

# Fuzzy Bayesian Condition Monitoring Model based on Exponential Distribution

K. Jenab and K. Rashidi

**Abstract**—Condition-based strategy refers to avoiding unnecessary maintenance and making timely actions through analyzing the received signals from monitoring devices. These signals sometimes may not be sensed, transmitted, or received precisely due to unexpected situations. Therefore, the fuzzy Bayesian model for condition monitoring of a system is proposed in this study. In order to apply the Bayesian concept, the fuzzy signals are assumed as fuzzy random variables with fuzzy prior distribution. Using the fuzzy signals, the newly developed model calculates the risk of operation for the system that results in determining the service time at minimum cost. A numerical example is also presented to demonstrate the application of the model.

**Index Terms**—Maintenance management, Condition-based maintenance, Fuzzy Bayesian decision theory

## I. INTRODUCTION

Manufacturing performance depends on safe, reliable and productive operations of assets. It is estimated that approximately 80% of the manufacturing expenditure is spent to correct chronic failures of machines, systems, and people. For example, each year the U.S. industry spends well over \$300 billion on plant maintenance and operation [1]. To reduce the maintenance cost, several maintenance strategies have been developed. These strategies can be categorized into two major classes: with or without Preventive Maintenance (PM). PM based strategies include 1) Age-dependent strategy refers to the replacement of the Line Replaceable Unit (LRU) in its upcoming operation age or upon failure, 2) Periodic PM strategy means that a LRU is preventively maintained at fixed time intervals independent of the failure history of the LRU, and repaired at intervening failures, 3) Failure limit strategy dictates PM performed only when the failure rate or other reliability measures of the LRU reach a predetermined level and intervening failures are corrected by repairs, 4) Sequential PM strategy means a LRU is preventively maintained at unequal time interval under the sequential PM strategy unlike the periodic PM strategy, and 5) Repair limit strategy refers to the case where the repair takes place if estimated repair cost is less than predetermined limit. Otherwise, the LRU is replaced. On the

other hand, there are strategies with reference time (i.e., without PM) that yield a lower maintenance costs than strategies with PM that include 1) Group and opportunistic maintenance strategies establishes groups of LRU's that should be replaced when a failure occurs. However, due to dependencies among LRUs, PM can be performed on non-failed LRU's as well, 2) Repair number counting and reference time strategy is a policy for costly operating LRU replacement with unknown operating time, and 3) Condition Based Maintenance (CBM) strategy refers to avoiding unnecessary maintenance and making timely actions through analyzing the received signals from monitoring devices. In conjunction with CBM, there exist several monitoring techniques such as vibration analysis, lubricant analysis, thermography, ultrasound, etc [2]. It is witnessed that the vibration analysis has been employed more than other approaches in the literature on CBM for the rotary equipment [2, 3, 4, 5]. Actually, the vibration is inevitable phenomenon in the rotary equipment that can be used as a healthy measure of the rotary equipment. When the equipment is working properly, the vibration produced by the equipment is small and constant. However, if a failure happens in the equipment, some dynamic processes in the system change. As a result, the produced vibration changes as well. In fact, if anomalies such as unbalance or wear in bearings develop in equipment, the global vibration level increases. Therefore, if something goes wrong in the rotary equipment, CMM is able to diagnose the problem. Many researches have been conducted in this field. Renwick (1984) proposed different steps of developing a condition monitoring program for the rotary equipment based on vibration signals, however, no specific processing method was proposed [4]. Kohda and Cui (2006) studied a general framework for the risk based configuration of a safety monitoring system using Dynamic Bayesian Network (DBN) [6]. However, calculations of the conditional state probabilities are complicated and difficult to obtain. Pedregal and Carnero (2006) set up a condition monitoring model for vibration data coming from a turbine driving a centrifugal compressor located at a petrochemical plant based on State Space framework [2]. Also, Orhan et al. (2006) used vibration monitoring and spectral analysis for diagnosis of the ball and cylindrical roller element bearing defects as a predictive maintenance tool [3]. Although three cases were investigated, the proposed method was not applicable for all rotary equipment. Chinnam and Baruah (2008) discussed an unsupervised algorithm for diagnostics and prognostics in machining processes [7].

This study aims at developing a fuzzy Bayesian condition

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monitoring model based on exponential distribution for the signals that sometimes may not be sensed, transmitted, or received precisely due to unexpected situations. It is assumed the fuzzy signals are fuzzy random variables with fuzzy prior distribution. Using the fuzzy signals, the dynamic risk of operation will be determined resulting in minimum cost and optimum service time. The remainder of the paper is organized as follows. The next section is devoted to the basic concept of fuzzy set. In sections II and III, we specify the notation and the assumptions concerning the fuzzy random variables. In sections IV and V, we discuss the fuzzy Bayesian monitoring model. Section VI presents an illustrative example. The results are discussed in section VII. Also, Concluding remarks are made in section VIII.

## II. THE BASIC CONCEPT OF FUZZY SET AND CONDITION MONITORING

Generally, in condition monitoring systems, the signals (i.e., mechanical signals, electrical signals, climatic signals), cannot be considered as precise inputs for making maintenance decision. For example, the vibration of the shaft in a rotary machine cannot be measured in an exact way because of fluctuation of the machine parameters and sensor failure due to unexpected situation. Therefore, the appropriate way to determine the vibration level is to say that the vibration level is around. Let  $X$  be a universal set and  $N$  be a subset of  $X$ . We can define a characteristic function  $c_N : X \longrightarrow \{0,1\}$  with respect to  $N$  by

$$c_N(v) = \begin{cases} 1 & \text{if } v \in N \\ 0 & \text{if } v \notin N \end{cases} \quad (1)$$

The concept of characteristic function can be extended to membership function  $m_{\tilde{N}} : X \longrightarrow [0,1]$  for fuzzy subset  $\tilde{N}$  of  $X$ . The value of  $m_{\tilde{N}}(v)$  can be interpreted as the membership degree of a signal in the set  $\tilde{N}$ . Let  $u$  be a signal received from sensor. The fuzzy number  $\tilde{u}$  corresponding to  $u$  can be interpreted as "around normal level". The graph of the membership function  $m_{\tilde{N}}(v)$  is bell-shaped and  $m_{\tilde{N}}(v)=1$  when  $u$  is close to normal level.

In condition monitoring, the signal should be classified to either normal or abnormal fuzzy class after measuring feature  $u$  (vibration). Therefore, two states of the classes can be defined:  $W_1$ =normal and  $W_2$ =abnormal. In order to classify the observed signals, a pattern recognition system is used [9], which is composed of four sections that are shown in Fig. 1. The input of the pattern recognition system is a signal and the output is the class.

It is assumed that the signal is classified to the normal class if  $u < T$  where  $T$  is the normal threshold for the signal, "<" is a fuzzy sense, and  $u_0$  is safe level of normal vibration. In this case, the exponential membership function can be expressed by

$$m_{\tilde{N}}(v) = \begin{cases} 1 & \text{if } v < v_0 \\ 1 - e^{-l(T-v)} & \text{if } v_0 < v < T \\ 0 & \text{if } v > T \end{cases} \quad \text{for } l > 0 \quad (2)$$

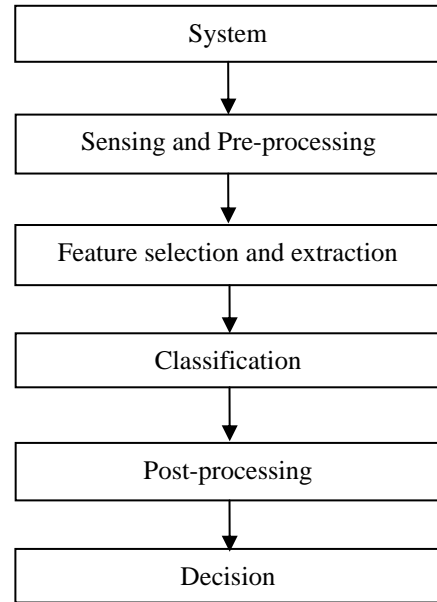


Fig. 1 Different stages of a pattern recognition system

Using (1), Fig. 2 shows the membership function in Table I for vibration signals received from a rotary shaft assuming the normal vibration threshold, lambda, and  $u_0$  values are 400, 0.001, and 340, respectively.

**Table I** Membership function of vibration signal for a rotary shaft

$u$	$m_{\tilde{N}}(v)$	Normalized $m_{\tilde{N}}(v)$
335	1	1
340	0.058235466	1
350	0.048770575	0.84
355	0.044002518	0.76
360	0.039210561	0.67
365	0.034394584	0.59
370	0.029554466	0.51
375	0.024690088	0.42
380	0.019801327	0.34
385	0.01488806	0.26
390	0.009950166	0.17
395	0.004987521	0.09
399	0.0009995	0.02
400	0	0



Fig. 2 Exponential membership function for vibration signals of a rotary shaft

If  $F(v)$  be the signal distribution function of  $\mathbf{u}$  and  $m_{\tilde{N}}(v)$  be the equipment condition status ( $\mathbf{w}_i$ : Normal, Abnormal) membership function, the fuzzy probability of the normal class is defined.

$$\tilde{P}(v | \mathbf{w}_1) = \tilde{P}(v < T) = \int_0^{\infty} m_{\tilde{N}}(v) . dF(v) \quad (3)$$

Similarly, for fuzzy signals  $\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n$  with corresponding membership  $m_{\tilde{N}}(v_1), m_{\tilde{N}}(v_2), \dots, m_{\tilde{N}}(v_n)$ , the fuzzy combined membership function can be defined in two ways as follows:

$$m_{\tilde{N}}(\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n) = \min\{m_{\tilde{N}}(v_1), m_{\tilde{N}}(v_2), \dots, m_{\tilde{N}}(v_n)\} \quad (4)$$

$$m_{\tilde{N}}(\tilde{u}_1, \tilde{u}_2, \dots, \tilde{u}_n) = \prod_{i=1}^n m_{\tilde{N}_i}(v_i) \quad (5)$$

Although there are different rules to combine membership functions [8], (4) and (5) present rules that are suitable for classifying features and making decisions. In this study, we are going to introduce the notation of fuzzy random variables for the monitoring signal (i.e., vibration) and also consider the fuzzy parameters.

### III. FUZZY RANDOM VARIABLES FOR MONITORING SIGNALS

Let  $\tilde{\mathbf{u}}$ , signal, be a real number and  $\tilde{N}$  be a fuzzy subset of  $\mathbf{R}$ . We denote normal class ( $\mathbf{w}_1$ ) by  $\tilde{N}_a(v) = \{v : m_{\tilde{N}}(v) \leq a\}$  the  $a$ -level set of  $\tilde{N}$  for  $a \in (0,1]$ .  $\tilde{N}$  is called a normal fuzzy set if there exists an  $\tilde{\mathbf{u}}$  such that  $m_{\tilde{N}}(v) = 1$ . Also, it is convex fuzzy set if  $m_{\tilde{N}}(kv_1 + (1-k)v_2) \geq \min\{m_{\tilde{N}}(v_1), m_{\tilde{N}}(v_2)\}$  for  $k \in [0,1]$

### IV. CONDITION MONITORING MECHANISM

By installing sensors, the vibration signals can be transferred from the equipment to the condition monitoring system, which processes and interpreters the signals. The condition monitoring system classifies the received signals to either 'Normal' or 'Abnormal' classes used for making maintenance decision as shown in Fig. 3.

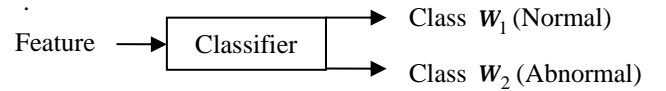


Fig. 3 Signal classifier

However, there exist some sensor errors and unexpected situations that result in not having precise information from equipment. As result, the fuzzy classifier that uses Bayesian Decision Theory is developed. Also, the membership function used in Bayesian Decision Theory is developed based on exponential distribution described in (2) for the equipment condition associated with exponentially distributed signals.

### V. FUZZY BAYESIAN CONDITION MONITORING MODEL

Bayesian Decision Theory is a fundamental approach for the pattern recognition problem. This approach proposed by Duha et al., (2001) aims at making decisions based upon probabilistic expressions and pre-defined class probabilities (prior probabilities) [9]. There are two strategies for prior probabilities in this study as follows:

- 1) Static Prior Probabilities (SPP) do not change over the time and are constant. They can be calculated based on the previous records of equipment.
- 2) Dynamic Prior Probabilities (DPP) change over the time based on equipment condition that may lead to

more precise classifications.

After determining the classes denoted by  $w_i$ , the prior probabilities,  $p(w_i)$ , can be estimated based on historic data. Consider a fuzzy random variable,  $\tilde{u}$ , with membership function,  $m_{\tilde{N}}(v)$  and the fuzzy subset  $\tilde{N}$  for Normal condition of the equipment. These prior probabilities can be used for computing the fuzzy posterior probability as follows:

$$\tilde{p}(w_i | v) = \frac{\tilde{p}(v | w_i) p(w_i)}{\tilde{p}(v)} \quad (6)$$

For normal condition class (i.e.,  $i=1$ )

$$\tilde{p}(w_1 | v) = \frac{\tilde{p}(v < T) p(w_1)}{\tilde{p}(v)} \quad (7)$$

Using (3), we get

$$\tilde{p}(w_1 | v) = \frac{p(w_1) \cdot \int_0^{\infty} m_{\tilde{N}}(v) \cdot dF(v)}{\sum_{i=1}^2 p(w_i) \cdot \int_0^{\infty} m_{\tilde{N}}(v) \cdot dF(v)} \quad (8)$$

Considering exponential membership function for the normal class and exponential vibration distribution, we have

$$\tilde{p}(w_1 | v) = \frac{p(w_1) \cdot \int_0^{\infty} m_{\tilde{N}}(v) \cdot I_n e^{-I_n v} dv}{p(w_1) \cdot \int_0^{\infty} m_{\tilde{N}}(v) \cdot I_n e^{-I_n v} dv + p(w_2) \cdot \int_0^{\infty} m_{\tilde{A}}(v) \cdot I_a e^{-I_a v} dv}$$

Similarly,  $\tilde{p}(w_2 | v)$  used for making decision can be calculated. The condition status of the equipment is *Normal* if  $\tilde{p}(w_1 | v) \geq \tilde{p}(w_2 | v)$ , otherwise it is considered as *Abnormal* condition. Knowing the condition of the equipment may help making timely actions and avoiding unnecessary maintenance.

However, this may not be the best decision because the classification errors may be associated with different cost. Thus, to reach to a better decision, a loss function,  $\Psi$ , is proposed to combine the posterior probability of each class with associated cost. Therefore, continue operation or stop operation option denoted by  $a_1$  and  $a_2$  for the equipment are incorporated in the loss function. As depicted in Table II, there is no cost associated with either continuing operation with the normal condition class,  $\Psi(w_1 | a_1) = 0$  or stop operation with the abnormal condition class,  $\Psi(w_2 | a_2) = 0$ . Furthermore, the loss function is enhanced by incorporating two types of fault known as failed-dangerous (FD) and failed-safe (FS) [10]. In failed-dangerous fault case, the condition monitoring system does not warn that the equipment is working under an abnormal condition ("Real Abnormal, Predicted Normal"). However, in failed-safe fault case, the condition monitoring system raises an alarm for abnormal condition while the equipment is working properly ("Real Normal, Predicted Abnormal"). Thus, the cost associated with FD faults is more than that of FS faults because an FD leads to serious accidents while an FS causes unnecessary maintenance. Table II shows an example of a

loss table in which cost of an FD fault is ten times more than the cost of an FS.

**Table II** Loss Table

Decision	Condition	
	Normal ( $w_1$ )	Abnormal ( $w_2$ )
Continue ( $a_1$ )	\$ 0	\$ 15000
Stop ( $a_2$ )	\$ 1500	\$ 0

Therefore, the expected loss is defined as follows:

$$L(a_j | v) = \sum_{i=1}^2 \Psi(a_j | w_i) p(w_i) = 0 \quad \forall j$$

Where  $\{a_1, a_2\} = \{\text{Continue, Stop}\}$  and  $\{w_1, w_2\} = \{\text{Normal, Abnormal}\}$

Thus, we have:

$$L(\text{Continue} | v) = \Psi(\text{Continue} | \text{Normal}) p(\text{Normal} | v) + \Psi(\text{Continue} | \text{Abnormal}) p(\text{Abnormal} | v) \quad (11)$$

And

$$L(\text{Stop} | v) = \Psi(\text{Stop} | \text{Normal}) p(\text{Normal} | v) + \Psi(\text{Stop} | \text{Abnormal}) p(\text{Abnormal} | v) \quad (12)$$

Because  $\Psi(w_1 | a_1) = 0$  and  $\Psi(w_2 | a_2) = 0$ , (11) and (12) can be simplified as follows:

$$L(\text{Continue} | v) = \Psi(\text{Continue} | \text{Abnormal}) p(\text{Abnormal} | v) \quad (13)$$

$$L(\text{Stop} | v) = \Psi(\text{Stop} | \text{Normal}) p(\text{Normal} | v) \quad (14)$$

Having the expected values of loss function for all cases, we are able to refine the decision rule in more efficient way that minimizes the conditional risk:

Decide  $w_1$  if  $\Psi(w_1 | v) < \Psi(w_2 | v)$

Decide  $w_2$  if  $\Psi(w_2 | v) < \Psi(w_1 | v)$

Based on the output of the classifier, we can classify each frequency to either normal or abnormal class. However, we define two more criteria to enhance the decision procedure. These criteria are:

- 4) The number of abnormal frequencies in the last 100 ones. This value can be interpreted as the percentage of abnormal frequencies in a period of time.
- 5) The number of consecutive abnormal frequencies. For example, five or more consecutive abnormal vibration frequencies can be considered dangerous.

## VI. ILLUSTRATIVE EXAMPLE

Using the fuzzy Bayesian CMM, our objective is to determine whether rotating equipment is in normal or abnormal condition based on its vibration signals. A program coded in Visual Basic is developed that uses the text files including normal and abnormal frequencies. These text files include 250 normal and 250 abnormal frequencies. The software determines the membership function of frequencies for each class (normal/abnormal) by using (2). Assuming the

normal vibration threshold, lambda, and  $u_0$  values are 400, 0.001, and 340, Fig. 4 and Fig. 5, depict the membership function of frequencies of the classes

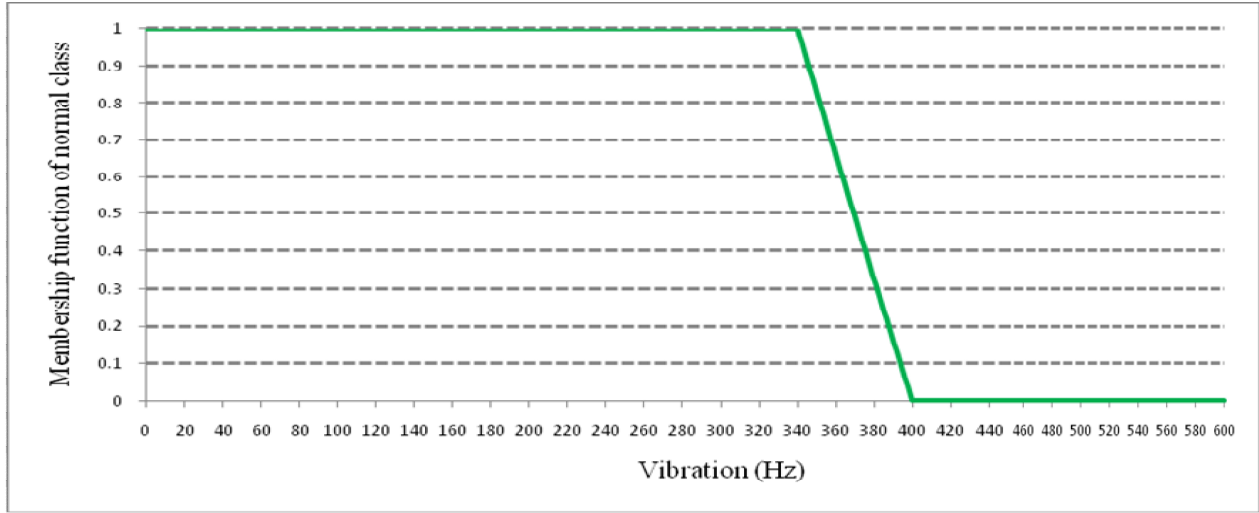


Fig. 4 Membership function of the normal class

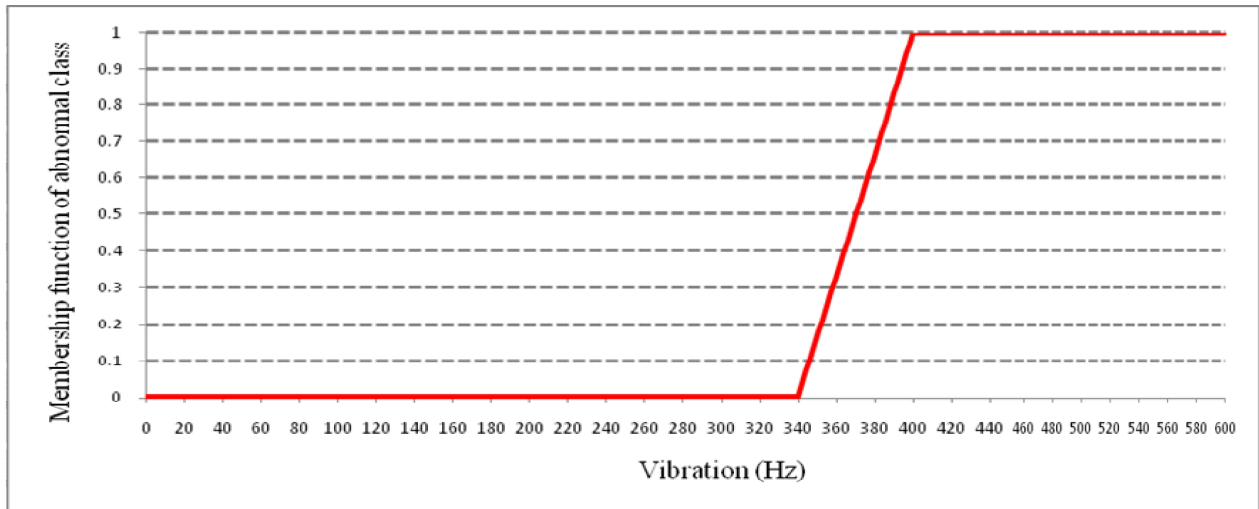


Fig. 5 Membership function of the abnormal class

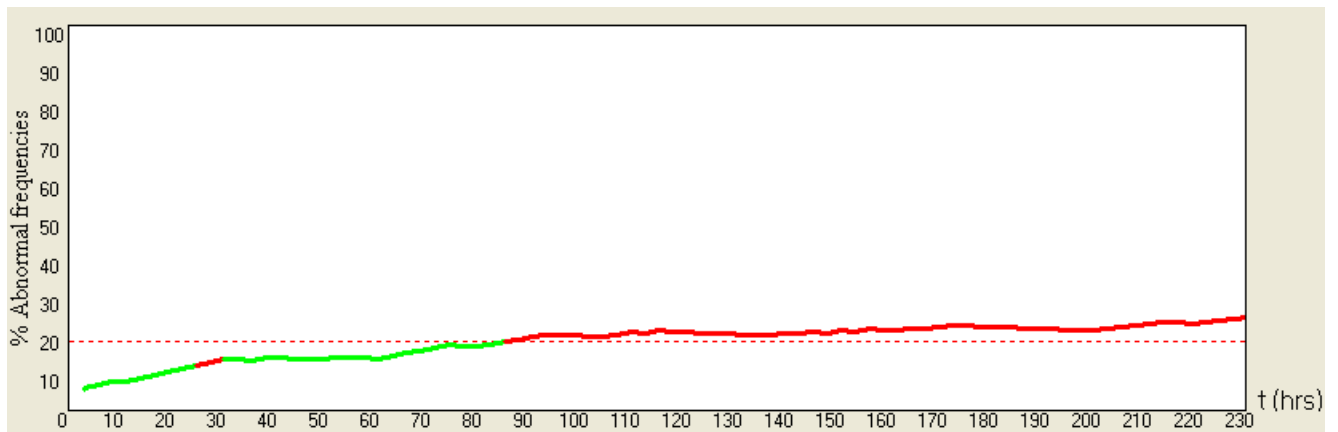


Fig. 6 Percentage of abnormal frequencies in the last 100 frequencies

There are two text files which restore normal and abnormal frequencies. The program first reads these data and calculates the parameters of both distributions (i.e.,  $I_n, I_a$ ).

$$P(v|\text{Normal}) = \mu_N(v) \lambda_N e^{-\lambda_N v} \quad (15)$$

$$P(x|\text{Abnormal}) = \mu_A(x) \lambda_A e^{-\lambda_A x} \quad (16)$$

Consider constant prior probabilities (i.e.,  $P(\text{Normal})=0.9$ )

and  $P(\text{Abnormal})=0.1$ ) and the loss of letting the equipment continue working when it is in an abnormal condition is 10 times more than the loss of stopping the equipment while working properly (i.e.,  $\psi(\text{Continue}|\text{Abnormal}) = 10 * \psi(\text{Stop}|\text{Normal})$ ) as shown in Table III. Therefore, using (15) and (16), the conditional risk loss values for both decisions (i.e., Continue, Stop) can be calculated based on a received signal from the equipment.

**Table III** Loss Table

	Normal	Abnormal
Continue	\$ 0	\$ 15000
Stop	\$ 1500	\$ 0

$$L(\text{Continue}|x) = 15000 * \frac{\mu_N(v) \lambda_N e^{-\lambda_N v}}{\mu_N(v) \lambda_N e^{-\lambda_N v} + \mu_A(v) \lambda_A e^{-\lambda_A v}} \quad (17)$$

$$L(\text{Stop}|x) = 1500 * \frac{\mu_A(v) \lambda_A e^{-\lambda_A v}}{\mu_N(v) \lambda_N e^{-\lambda_N v} + \mu_A(v) \lambda_A e^{-\lambda_A v}} \quad (18)$$

After reading each vibration frequency,  $v$ , from the sensor installed on the equipment, it will be classified to the class with minimum risk as follows.

If  $L(\text{Continue}|v) < L(\text{Stop}|v)$ ,  $v$  is classified as “Normal” (19)

If  $L(\text{Stop}|v) < L(\text{Continue}|v)$ ,  $v$  is classified as “Abnormal” (20)

By classifying each frequency, decisions regarding maintenance can be made based on two criteria as follows:

1) The number of abnormal frequencies in the last 100 received signals: We assume that if this value exceeds 20 abnormal frequencies, the equipment is in an abnormal condition and should be stopped for maintenance. As depicted in Fig. 6, the equipment is working under abnormal condition after  $t=88$  hours of operation. Furthermore, there is a possibility that the equipment works under abnormal condition while the number of abnormal frequencies does not exceed 20 as shown in Fig. 6 at  $t=26$  hours.

2) The number of consecutive abnormal frequencies: If this number is equal to or more than five, the condition of the equipment is considered unsafe. Therefore, it has to be stopped for maintenance. For example, by using (19) and (20), we observe five consecutive frequencies have been classified as abnormal signals even though four frequencies are below predefined threshold at  $t=26$  hour. Fig. 7 shows the output of the program for this criterion.

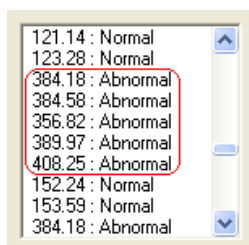


Fig. 7 Consecutive abnormal frequencies

## VII. RESULTS AND DISCUSSION

Applying the decision policies, the program finds the time of maintenance at minimum cost. The threshold policy may help detecting the problem in a certain time interval. However, the consecutive policy can detect the assignable causes regardless of the overall equipment condition. As shown in Fig. 6, the number of abnormal frequencies in the last 100 frequencies is less than 20 until  $t=88$  hour. Therefore, we may conclude that the equipment is working properly. However, there exist a point at  $t=26$  hour that the equipment works under abnormal condition resulting in damage to the equipment in a long run. This case resulting from assignable causes can be detected by the consecutive policy. In both case, the equipment should be scheduled for maintenance.

## VIII. CONCLUSION

Traditional methods of maintenance such as Corrective Maintenance (CM) and Preventive Maintenance (PM) have some disadvantages. CM leads to cost while the failed equipment is down and PM results in unnecessary and costly periodic maintenance. Also, PM may cause extra wear out to the equipment because of untimely action. In order to prevent additional maintenance costs and also maintaining the equipment effectively, CBM needs an efficient CMM. In this study, the fuzzy intelligent CMM model has been developed which uses vibration signals of equipment in order to find out whether the equipment is in normal or abnormal condition. Decision criteria have been defined to determine the maintenance time based on the output of the model coded in Visual Basic. This program helps efficiently avoid unnecessary maintenance and make timely actions through analyzing the received vibration signals from the equipment based on Bayesian Decision Theory and fuzzy logic. The application of Bayesian and fuzzy logic results in managing the signals that sometimes may not be sensed, transmitted, or received precisely due to unexpected situations. For future work, one can develop a fuzzy Bayesian Condition Monitoring Model based on other distribution (e.g., Weibull, Reighley) and perform comparative analysis of the results.

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