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Finding a Shortest Path Using an Intelligent Technique

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Abstract— **This paper, presents a method to find the shortest distance path between two vertices on a *fuzzy weighted graph*, that is vertices (or nodes) and edges (or links) remain crisp, but the edge weights will be fuzzy numbers.. We propose an algorithm to deal with fuzzy shortest path problem. The algorithm first finds the shortest path length and then a similarity measure degree is taken to find out the shortest paths.**

***Index Terms*— Fuzzy sets, Shortest path problem, Similarity measure, Weighted graph.**

I. INTRODUCTION

Shortest path problems are very helpful in road network applications namely, transportation, communication routing and scheduling. We consider a directed network consisting of a finite set of vertices and a finite set of directed edges. It is assumed that there is only one directed edge between any two vertices. Now in any network path the arc length may represent time or cost .Therefore in real world, it can be considered to be a fuzzy set.

Fuzzy sets theory, proposed by Zadeh, is frequently utilized to deal with the uncertainty problem.

II. RELATED WORK

The fuzzy shortest path problem was first analyzed by Dubois and Prade. He used Floyd's algorithm and Ford's algorithm to treat the fuzzy shortest path problem. Although in their method the shortest path length can be obtained, maybe the corresponding path in the network doesn't exist. Klein [5] proposed a dynamical programming recursion-based fuzzy algorithm. Lin and Chen [6] found the fuzzy shortest path length in a network by means of a fuzzy linear programming approach. Chuang and kung [7], proposed fuzzy shortest path length procedure that can find fuzzy shortest path length among all possible paths in a network. It is based on the idea that a crisp number is the minimum if and only if any other number is larger than or equal to it.

III. PRELIMINARY DEFINITIONS

A. Fuzzy Set Theory

A fuzzy set $\tilde{a}$ in a universe of discourse X is characterized by a membership function $m_{\tilde{a}}(x)$ which associates with each element x in X, a real number in the interval [0, 1]. The function value $m_{\tilde{a}}(x)$ is termed the grade of membership of x in $\tilde{a}$.

B. Fuzzy number and its arithmetic

A fuzzy number is a quantity whose value is imprecise, rather than exact as is the case with "ordinary" (single-valued) numbers. Any fuzzy number can be thought of as a function whose domain is a specified set usually the set of real numbers, and whose range is the span of non-negative real numbers between, and including, 0 and 1. Each numerical value in the domain is assigned a specific "grade of membership" where 0 represents the smallest possible grade, and 1 is the largest possible grade.

In this article trapezoidal fuzzy numbers are used. In general, a trapezoidal membership function is described by a Quadruple A (*a*, *b*, *c*, *d*) as shown in fig 1:

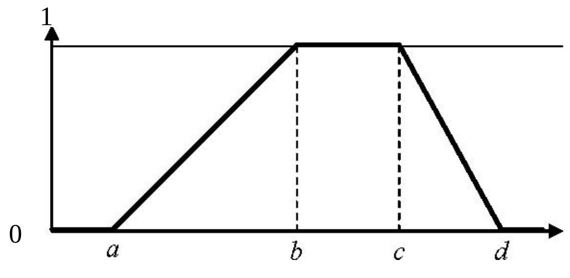

Fig.1

Where c and d are respectively the lower and the upper bounds of the fuzzy number, and [a, b] is the core. A trapezoidal fuzzy number A(a1, a2, a3, a4) is defined by the following membership function:

$$A(x)=\begin{cases}1-\dfrac{a_1-x}{a_3} & if \quad a_1-a_3\le x<a_1,\\ 1 & if \quad a_1\le x\le a_2,\\ 1-\dfrac{x-a_2}{a_4} & if \quad a_2<x\le a_2+a_4,\\ 0 & otherwise\end{cases}$$

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Let $A=(a_1,a_2,a_3,a_4)$ and $B=(b_1,b_2,b_3,b_4)$ be two trapezoidal fuzzy numbers written in quadruple form, then the fuzzy sum of these two is

$$A+B=(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4). \tag{1}$$

And in the same way other operations can be defined.

C. Fuzzy Weighted Graph

A fuzzy weighted graph $G=(V, E, c)$ consists of a set V of vertices or nodes v*i* and a binary relation E of edges $ek=(vi, vj) \in V*V$; we denote *tail*(*ek*)= v*i* and *head*(*ek*)= v*j* . v*i* is sometimes called a parent of v *j*, whereas v*j* is a child of v*i* . With each edge (v*i*, v*j*), a weight or cost *ci*, *j*= *c(*v*i*, v*j*)=(*c(*v*i*, v*j*)*1*, ……*c(* v*i*, v*j*) *r*) a vector of fuzzy numbers with $r>=1$, is associated.

Each fuzzy number can be seen as the evaluation of a given criterion.

IV. THE ALGORITHM

A path from a vertex *u* to another vertex *v in* a graph *G = (V, E), where V is* set of vertices and *E is* the set of edges, is a sequence of vertices $u, v_1, v_2, \ldots, v_n, v$ such that (u, v_1), $(v_1, v_2), \ldots, (v_n, v)$ are edges in *E*. The length of a path is the sum of the length of the edges on the path. A shortest path between two vertices *u* and *v* is the path whose length is minimum among all other paths between *u* and *v*. Now, if the length of each edge is an imprecise number, namely, trapezoidal fuzzy number then the length of the paths between two specified vertices will be an imprecise number of same kinds. In this case, the shortest path problem is addressed as fuzzy shortest path problem and since the problem involves comparison between the lengths of paths or their components, a suitable ranking method is required to compare the numbers. At this point, fuzzy shortest path problem is completely different from its crisp problem.

Various order relations for trapezoidal fuzzy numbers are available in the literature. One is that the fuzzy shortest path length (FSPL) formula on two path length by using half-inverse membership function as follows:

For two fuzzy path length $\tilde{L}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{L}_2=(a_2,b_2,c_2,d_2)$; $\tilde{L}_{\min}=(a,b,c,d)$

$$a=\min(a_1,a_2)$$

$$b=\begin{cases}\min(b_1,b_2) & if \quad \min(b_1,b_2)\le \max(a_1,a_2)\\ \dfrac{(b_1\times b_2)-(a_1\times a_2)}{(b_1+b_2)-(a_1+a_2)} & if \quad \min(b_1,b_2)>\max(a_1,a_2)\end{cases}$$

$$c=\min[\min(c_1,c_2),\max(b_1,b_2)]$$

$$d=\min[\min(d_1,d_2),\max(c_1,c_2)]$$

We introduce $\tilde{L}^{\min}$ as follows:

For two trapezoidal fuzzy numbers $\tilde{L}_1=(a_1,b_1,c_1,d_1)$ and $\tilde{L}_2=(a_2,b_2,c_2,d_2)$:

$$\tilde{L}^{\min}=\sup\{L_k \mid L_k=\min(L_1,L_2)\ ;\ k=1,2,\ldots,n\} \tag{2}$$

$$\tilde{L}^{\min}(a,b,c)=\text{Min}(\tilde{L}_1,\tilde{L}_2)=(\min(a_1,a_2),\min(b_1,b_2),\ \min(c_1,c_2)) \tag{3}$$

In many practical situations, we often encounter how to distinguish between two similar sets or groups. That is to say, we need to employ a measurement tool to measure similarity degree between them. Several similarity measures had been presented to evaluate the similarity degree between two fuzzy sets. We introduce a new method for finding similarity degree between two trapezoidal fuzzy numbers. In order to, we use the intersection area of two trapezoidal fuzzy sets to measure the similarity degree between L_i and fuzzy shortest path length. The larger the intersection area of two trapezoidal, the higher the similarity degree between them is.

Let the *ith* fuzzy path length $\tilde{L}_i=(a_i,b_i,c_i,d_i)$ and the fuzzy shortest path length $\tilde{L}^{\min}=(a,b,c,d)$ then the similarity degree S_i between L_i and L_{min} can be calculated as:

$$S(\tilde{L}_i,\tilde{L}^{\min})=\begin{cases}0 & ;\ \tilde{L}_i\cap\tilde{L}^{\min}=f\\ \dfrac{100(d-a_i)^2}{2(d_i-a_i)[(c-b)+(d-c)+(b_i-a_i)+(c_i-b_i)]} & ;\ \tilde{L}_i\cap\tilde{L}^{\min}\neq f\end{cases} \tag{4}$$

As mentioned previously, the similarity measure defined in (4) will help decision makers to decide which path is the shortest one. The proposed algorithm can be shown as follows:

Step 1. Form the possible paths from source vertex *s* to destination vertex *d* and compute the corresponding path lengths L_i, $i = 1, 2. . . m$, for possible *m* paths.

Step 2. Find the fuzzy shortest length $\tilde{L}^{\min}$ by using formula (5).

Step 3. Employ fuzzy similarity measure defined in (4) to yield the similarity degree $S(\tilde{L}_i,\tilde{L}^{\min})$ between $\tilde{L}^{\min}$ and $\tilde{L}_i$ for $i = 1, 2. . . m$.

Step 4. Obtain the shortest path with the highest $S(\tilde{L}_i,\tilde{L}^{\min})$.

V. IMPLEMENTATION

In this section, we will execute the proposed algorithm on the Fig 2. A classical weighted graph with trapezoidal fuzzy lengths is shown in Fig.2 It is our purpose to determine fuzzy shortest path length and shortest path from vertex 1 to 6 in this network.

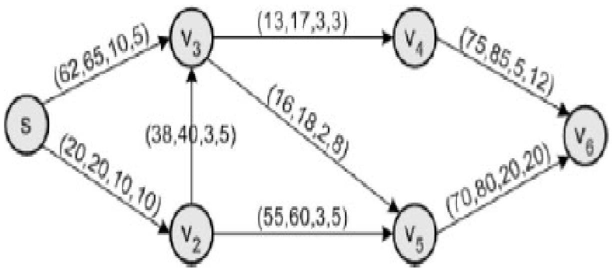

Fig.2

There are five paths as follows:

P_1: S- V_3 – V_4 - V_6 $\longrightarrow$ L_1 = (150, 167, 18, 20),
P_2: S- V_3 – V_5 - V_6 $\longrightarrow$ L_2 = (148, 163, 32, 33),
P_3: S- V_2 – V_5 - V_6 $\longrightarrow$ L_3 = (145, 160, 33, 35),
P_4: S- V_2 – V_3 - V_5- V_6 $\longrightarrow$ L_4 = (144, 158, 35, 43),
P_5: S- V_2 – V_3 - V_4 - V_6 $\longrightarrow$ L_5 = (146, 162, 21, 30).

We can obtain $\tilde{L}^{\min}$ = (144, 158, 21, 30) through the proposed algorithm. Now using (4), we can get the similarity degree S_i between $\tilde{L}^{\min}$ and $\tilde{L}_i$. Finally, we choose P_4 as the shortest path, since the corresponding L4 has the highest similarity degree (=26.91) to $\tilde{L}^{\min}$.

TABLE 1

Paths	$S(\tilde{L}_i, \tilde{L}^{\min})$	*Ranking*
P_1: S- V_3 – V_4 - V_6	20.36	4
P_2: S- V_3 – V_5 - V_6	17.14	5
P_3: S- V_2 – V_5 - V_6	24.49	2
P_4: S- V_2 – V_3 - V_5- V_6	26.91	1
P_5: S- V_2 – V_3 - V_4 - V_6	22.74	3

VI. CONCLUSION

In this paper we have developed an algorithm to find optimal paths in a fuzzy weighted graph with its edge lengths as trapezoidal fuzzy numbers. Fuzzy shortest path length and shortest path are the useful information for the decision makers. We have tried to accumulate most of the existing ideas on comparison of trapezoidal fuzzy numbers, and proposed a new approach to imprecise numbers. An illustrative example is included to demonstrate the proposed method.

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