table VI shows amount of variable $y_{i,o}^t$. The results indicate that capacity option 2 should be added to plant 3 at period 2.

TABLE VI. AMOUNT OF VARIABLE $y_{i,o}^t$

	Time periods	1	2	3	4	5
Plant	Capacity option					
3	2		1	1	1	1

Table VII shows amount of raw material transferred from each supplier to each plant in each period. Transportation cost from supplier to plants has not been taken into consideration in the price of unit raw material, thus it can be a criterion for the selection of suppliers and procurement planning.

TABLE VII . AMOUNT OF RAW MATERIAL TRANSFERRED FROM EACH SUPPLIER TO EACH PLANT

			Time period				
Supplier	Plant	Raw material	1	2	3	4	5
1	2	1			8497	5400	13907
1	2	3			16995	10800	27815
1	2	4	22096				13907
1	3	1	11406		52531	38454	44720
1	3	2		42340	1000		
1	3	3		35841	28027	35667	19965
1	3	4	20250	32100	45206		38719
2	2	1	24021	3000			
2	2	2	11010				13907
2	2	3	48042	6000			
2	2	4		9000		5400	
2	3	1	8844	56829			
2	3	2	10125		73555	25575	58829
2	3	3	40500	77818	53455		52765
2	3	4		54265	23694	38454	12611
3	2	2	1000	9000	25492	5400	
3	2	4	1925		25492		
3	3	1					33060
3	3	2		44025	22148	12879	
3	3	3			23580	41241	82831
3	3	4			27803		26450

Table VIII Shows quantity of manufactured products in each plant during the planning horizon. Products 1 and 2 will be produced in all periods in all plants. As it can be seen in this table there is a reduction in production at period 4 and it can be considered as main reason in decreasing net profit at this period has been illustrated in Fig.1. Trends of quantity of manufactured products during the planning horizon have been illustrated in Fig.2.

TABLE VIII. QUANTITY OF MANUFACTURED PRODUCT IN EACH PLANT

		Time period				
Finished product	Plant	1	2	3	4	5
1	2	12010	1500	4248	2700	6953
1	3	10125	28414	26265	19227	38890
2	2	1489	3000	8497	1800	4635
2	3	3375	28788	32234	12818	19609

Table IX shows quantity of transferred finished products from plants to each warehouse in each period. As it can be seen from this table, at the first period both plants send their product to hired public warehouse but in the following periods no products will be sent to this warehouse. According to the predetermined condition for hiring public warehouse that it cannot be hired for less than 3 periods, it still remain opened at periods 2 and 3 but due to establishment of a private warehouse at the second period and with respect to its low variable storage cost, no products will be sent to public warehouse in periods 2 and 3.

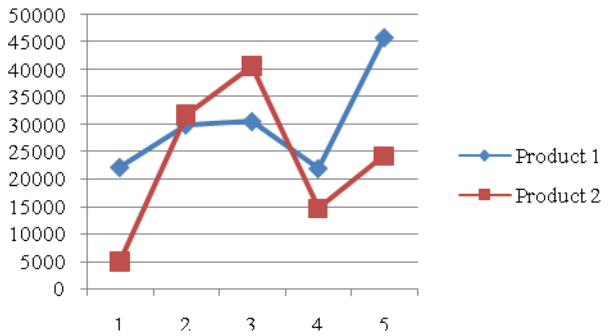

Fig.2. Trends of quantity of manufactured product in each period

TABLE IX. TRANSFERRED FINISHED PRODUCTS FROM PLANTS TO EACH WAREHOUSE

			Time period				
Plant	warehouse	Finished product	1	2	3	4	5
2	5	1		1500	4248	2700	6953
2	5	2		3000	8497	1800	4635
2	6	1	12010				
2	6	2	1489				
3	5	1		28414	26265	19227	38890
3	5	2		28788	32234	12818	19609
3	6	1	10125				
3	6	2	3375				

In this paper demands are consider as total needs in market, so it's not necessary to satisfy all the demands. The proposed model aims to make a planning to send products to markets in order to maximize total profit. Table X shows quantity of finished product transferred from each warehouse to each customer zone. With respect to these results customer zone 3 received products only at the first period.

Fig.3 illustrates four echelons of the current numerical example. It can be seen in this figure that all suppliers are active in all periods. In the case of plants, locations 2 and 3 have been selected for establishment of production units at the first period. Public warehouse 3 will be hired at the first period for duration of 3 periods; also location 2 has been selected for establishment of a private warehouse at the second period. Customer zone 3 received products only at the first period, customer zone 1 received products in all periods except first period and customer zone 2 received products in all periods. Also supply chain network in the first period of the current numerical example has been illustrated in Fig.4.

TABLE X. FINISHED PRODUCT TRANSFERRED FROM EACH WAREHOUSE TO EACH CUSTOMER ZONE

			Time period				
warehouse	Customer zone	Finished product	1	2	3	4	5
2	1	2		31788	40732	14618	24245
2	2	1		29914	30514	21927	45844
3	2	2	4864				
3	3	1	22135				

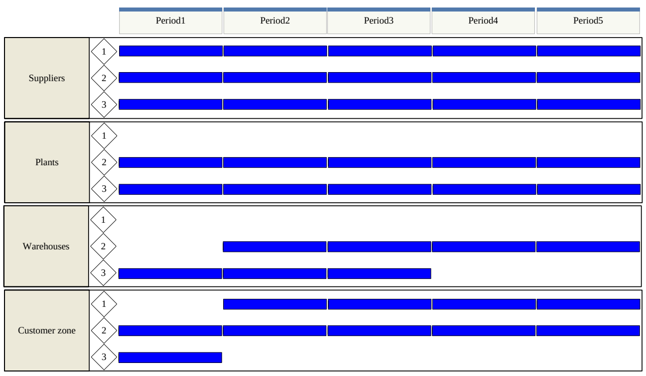

Fig.3. A schematic view on four echelons in the current numerical example

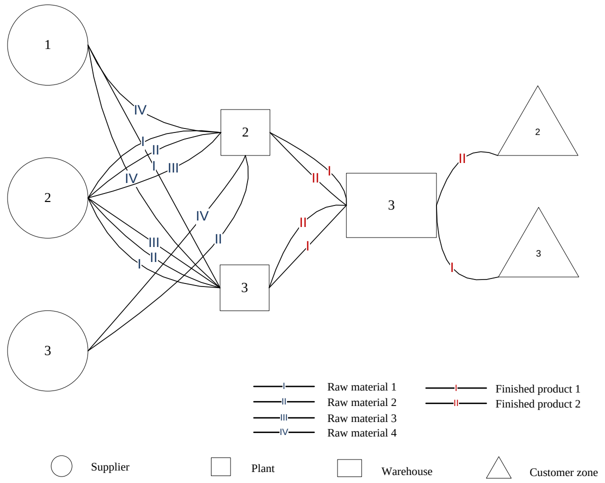

Fig.4. Supply chain network for the first period

IV. CONCLUSIONS

Supply chain master planning is one of the most important strategic decisions in the current competitive business environment. Since that top managers try to make the best decisions about their company to ensure long term survival. Good supply chain design contributes them to reduce costs, increase products quality, delivering products to customers timely and as a result increasing total profit. In this regard planning supply chain expansion for a long term horizon is one of the most important strategic decisions.

This paper proposes a model for supply chain design, production and distribution planning and expansion planning. A four echelon, multiple commodity supply chain has been considered in a dynamic time horizon with objective function of maximization of total profit. Echelons include suppliers, production units, distribution centers and customer zones. The propose model makes strategic decisions including plant and warehouse location, type of warehouses (public or private) and decisions on adding capacity to opened facilities. Some tactical decisions are made such as quantity of raw material to be transferred from each supplier to plants, quantity of each product to be manufactured in each plant, quantity of finished product to be transferred from each production unit to each established warehouse and quantity of product to be transferred from each warehouse to each customer zone. One of the most important features of the propose model is that expansion of the supply chain is being planned with respect to cumulative net profit after tax and stakeholder's share. Also some constraints have been considered to make the proposed model more realistic. A numerical example has been designed, solved and analyzed to illustrate application of the proposed model.

APPENDIX

TABLE XI. SOME INPUT PARAMETERS FOR NUMERICAL EXAMPLE

	Plants			Private warehouse	
	1	2	3	1	2
Initial capacity	15000	15000	15000	100000	100000
Maximal installable capacity	100000	100000	200000	200000	200000
Initial storage capacity	-	-	-	100000	100000
fixed cost for opening a facility	8000000	9000000	9000000	1700000	2000000
fixed cost for operating a facility i	200000	220000	270000	80000	70000
fixed cost for operating facility l for capacity option 1	30000	30000	30000	20000	20000
fixed cost for operating facility l for capacity option 2	70000	70000	70000	40000	40000

TABLE XII. AVAILABLE CAPACITY OF SUPPLIER S FOR RAW MATERIAL P AT PERIOD T

		Periods				
Supplier	Raw material	1	2	3	4	5
1	1	11406	0	67822	83698	58628
1	2	57224	42340	22261	56823	0
1	3	0	35841	45022	94196	47781
1	4	42346	32100	45206	0	52627
2	1	78443	74861	22245	85521	0
2	2	56832	0	73555	25575	72737
2	3	88542	83818	53455	0	52765
2	4	0	83531	23694	43854	12611
3	1	72912	52716	0	82368	62679
3	2	87344	53025	75842	92373	0
3	3	0	0	23580	41241	82831
3	4	80613	62371	53296	0	61238

TABLE XIII. DEMAND OF CUSTOMER C FOR PRODUCT P AT T

		Periods				
Customer	Product	1	2	3	4	5
1	1	119025	154733	201152	261498	339947
1	2	108685	141291	183678	238781	310415
2	1	121008	157310	204504	265855	345611
2	2	110186	143242	186214	242079	314702
3	1	125705	163417	212441	276174	359026
3	2	107993	140391	182508	237261	308439

TABLE XIV. QUANTITY OF RAW MATERIAL P' NECESSARY TO MANUFACTURE A UNIT OF FINAL PRODUCT P

Final product	Raw material			
	1	2	3	4
1	2	1	4	2
2	1	3	2	3

TABLE XV. WORKLOAD FOR THE TREATMENT OF A UNIT P AT FACILITY I

Final product	Facility				
	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	1

TABLE XVII. NUMBER OF DELIVERIES FROM PLANT I TO WAREHOUSE J IN ONE PERIOD

Plant	Warehouse		
	1	2	3
1	2	3	3
2	1	2	2
3	2	3	2

TABLE XVIII. SELLING PRICE OF ITEM OF UNIT P TO CUSTOMER C

Final product	Customer		
	1	2	3
1	2500	2500	2500
2	2300	2300	2300

TABLE XIX. PRICE OF RAW MATERIAL P FROM SUPPLIER S

Raw material	Supplier		
	1	2	3
1	23	27	29
2	17	14	14
3	19	15	18
4	10	12	13

TABLE XX. FIXED COST FOR ADDING CAPACITY OPTION O TO FACILITY I

Capacity option	Facility				
	1	2	3	4	5
1	800000	700000	700000	250000	300000
2	1500000	1350000	1200000	400000	540000

TABLE XXI. TREATMENT COST OF A UNIT P AT PLANT I

Final product	Plant		
	1	2	3
1	35	30	25
2	40	35	30

TABLE XXII. STORAGE COST OF A UNIT OF P AT WAREHOUSE J

Final product	Warehouse		
	1	2	3
1	8	7	45
2	6	5	35

TABLE XXIII. TRANSPORTATION COST OF A UNIT OF P FROM SUPPLIER S TO PLANT I

Raw material	supplier	Plant		
		1	2	3
1	1	6	4	5
1	2	7	3	6
1	3	9	9	8
2	1	4	8	8
2	2	3	5	4
2	3	8	5	9
3	1	6	4	5
3	2	7	3	6
3	3	9	9	5
4	1	4	4	8
4	2	3	5	4
4	3	8	3	6

TABLE XXIV. TRANSPORTATION COST OF A UNIT OF P FROM PLANT I TO WAREHOUSE J

Final product	Plant	Warehouse		
		1	2	3
1	1	5	7	6
1	2	6	8	8
1	3	8	4	5
2	1	5	8	4
2	2	8	4	8
2	3	3	6	5

TABLE XXV. TRANSPORTATION COST OF A UNIT OF P FROM WAREHOUSE J TO CUSTOMER ZONE C

Final product	Warehouse	Customer zone		
		1	2	3
1	1	6	4	5
1	2	7	3	6
1	3	9	9	8
2	1	4	8	8
2	2	3	5	4
2	3	8	5	9

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