International Journal of Engineering and Technology Vol. 1, No. 1, April, 2009
1793-8236

A New Technique for Load–Flow Analysis of Radial Distribution Networks

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***Abstract*—A simple method to solve the load-flow problem of radial distribution networks is reported in this paper. This algorithm easily computes the power flow through any branch exploiting the radial feature of the distribution networks. The proposed method also reduces the data preparation and can handle arbitrary node numbering scheme very easily. The proposed method is compared with the other methods proposed by Das *et al.* [15], Ghosh *et al*. [16] and Ranjan *et al.* [19] to demonstrate its effectiveness. The detailed load-flow results for three examples with different kinds of load-modelling along with conclusion are presented in this paper.**

***Index Terms*—Load-flow, Radial, Load-modelling, Feeder, Lateral**

I. INTRODUCTION

The load–flow study in a power system has paramount importance because it reveals the electrical performance and power–flows of the system operating under steady state and it provides the real and reactive power losses and voltages at different nodes of the system. An efficient load–flow study becomes very effective for optimal conductor selection during planning of the system and also for the stability analysis of the system.

Distribution networks are structurally weakly meshed but are operated with a radial in nature and it has simplicity in design and cost. On the other hand, the transmission system is loop in nature. Distribution networks have high R/X ratio whereas the transmission networks have high X/R ratio. The distribution networks are ill–conditioned in nature. The variables for the load–flow analysis of distribution systems are different from that of transmission systems.

The load–flow methods proposed by the researchers Tinney *et al.* [1] and Scott *et al.* [2] were unable to converge for the ill–conditioned networks. The methods proposed by Iwamoto *et al.* [3] andRajjic *et. al.* [4] were very time consuming and increases the complexity. The following methods were proposed for load–flow analysis of distribution systems.

The load–flow techniques developed by Kersting and Mendive [5] and Kersting [6] for solving radial distribution networks updated voltages and currents during the backward and forward sweeps with the help of ladder–network theory. Stevens *et al.* [7] showed that the method proposed by Kersting and Mendive [5] and Kersting [6] was the fastest but did not converge in five out of twelve cases studied.

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Shirmohammadi *et al.* [8] proposed a method for solving radial distribution networks applying the direct voltage application of Kirchhoff's laws and presented a branch–numbering scheme to enhance the numerical performance of the solution method. Their method needed exhaustive data preparation. They also extended their method for solving the weakly meshed distribution networks. Baran and Wu [9] presented the load–flow solution of radial distribution networks by iterative solution of three fundamental equations representing the real power, reactive power and voltage magnitude. Chiang *et al*. [10] showed the uniqueness of load–flow solution for radial distribution networks. Renato [11] proposed one method for obtaining load–flow solution of radial distribution networks computing the electrical equivalent for each node summing all the loads of the network fed through the node including losses and then starting from the source node, voltage of every receiving end node was computed. Chiang [12] had shown three different algorithms for solving radial distribution networks based on the method of Baran and Wu [9]. Goswami and Basu [13] presented an approximate method using sequential numbering scheme for solving radial and meshed distribution networks with a condition that any node in the network could not be the junction of more than three branches i.e., one incoming and two outgoing. Jasmon and Lee [14] presented a load–flow method for obtaining the load–flow solution of radial distribution networks using the three fundamental equations representing the real power, reactive power and voltage magnitude that had been proposed by Baran and Wu [9]. Das *et al.* [15] proposed a load–flow method using sequential numbering scheme. A number of coding is to be supplied when the lateral and sub laterals exist. For large system this increases the complexity of the computation. Ghosh and Das [16] developed a load–flow method for solving radial distribution networks based on the technique with nodes beyond branches using the voltage convergence and had shown the proof of convergence. They had also shown that the incorporation of charging admittances reduces losses and improves voltage profile. The main draw back of this method is that it stores the nodes beyond each branch for every branch. This method calculates the current for each branch by adding the load currents of the nodes beyond the respective branch. Aravindhababu *et al.* [17] developed a simple and efficient branch-to-node matrix-based power flow (BNPF) for radial distribution systems and this method is unsuitable for extension to optimal power flow for which the NR method seems more appropriate. In this method, presence of any sub laterals makes complicate the matrix formation. Mekhamer *et al*. [18] proposed a method for load–flow solution of radial distribution networks using the terminal conditions. Ranjan *et al*. [19] proposed a new load–flow technique using power convergence characteristic. They claimed that their algorithm can easily accommodate the composite load modelling if the composition of load is known. They also claimed that this algorithm has good convergence property for practical radial distribution networks. They have used the sum of real and reactive power load at each node and they have reduced the network into its equivalent network. Ghosh [20] proposed a method to solve the load–flow solution of radial distribution networks using the radial feature of the network and reduced the complexity of calculations.

The main aim of author in this paper is to present an efficient load–flow for the radial distribution networks exploiting the radial feature of the distribution network without any requirement of the exhaustive data preparation for branch number, sending–end node and receiving–end node. The numbering of feeder, laterals(s) and sub lateral(s) are done in ascending order. The proposed method can handle arbitrary numbering scheme also. Three examples (29–node, 33–node and 69–node radial distribution networks) with constant power (CP), constant current (CI), constant impedance (CZ), composite and exponential load modelling for each of these examples are selected to test the efficiency of the load–flow. The proposed method has been compared with the other existing methods proposed by Das *et al.* [15], Ghosh *et al.* [16] and Ranjan *et al.* [19].

II. Assumptions

The three-phase radial distribution networks are assumed to be balanced and hence represented by their single-line diagram. The effect of charging capacitances are also neglected.

III. Solution Methodology

A single-line diagram of a radial distribution network is shown in Fig. 1 and Table I shows the branch number, sending–end node and receiving–end node of Fig. 1.

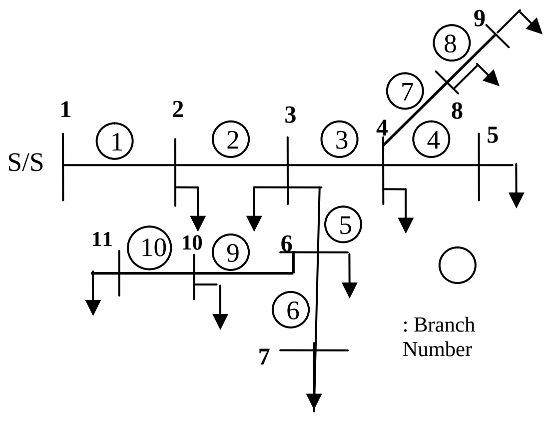

Fig. 1 Single-line diagram of a radial distribution network

TABLE I
BRANCH NUMBER (JJ), SENDING END NODE (M1 = IS(JJ)), RECEIVING END NODE (M2 = IR(JJ)) AND NODES BEYOND BRANCHES 1, 2, 3, …, 10 OF FIG. 1

Branch Number (jj)	Sending end m1 = IS(jj)	Receiving end m2 = IR (jj)
1	1	2
2	2	3
3	3	4
4	4	5
5	3	6
6	6	7
7	6	8
8	7	9
9	4	10
10	10	11

The proposed method needs only the first node of the feeder, lateral(s) and sub lateral(s) and their total number of nodes for sequential numbering. The proposed method does not need the branch numbers, sending–end nodes and receiving–end nodes. Hence the proposed method reduces the data preparation which the other all existing methods could not. The proposed logic puts the numbers of the remaining nodes of feeder, lateral(s) and sub lateral(s) automatically. For Fig. 1, the following data are required for sequential numbering scheme:

1(first node), 5(total number of node, N(1)) ¬ *FEEDER*
3(first node), 3(total number of node, N(2)) ¬ *LATERAL 1*
4(first node), 3(total number of node, N(3)) ¬ *LATERAL 2*
6(first node), 3(total number of node, N(4)) ¬ *SUB LATERAL*

Beside these, the proposed method needs the following:

Column I	*Column II*
2 (branch of feeder)	5 (first branch of lateral 1)
4 (branch of feeder)	7 (first branch of lateral 2)
5 (branch of lateral 1)	9 (first branch of sub lateral)

Resistance and reactance of each branch
Real and reactive power load of each node

If the node numbers are not sequential, they are stored in arrays as follows:

The node numbers of feeder, lateral(s) and sub lateral(s) are stored in the array with variable name NN. Store the nodes of feeder in array NN(jj) from jj =1 to N(1). Next store the nodes of lateral 1 in NN(i) from i = N(1) + 1 to N(1) + N(2). Next store the nodes of lateral 2 in NN(i) from i = N(1) + N(2) + 1 to N(1) + N(2) + N(3). Next store the nodes of sub lateral in NN(i) from i = N(1) + N(2) + N(3) + 1 to N(1) + N(2) + N(3) + N(4).

Only the node numbers are required when they are sequential. The proposed method does not need the details of branch numbers, sending–end nodes and receiving–end nodes which the other methods need.

To calculate the branch currents of each branch, the following logic is proposed:

The proposed method calculates branch currents of sub lateral (s), then lateral(s) and then feeder.

To calculate the voltage of each node let us consider Fig. 2 shown below.

Fig. 2 Sending–end node and receiving–end node of a branch

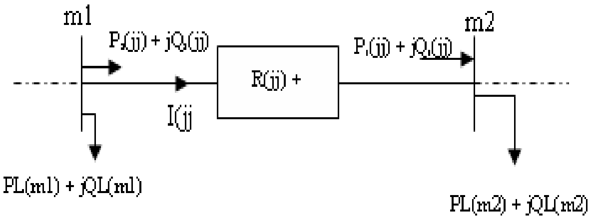

Fig. 2 Sending–end node and receiving–end node of a branch

The following notations were used in this paper work:

jj: Branch number

N(i): Total number of nodes of feeder, lateral(s) or sub lateral(s)

NB(i): Total number of branches of feeder, lateral(s) or sub lateral(s)

NN(i): Array for storing the nodes

PR(jj): Active power at the branch–jj entering the node NN(jj+1)

QR(jj): Reactive power at the branch–jj entering the node NN(jj+1)

PS(jj): Active power at the branch–jj coming out of the node NN(jj)

QS(jj): Reactive power at the branch–jj coming out of the node NN(jj)

I(jj): Current through the branch–jj

LP(jj): Real power loss of the branch–jj

LQ(jj): Reactive power loss of the branch–jj

PL(m2): Real power load at the node m2

QL(m2): Reactive power load at the node m2

V(m2): Complex value of the voltage at the node m2

Z(jj): Impedance of the branch–jj

Let $V(m1) = |V(m1)|\angle\delta_1$

$V(m2) = |V(m2)|\angle\delta_2$

$Z(jj) = |Z(jj)|\angle\theta$

From Fig. 2, we have

$$P_s(jj) + jQ_s(jj) = V(m1)I^*(jj) \qquad (1)$$

and

$$I(jj) = \frac{V(m1) - V(m2)}{Z(jj)} = \frac{|V(m1)|\angle\delta_1 - |V(m2)|\angle\delta_2}{|Z(jj)|\angle\theta} \qquad (2)$$

From Equation (1) and Equation (2), we have

$$\therefore P_s(jj) + jQ_s(jj)$$

$$= |V(m1)|\angle\delta_1 \left[\frac{|V(m1)|\angle-\delta_1 - |V(m2)|\angle-\delta_2}{|Z(jj)|\angle-\theta}\right]$$

$$= \frac{|V(m1)|^2\angle\theta - |V(m1)||V(m2)|\angle\theta + \delta_1 - \delta_2}{|Z(jj)|} \qquad (3)$$

From Equation (3), we have

$$P_s(jj) = \frac{|V(m1)|^2\cos\theta - |V(m1)||V(m2)|\cos(\theta + \delta_1 - \delta_2)}{|Z(jj)|}$$

(4)

$$\text{and } Q_s(jj) = \frac{|V(m1)|^2\sin\theta - |V(m1)||V(m2)|\sin(\theta + \delta_1 - \delta_2)}{|Z(jj)|}$$

(5)

Similarly,

$$\therefore P_r(jj) + jQ_r(jj) = V(m2)I^*(jj)$$

$$= |V(m2)|\angle\delta_2 \left[\frac{|V(m1)|\angle-\delta_1 - |V(m2)|\angle-\delta_2}{|Z(jj)|\angle-\theta}\right]$$

$$= \frac{|V(m1)||V(m2)|\angle\theta + \delta_2 - \delta_1 - |V(m2)|^2\angle\theta}{|Z(jj)|} \qquad (6)$$

From Equation (6), we have

$$P_r(jj) = \frac{|V(m1)||V(m2)|\cos(\theta + \delta_2 - \delta_1) - |V(m2)|^2\cos\theta}{|Z(jj)|}$$

(7)

and

$$Q_r(jj) = \frac{|V(m1)||V(m2)|\sin(\theta + \delta_2 - \delta_1) - |V(m2)|^2\sin\theta}{|Z(jj)|} \qquad (8)$$

Since δ_1 and δ_2 are very small, hence the difference $\delta_1 \sim \delta_2$ can be neglected.

From Equation (4) and Equation (7), we have

$$P_s(jj) + P_r(jj) = \left[\frac{|V(m1)|^2 - |V(m2)|^2}{|Z(jj)|}\right]\cos\theta \qquad (9)$$

From Equation (5) and Equation (8), we have

$$Q_s(jj) + Q_r(jj) = \left[\frac{|V(m1)|^2 - |V(m2)|^2}{|Z(jj)|}\right]\sin\theta \qquad (10)$$

From Equation (9) and Equation (10), we have

$$[P_s(jj) + P_r(jj)]^2 + [Q_s(jj) + Q_r(jj)]^2 = \left[\frac{|V(m1)|^2 - |V(m2)|^2}{|Z(jj)|}\right]^2$$

i.e., $|V(m1)|^2 - |V(m2)|^2$

$$= |Z(jj)|\sqrt{[P_s(jj) + P_r(jj)]^2 + [Q_s(jj) + Q_r(jj)]^2}$$

i.e., $|V(m2)|^2$

$$= |V(m1)|^2 - |Z(jj)|\sqrt{[P_s(jj) + P_r(jj)]^2 + [Q_s(jj) + Q_r(jj)]^2}$$

i.e., $|V(m2)|$

$$= \sqrt{|V(m1)|^2 - |Z(jj)|\sqrt{[P_s(jj) + P_r(jj)]^2 + [Q_s(jj) + Q_r(jj)]^2}}$$

(11)

where $P_s(jj) = P_r(jj) + |I(jj)|^2R(jj)$ (12)

and $Q_s(jj) = Q_r(jj) + |I(jj)|^2X(jj)$ (13)

The receiving–end power $P_r(jj)$ (denoted as PR below) of each node for feeder, lateral(s) and sub lateral(s) is computed at first with assumption that they are separated.

The branch number of feeder, lateral 1, lateral 2 and sub lateral are NB(1) = N(1) – 1, NB(2) = N(2) – 1, NB(3) = N(3) – 1, and NB(4) = N(4) – 1 respectively.

Total number of branches (NB) = NB(1) + NB(2) + NB(3) + NB(4)

Real and reactive power losses of each branch are

$$LP(jj) = |I(jj)|^2 R(jj) \qquad (14)$$

$$\text{and } LQ(jj) = |I(jj)|^2 X(jj) \qquad (15)$$

respectively.

For Sub lateral: jj = 9, 10 [loop jj = 10 to 9 i.e., NB to NB−NB(4)+1]

PR(10) = PL(NN(11))

PR(9) = PL((NN(10)) + LP(10) + PR(10)

For Lateral 2: jj = 7, 8 [loop jj = 8 to 7 i.e., NB−NB(4) to NB−NB(4) −NB(3)+1]

PR(8) = PL(NN(9))

PR(7) = PL((NN(8)) + LP(8) + PR(8)

For Lateral 1: jj = 5, 6 [loop jj = 6 to 5 i.e., NB−NB(4) −NB(3) to NB−NB(4) −NB(3)−NB(2) + 1]

PR(6) = PL(NN(7))

PR(5) = PL((NN(6)) + LP(6) + PR(6)

For Feeder: jj = 1, 2, 3, 4 [loop jj = 4 to 1 i.e., NB−NB(4) −NB(3)−NB(2) to NB−NB(4) −NB(3)−NB(2) −NB(1) +1]

PR(4) =PL(NN(5))

PR(3) = PL((NN(4)) + LP(4) + PR(4)

PR(2) = PL((NN(3)) + LP(3) + PR(3)

PR(1) = PL((NN(2)) + LP(2) + PR(2)

In general, we have

$$PR(jj)=PL(NN(jj + 1)) + LP(jj + 1) + PR(jj+1) \quad (16)$$

Similarly, for Q_r (QR) we have

$$QR(jj)=QL(NN(jj + 1)) + LQ(jj + 1) +PR(jj+1) \quad (17)$$

Next, the proposed method during computation of PR(jj), checks the branch number with the branch numbers 2, 4 and 5 i.e., branch number under column I. If it matches, then the proposed logic immediately adds the corresponding branch current under column II.

$$PR_{new}(5) = PR_{old}(5) + PR(9) \quad (18)$$

$$\text{and} \quad QR_{new}(5) = QR_{old}(5) + QR(9) \quad (19)$$

The sending−end line powers (denoted by PS for P_s) are expressed below.

$$PS(jj) = PR(jj) + LP(jj) \quad (20)$$

$$QS(jj) = QR(jj) + LQ(jj) \quad (21)$$

The branch current I(jj) is expressed by

$$\left|I(jj)\right| = \frac{\sqrt{P_r^2(jj) + Q_r^2(jj)}}{\left|V(m2)\right|} \quad (22)$$

The algorithm for computation of PR(jj) and QR(jj) and also PS(jj) and QS(jj) is shown below.

Step 1 : Read the number of feeder(A), lateral(s) (B) and sub lateral(s) (C).

Step 2 : TT = A + B + C

Step 3 : Read total number of nodes of feeder, each lateral and sub lateral respectively i.e., NN(i) for i = 1, 2, ….., TT.

Step 4 : Read the branch(es) connected to the common node of feeder and lateral or to the common node of lateral and sub lateral.

Step 5 : Compute the number of branches of feeder, lateral(s) or sub lateral(s) respectively.

Step 6 : Get the status of numbering scheme.

Step 7 : If it is sequential, ask for the starting node of feeder, each lateral and sub lateral respectively. Go to Step 9.

Step 8 : If it is not sequential, read the set of nodes as well as branches of feeder, each lateral and sub lateral respectively.

Step 9 : Calculate PR(jj) and QR(jj) for each feeder, lateral(s) and sub lateral(s) using Equations (16) and (17) respectively. Modify the values of PR(jj) and QR(jj) using Equations (18) and (19) respectively for the branches where applicable.

Step 10 : Calculate the values of PS(jj) and QS(jj) for branches of feeder, lateral(s) and sub lateral(s) using Equations (20) and (21) respectively.

IV. Load Modelling

A balanced load that can be represented either as constant power, constant current, constant impedance or as an exponential load is considered here. The general expression of load is shown below.

$$P(m2) = P_n\,[a_0 + a_1V(m2) + a_2V^2(m2) + a_3V^{e1}(m2)] \quad (23)$$

$$Q(m2)= Q_n[b_0 + b_1V(m2) + b_2V^2(m2) + b_3V^{e1}(m2)] \quad (24)$$

where, P_n and Q_n are nominal real and reactive power respectively and V(m2) is the voltage at node m2.

For all the loads, Equation (23) and Equation (24) are modeled as

$$a_0 + a_1 + a_2 + a_3 = 1.0 \quad (25)$$

$$b_0 + b_1 + b_2 + b_3 = 1.0 \quad (26)$$

For constant power (CP) load $a_0 = b_0= 1$ and $a_i = b_i = 0$ for i = 1, 2, 3. For constant current (CI) load $a_1 = b_1= 1$ and $a_i = b_i = 0$ for i = 0, 2, 3. For constant impedance (CZ) load $a_2 = b_2 = 1$ and $a_i = b_i = 0$ for i = 0, 1, 3. Composite load modelling is combination of CP, CI and CZ. For exponential load $a_3 = b_3 = 1$ and $a_i = b_i = 0$ for i = 0, 1, 2 and e1 and e2 are 1.38 and 3.22 respectively [22].

V. Algorithm For Computation Of Load−Flow

To calculate the node voltages and branch currents and the total system loss, an initial guess of zero real and reactive power loss is assumed. Also flat voltage start is used. The convergence criteria is such that if $\text{Max}|V_{old}[FN(i, j)] - V_{New}[FN(i, j)]| < \varepsilon$, for i = 1, 2, .., TN and j = 1, 2, …., N(i)=total number of nodes of FN(i). The following are the steps for load flow calculation:

Step 1 : Get the number of Feeder(A), lateral(s) (B) and sub lateral(s) (C).

Step 2 : TT = A + B + C

Step 3 : Read the total number of nodes N(i) of feeder, lateral(s) and sub lateral(s) for i = 1, 2, …, TT and also the starting node for sequential numbering scheme else read the nodes of feeder, lateral(s) and sub lateral(s) i.e., FN(i, j) for j =1, 2, …, N(i) and i = 1, 2, …., TT if these are not sequential.

Step 4 : Read real and reactive power load at each node i.e., PL[NN(i)] and

QL[NN(i)] for all nodes i.

Step 5 : Initialize PL[NN(1)] = 0.0 and QL[NN(1)] = 0.0.

Step 6 : Read the branches of feeder, lateral(s) and sub lateral(s) i.e., NB(jj) for all jj.

Step 7 : Read resistance and reactance of each branch i.e., R[NB(jj)] and X[NB(jj)] for all jj.

Step 8 : Read the incoming branch (feeder/lateral) and outgoing branch of the common node (lateral/sub lateral).

Step 9 : Read base kV and base MVA, Total number of iteration (ITMAX), ε (0.00001).

Step 10 : Compute the per unit values of PL[NN(i)] and QL[NN(i)] for all values of i as well as R[NB(jj)] and X[NB(jj)] for all j.

Step 11 : Set PL1[NN(i)] = PL[NN(i)] and QL1[NN(i)] = QL[NN(i)] for i.

Step 12 : Set LP[jj] = 0.0 and LQ[jj] = 0.0 for all jj.

Step 13 : Set V[NN(i)] = 1.0 + j0.0 for all i and set V1[NN(i)] = V[NN(i)] for all i

Step 14 : Use the **Step 9 and Step 10** (Art 3.0) to calculate the values of PR(jj), QR(jj) and PS(jj) and QS(jj) of each feeder, lateral(s) and sub lateral(s) respectively.

Step 15 : Set IT = 1

Step 16 : Set PL[NN(i)] = PL1[NN(i)] and QL[NN(i)] = QL1[NN(i)] for all i.

Step 17 : Use proper load–modeling using Equations (23) and (24).

Step 18 : Compute voltage |V[NN(i)]| using Equation (12) all i starting from NN(2).

Step 19 : Compute |ΔV[NN(i)]| = |V1[NN(i)]| – |V[NN(i)]| for all i.

Step 20 : Compute current |I[NB(jj)]| using Equation (22) for all jj.

Step 21 : Set |V1[NN(i)]| = |V[NN(i)]| for all i.

Step 22 : Compute LP[NB(jj)] and LQ[NB(jj)] for all jj using Equations (20) and (21) respectively.

Step 23 : Find ΔV_{max} from |ΔV[NN(i)]| for all i starting from NN(2).

Step 24 : If $\Delta V_{min} \leq 0.00001$ go to Step 27 else go to Step 25.

Step 25 : IT = IT + 1

Step 26 : If IT ≤ ITMAX go to Step 17 else write “NOT CONVERGED” and go to Step 28.

Step 27 : Write “CONVERGED” and display the results: Total Real and Reactive Power Losses, Voltages of each node, minimum value of voltage and its node number and total real and reactive power load for CP, CI, CZ, Composite and Exponential Load Modelling.

Step 28 : Stop

VI. Examples

The following three examples have been considered to demonstrate the effectiveness of the proposed method.

The first example is **29-node** radial distribution network shown in Fig. 3. Data for this system are available in [16]. Real and reactive power losses of this system are 872.74 kW and 348.08 kVAr, 331.18 kW and 133.62 kVAr, 216.30 kW and 88.26 kVAr, 372.05 kW and 149.94 kVAr, and 253.50 kW and 103.13 kVAr respectively for CP, CI, CZ, Composite and Exponential load modelling. The minimum voltage occurs at node number 18 in all cases. Base values for this system are **11 kV and 100 MVA** respectively. Table II shows the load low results for constant power, constant current, constant impedance, composite and exponential load modelling for sequential numbering (Case A) and non sequential numbering (Case B) of nodes shown in Fig. 4.

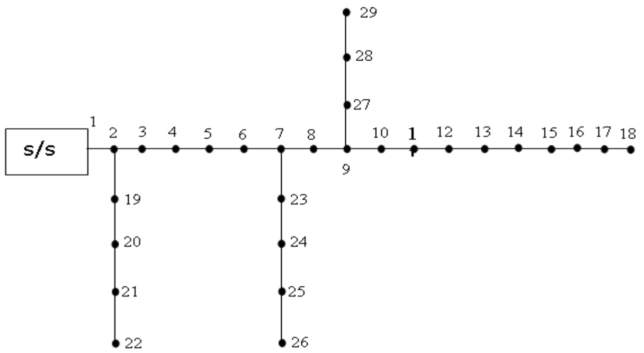

Fig. 3 29 Node Radial Distribution Network [16] for CASE A

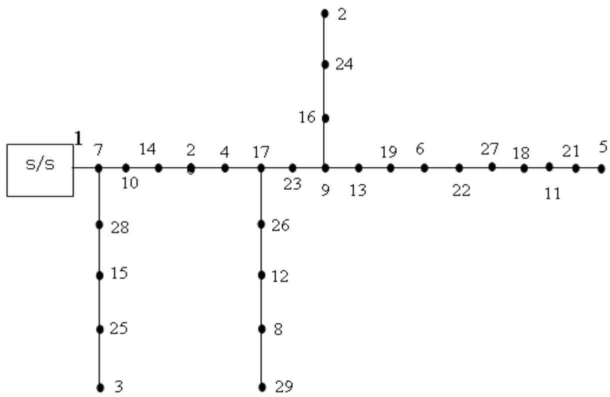

Fig. 4 29 Node Radial Distribution Network [16] for CASE B (renumbered)

TABLE II LOAD FLOW SOLUTION FOR FIRST EXAMPLE [16] FOR CASE A AND CASE B

Case A	Voltage Magnitude (p.u.) for					Case B
	CP	CC	CZ	CC	Exp.	
1	1.000000	1.000000	1.000000	1.000000	1.000000	1
2	0.948793	0.963928	0.968168	0.962432	0.966700	7

3	0.894578	0.927882	0.937326	0.924577	0.933955	10
4	0.865165	0.909196	0.921731	0.904820	0.917207	14
5	0.846357	0.897479	0.912087	0.892394	0.906767	20
6	0.776098	0.854810	0.877944	0.846981	0.869381	4
7	0.731592	0.829172	0.858261	0.819485	0.847462	17
8	709607	0.816795	0.849020	0.806170	0.837089	23
9	0.671772	0.796000	0.833948	0.783728	0.820040	9
10	0.625878	0.771350	0.816902	0.757062	0.800451	13
11	0.596485	0.755901	0.806732	0.740314	0.788600	19
12	0.583641	0.749275	0.802502	0.733117	0.783611	6
13	0.550515	0.732519	0.792202	0.714880	0.771362	22
14	0.523982	0.719353	0.784542	0.700529	0.762123	27
15	0.508507	0.711758	0.780314	0.692247	0.756960	18
16	0.498177	0.706734	0.777598	0.686765	0.753607	11
17	0.488407	0.702021	0.775109	0.681619	0.750491	21
18	0.485140	0.700449	0.774293	0.679902	0.749465	5
19	0.942704	0.958219	0.962745	0.956688	0.961218	28
20	0.937617	0.953460	0.958239	0.951900	0.956660	15
21	0.936317	0.952245	0.957090	0.950676	0.955496	25
22	0.935187	0.951189	0.956092	0.949614	0.954482	3
23	0.885954	0.920290	0.930462	0.916889	0.926878	26
24	0.881739	0.916587	0.927126	0.913137	0.923421	12
25	0.879756	0.914847	0.925562	0.911374	0.921806	8
26	0.765837	0.847034	0.871706	0.838963	0.862633	29
27	0.762085	0.844195	0.869445	0.836036	0.860178	16
28	0.760902	0.843301	0.868734	0.835113	0.859402	24
29	0.760411	0.842929	0.868440	0.834730	0.859076	2

The second example is **33-node** radial distribution network (nodes have been renumbered with Substation as node 1) shown in Fig.5. Data for this system are available in [21]. Real and reactive power losses of this system are 202.32 kW and 135.04 kVAr, 176.14 kW and 117.24 kVAr, 154.29 kW and 102.37 kVAr, 178.71 kW and 119.04 kVAr, and 155.07 kW and 102.90 kVAr respectively for CP, CI, CZ, Composite and Exponential load modelling. The minimum voltage occurs at node number 18 in all cases. Base values for this system are **12.66 kV and 100 MVA** respectively.

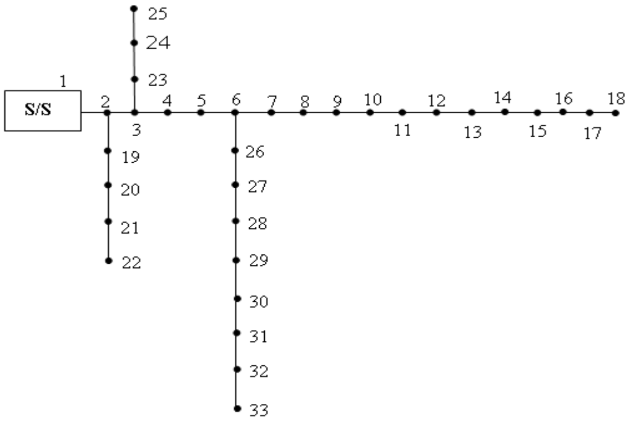

Fig. 5 33 Node Radial Distribution Network [20]

The third example is **69-node** radial distribution network (nodes have been renumbered with Substation as node 1) shown in Fig.6. Data for this system are available in [9]. Real and reactive power losses of this system are 225.09 kW and 102.13 kVAr, 191.16 kW and 87.61 kVAr, 163.63 kW and 75.84 kVAr, 194.51 kW and 89.02 kVAr, and 185.24 kW and 84.86 kVAr respectively for CP, CI, CZ, Composite and Exponential load modelling. The minimum voltage occurs at node number 65 in all cases. Base values for this system are **12.66 kV and 100 MVA** respectively.

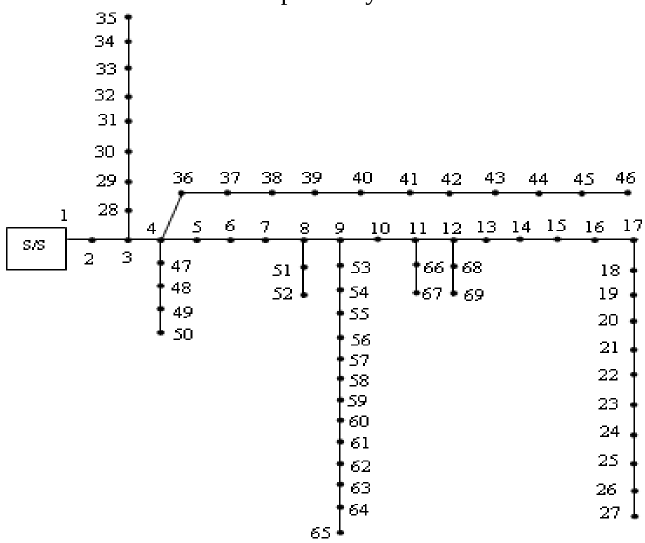

Fig. 6 69 node radial distribution network[9]

Table III shows the total real power loss, reactive power loss, minimum voltage and the corresponding node for constant power (CP), constant current (CI), constant impedance (CZ), composite load **(40% CP + 30% CI + 30% CZ)** and exponential load for 29–node, 33–node and 69–node radial distribution networks respectively.

TABLE III REAL POWER LOSS, REACTIVE POWER LOSS, MINIMUM VOLTAGE FOR CP, CI, CZ, COMPOSITE AND EXPONENTIAL LOAD MODELLING FOR 29–NODE, 33–NODE AND 69–NODE RESPECTIVELY

Example	Type of Load	Power Loss		Minimum Voltage **(p.u.)**
		Real **(kW)**	Reactive **(kVAr)**	
29–node radial	CP	872.74	348.08	V_{18} = 0.485140
	CI	331.18	133.62	V_{18} = 0.700449

distribution network **[16]**	CZ	216.30	88.26	V_{18} = 0.774293
	Composite	372.05	149.94	V_{18} = 0.679902
	Exponential	253.50	103.13	V_{18} = 0.749465
33–node radial distribution network **[21]**	CP	202.32	135.04	V_{18} = 0.909915
	CI	176.14	117.24	V_{18} = 0.916611
	CZ	154.29	102.37	V_{18} = 0.922623
	Composite	178.71	119.04	V_{18} = 0.915867
	Exponential	155.07	102.90	V_{18} = 0.921487
69–node radial distribution network **[9]**	CP	225.09	102.13	V_{65} = 0.906735
	CI	191.16	87.61	V_{65} = 0.914570
	CZ	163.63	75.84	V_{65} = 0.921480
	Composite	194.51	89.02	V_{65} = 0.913737
	Exponential	185.24	84.86	V_{65} = 0.919602

Table IV shows the comparison of relative CPU time of the proposed method with the methods of Das *et al.* [15], Ghosh *et al.* [16] and Ranjan *et al.* [19]. All simulation works have been done in **Celeron Processor 1GHz**.

TABLE IV COMPARISON OF RELATIVE CPU TIME OF THE PROPOSED METHOD WITH OTHER EXISTING METHODS [15, 16, 19] FOR CONSTANT POWER LOAD

Examples / Methods	Example 1	Example 2	Example 3
	Speed	Speed	Speed
Proposed method	1.00	1.00	1.00
D.Das *et al.* **[15]**	2.10	1.96	2.53
S.Ghosh and D.Das **[16]**	1.58	1.45	1.93
Ranjan and D.Das **[19]**	1.79	1.63	2.10

VII. CONCLUSION

A new method for load–flow analysis has been proposed in this paper for radial distribution networks that does not need the exhaustive line data preparation for branch number, sending–end node and receiving–end node. For sequential numbering scheme it needs the starting node of feeder, lateral(s) and sub lateral(s) only. Effectiveness of the proposed method has been tested by three examples (29–node, 33–node and 69–node radial distribution networks) with constant power load, constant current load, constant impedance load, composite load and exponential load for each of these examples where the voltage convergence has assured the satisfactory convergence in every case. The proposed method can handle arbitrary numbering scheme also. The superiority of the proposed method in terms of speed has been checked by comparing with the other methods proposed by Das *et al.* [15], Ghosh *et al.* [16] and Ranjan *et al.* [19] and the proposed method consumes less amount of memory compared to the above three other methods.

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